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ASPECTS OF COLOR SUPERCONDUCTIVITY*

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I discuss some aspects of recent developments in color superconductivity in high density quark matter. I calculate the Cooper pair gap and the critical points at high density, where magnetic gluons are not screened. The ground state of high density QCD with three light flavors is shown to be a color-flavor locking state, which can be mapped into the low-density hadronic phase. The meson mass at the CFL superconductor is also calculated. The CFL color superconductor is bosonized, where the Fermi sea is identified as a Q-matter and the gapped quarks as topological excitations, called superqualitons, of mesons. Finally, as an application of color supercoductivity, I discuss the neutrino interactions in the CFL color superconductor.

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1. Introduction

Matter exhibits several different phases, as shown in Fig. 1, depending on external parameters. At temperature, higher than the deconfinement temperature ($T > 100$ MeV), quarks confined in the nucleons get liberated and matter becomes a quark–gluon plasma, as happened in the very early universe. Similarly, also at extremely high density, where the Fermi momentum of nucleons in matter is larger than 1 GeV or so as in the core of compact stars like neutron stars, the wavefunction of quarks in nucleons will overlap with that of quarks in other nucleons due to asymptotic freedom. At such high density, quarks are no longer confined in nucleons and thus the nuclear matter will become a quark matter, where rather weakly interacting quarks move around [1].

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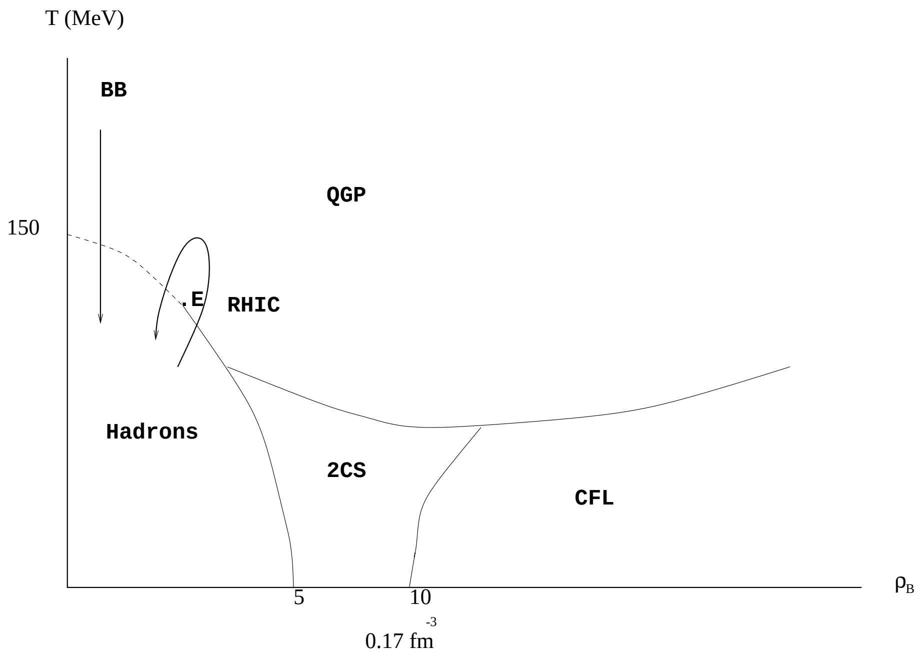

Fig. 1. A schematic phase diagram of matter.

The Fermi surface of quark matter at high density is unstable at low temperature, a phenomenon called Cooper instability, against forming pairs of quarks or holes, if attraction exists between a pair of quarks or holes with opposite momenta. No matter how small the attraction is, it will dominate any repulsive forces at low energy, since the attraction between a pair of quarks or holes with opposite momenta is a relevant operator while all repulsive forces become irrelevant as one scales down toward the Fermi sea [2, 3]. It turns out that color anti-triplet diquark condensates are energetically most preferred among possible pairings, including particle–hole parings or density waves [4].

Intense study on color superconductivity [7] shows that superconducting quark matter has two different phases, depending on density[1]. At intermediate density, the Cooper pair is color anti-triplet but flavor singlet, breaking only the color symmetry down to a subgroup, $\mathrm{SU}(3)_c \mapsto \mathrm{SU}(2)_c$:

$$\begin{aligned}\left\langle \psi^a_{\mathrm{L}i}(\vec{p})\psi^b_{\mathrm{L}j}(-\vec{p})\right\rangle &= -\left\langle \psi^a_{\mathrm{R}i}(\vec{p})\psi^b_{\mathrm{R}j}(-\vec{p})\right\rangle \\ &= \varepsilon_{ij}\varepsilon^{ab3}\Delta\,, \qquad (1)\end{aligned}$$

[1] New phases like the LOFF phase [5] or a chiral crystal phase [6] might exist at the intermediate density if one includes the Fermi surface mismatch due to the difference in quark mass.

where $i,j=1,2$ and $a,b=1,2,3$ are flavor and color indices, respectively. For high density where the chemical potential is larger than the strange quark mass, $\mu > m_s$, the strange quark participates in Cooper-pairing. At such a high density, the Cooper-pair condensate is predicted to take a so-called Color-Flavor Locking (CFL) form [8], breaking not only color symmetry but also flavor symmetry maximally:

$$\begin{aligned}\left\langle \psi_{\mathrm{L}i}^{a}(\vec{p})\psi_{\mathrm{L}j}^{b}(-\vec{p})\right\rangle &= -\left\langle \psi_{\mathrm{R}i}^{a}(\vec{p})\psi_{\mathrm{R}j}^{b}(-\vec{p})\right\rangle \\ &= k_1\delta_i^a\delta_j^b + k_2\delta_j^a\delta_i^b\,. \end{aligned} \tag{2}$$

At much higher density ($\mu \gg \Lambda_{\mathrm{QCD}}$), $k_1(\equiv \Delta_0) \simeq -k_2$ and the color–flavor locking phase is shown to be energetically preferred [9–11].

2. Cooper pair gap and the critical points

There are two kinds of the attractive forces that lead to Cooper instability, depending on the density. At the intermediate density, where $\mu < m_s$ or $\rho \sim (5\text{–}10)\times 0.17\ \mathrm{fm}^{-3}$, the QCD interaction is approximated with four-quark interactions,

$$\mathcal{L}_{\mathrm{QCD}}^{\mathrm{eff}} \ni \frac{g^2}{\Lambda^2}\bar{\psi}\psi\bar{\psi}\psi + \cdots, \tag{3}$$

since both electric and magnetic gluons are screened due to the medium effect. This short-range attraction is precisely the BCS type interaction, which is generated in metal by the exchange of massive phonons. The Cooper pair gap is then given by [18]

$$\Delta \simeq \varepsilon_{\mathrm{F}} \exp\left[-\frac{\Lambda^2}{g^2 p_{\mathrm{F}}^2}\right], \tag{4}$$

which is estimated to be $10 \sim 100$ MeV, for Λ and ε_{F} are of the order of Λ_{QCD} and g is of the order of one at the intermediate density. On the other hand, though electric gluons are screened in quark matter, the magnetic gluons are not screened at high density even at a nonperturbative level, as argued convincingly by Son [12]. Thus the Cooper-pairing force at high density is long-ranged and the gap equation is so-called the Eliashberg equation. The (long-range) magnetic gluon exchange interaction leads to an extra (infrared) logarithmic divergence in the gap equation, which is in hard-dense loop (HDL) approximation given as,

$$\Delta(p_0) = \frac{g_s^2}{36\pi^2}\int_{-\mu}^{\mu} dq_0 \frac{\Delta(q_0)}{\sqrt{q_0^2+\Delta^2}} \ln\left(\frac{\bar{\Lambda}}{|p_0-q_0|}\right), \tag{5}$$

where $\bar{\Lambda} = 4\mu/\pi \cdot (\mu/M)^5 e^{3/2\xi}$ with a gauge parameter ξ. By solving the gap equation (5), one finds the Cooper pair gap to be[2] [12–17]

$$\Delta_0 = 2^{9/2}\pi^4 N_f^{-5/2} e^{3\xi/2+1} \frac{\mu}{g_s^5} \exp\left(-\frac{3\pi^2}{\sqrt{2}g_s}\right) . \tag{6}$$

Though the ground state of quark matter is a color superconductor, one needs to know its criticality to observe color superconductivity in the laboratory or in stellar objects. The quark matter which might exist in the core of compact stars like neutron stars will be in the superconducting phase if the interior temperature of compact stars is lower than the critical temperature and the density is higher than the critical density. For the neutron stars, the inner temperature is estimated to be < 0.7 MeV and the core density is around 1.7 fm^{-3}, which is ten times higher than the normal nuclear matter density [23]. Since the critical temperature of BCS superconductivity is given as [18]

$$T_{\rm C} = \frac{1}{\pi} e^{\gamma} \Delta \simeq 0.57\Delta, \tag{7}$$

the critical temperature of color superconductivity at the intermediate density is quite large; $T_{\rm C} \sim$ 5-50 MeV. In dense QCD with unscreened magnetic gluons, the critical temperature turns out to take the BCS type value [10,21,22], $T_{\rm C} \simeq 0.57\Delta$, though the pairing force is very different from that of the BCS superconductivity. Since the unscreened magnetic gluons give a much bigger gap than the usual BCS type gap, the critical temperature of color superconductivity is quite large compared to the interior temperature of neutron stars, regardless of the form of attractive forces.

It is instructive to derive the critical temperature for the color superconductivity at high density, where the pairing is mediated by the unscreened magnetic gluons. We start with the zero temperature Cooper-pair gap equation, Eq. (5).

Following the imaginary-time formalism developed by Matsubara [19], the gap equation becomes at finite temperature T

$$\Delta(\omega_{n'}) = \frac{g_s^2}{9\pi} T \sum_{n=-\infty}^{+\infty} \int \frac{dq}{2\pi} \frac{\Delta(\omega_n)}{\omega_n^2 + \Delta^2(\omega_n) + q^2} \ln\left(\frac{\bar{\Lambda}}{|\omega_{n'} - \omega_n|}\right) , \tag{8}$$

where $\omega_n = \pi T(2n+1)$ and $q \equiv \vec{v}_{\rm F} \cdot \vec{q}$. We now use the constant (but temperature-dependent) gap approximation, $\Delta(\omega_n) \simeq \Delta(T)$ for all n. Tak-

[2] Were we to take the UV cutoff of the effective theory to be 2μ instead of μ, taken in [13], we would get the usual value, 2^8, instead of 2^2 for $N_f = 2$ in the prefactor.

ing $n' = 0$ and converting the logarithm into integration, we get

$$\Delta(T) = \frac{g_s^2}{18\pi} T \sum_{n=-\infty}^{+\infty} \int \frac{dq}{2\pi} \int_0^{\bar{\Lambda}^2} dx \frac{\Delta(T)}{\omega_n^2 + \Delta^2(T) + q^2} \frac{1}{x + (\omega_n - \omega_0)^2} \, . \quad (9)$$

Using the contour integral [20], one can in fact sum up over all n to get

$$1 = \frac{g_s^2}{36\pi^2} \int dq \int_0^{\bar{\Lambda}^2} \frac{dx}{2\pi i} \oint_c \frac{d\omega}{1+\mathrm{e}^{-\omega/T}} \frac{1}{[\omega^2 - q^2 - \Delta^2(T)]\,[(\omega - i\omega_0)^2 - x]} \, . \quad (10)$$

Since the gap vanishes at the critical temperature, $\Delta(T_\mathrm{C}) = 0$, we get, after performing the contour integration in Eq. (10),

$$\begin{aligned} 1 \; = \; & \frac{g_s^2}{36\pi^2} \int dq \int_0^{\bar{\Lambda}^2} dx \left\{ \frac{(\pi T_\mathrm{C})^2 + x - q^2}{\left[(\pi T_\mathrm{C})^2 + x - q^2\right]^2 + (2\pi T_\mathrm{C} q)^2} \frac{\tanh\left[q/(2T_\mathrm{C})\right]}{2q} \right. \\ & \left. + \frac{(\pi T_\mathrm{C})^2 + q^2 - x}{\left[(\pi T_\mathrm{C})^2 + q^2 - x\right]^2 + (2\pi T_\mathrm{C})^2 x} \frac{\coth\left[\sqrt{x}/(2T_\mathrm{C})\right]}{2\sqrt{x}} \right\} . \end{aligned} \quad (11)$$

At high density $\bar{\Lambda} \gg T_\mathrm{C}$, the second term in the integral in Eq. (11) is negligible, compared to the first term, and integrating over x, we get

$$\begin{aligned} 1 \; &= \; \frac{g_s^2}{36\pi^2} \int_0^{\lambda_\mathrm{c}} dy \frac{\tanh y}{y} \left[\ln\left(\frac{\lambda_\mathrm{c}^2}{(\pi/2)^2 + y^2} \right) + O\left(\frac{y^2}{\lambda_\mathrm{c}^2} \right) \right] \\ &= \; \frac{g_s^2}{36\pi^2} \left[\int_0^1 dy \frac{\tanh y}{y} \ln \lambda_\mathrm{c}^2 + \int_1^{\lambda_\mathrm{c}} dy \frac{\tanh y}{y} \ln \frac{\lambda_\mathrm{c}^2}{y^2} + \cdots \right] \\ &\simeq \; \frac{g_s^2}{36\pi^2} \left[\ln\left(\mathrm{e}^A \lambda_\mathrm{c} \right) \right]^2 , \end{aligned} \quad (12)$$

where we have introduced $y \equiv q/(2T_\mathrm{C})$ and $\lambda_\mathrm{c} \equiv \bar{\Lambda}/(2T_\mathrm{C})$ and A is given as

$$A = \int_0^1 dy \frac{\tanh y}{y} + \int_1^{\infty} dy \frac{\tanh y - 1}{y} = \ln\left(\frac{4}{\pi} \right) + \gamma \, , \quad (13)$$

where the Euler–Mascheroni constant $\gamma \simeq 0.577$. Therefore, we find the critical temperature

$$T_\mathrm{C} = \frac{\mathrm{e}^A}{2} \bar{\Lambda} \exp\left(-\frac{6\pi}{g_s} \right) \, . \quad (14)$$

Now, one can also solve the gap equation Eq. (5) in the same approximation used to find the critical temperature. Taking the gap independent of the energy, we get

$$\begin{aligned} 1 &\simeq \frac{g_s^2}{18\pi^2}\int_0^{\bar{\Lambda}} \frac{dq_0}{\sqrt{q_0^2+\Delta^2}}\ln\left(\frac{\bar{\Lambda}}{q_0}\right) \\ &= \frac{g_s^2}{18\pi^2}\int_0^{\bar{\lambda}} \frac{dx}{\sqrt{x^2+1}}\left(\ln\bar{\lambda}-\ln x\right) \\ &\simeq \frac{g_s^2}{36\pi^2}\left[\ln\left(2\bar{\lambda}\right)\right]^2 , \end{aligned} \tag{15}$$

where we have introduced $x = q_0/\Delta$, $\bar{\lambda} \equiv \bar{\Lambda}/\Delta$, and used the fact that the gap vanishes rapidly at energy higher than $\bar{\Lambda}$. In this constant gap approximation, the gap is given as

$$\Delta = 2\bar{\Lambda}\exp\left(-\frac{6\pi}{g_s}\right), \tag{16}$$

which is about 1.75 T_{C}. As comparison, we note in the BCS case, which has a contact four-Fermi interaction with strength $\bar{g}$, the critical temperature is given as

$$\begin{aligned} 1 &= \bar{g}\int_0^{\tilde{\omega}_{\mathrm{C}}} dz\frac{\tanh z}{z} \\ &\simeq \bar{g}\left[\int_1^{\tilde{\omega}_{\mathrm{C}}}\frac{dz}{z}+\int_0^1 dz\frac{\tanh z}{z}-\int_1^{\infty} dz\frac{1-\tanh z}{z}\right] \\ &= \bar{g}\ln\left(\mathrm{e}^{A}\tilde{\omega}_{\mathrm{C}}\right) , \end{aligned} \tag{17}$$

where $\tilde{\omega}_{\mathrm{C}}(\gg 1)$ is determined by the Debye energy, $\tilde{\omega}_{\mathrm{C}} = \omega_{\mathrm{D}}/(2T_{\mathrm{C}})$. Since the gap $\Delta = 2\omega_{\mathrm{D}}\exp(-1/\bar{g})$ in the BCS superconductivity, the ratio between the critical temperature and the Cooper-pair gap in both the color superconductivity at high density and the BCS superconductivity is given as $\mathrm{e}^{\gamma}/\pi \simeq 0.57$,

At high density, antiquarks are difficult to create due to the energy gap provided by the Fermi sea and thus it is energetically disfavored for antiquarks to participate in condensation. But, as the density becomes lower, one has to take into account the effect of antiquarks. In the high density effective theory, this effect is incorporated in the higher order operators [10]. First, we add the leading $1/\mu$ corrections to the gap equation Eq. (5) to see how the formation of Cooper pair changes when the density decreases. The leading $1/\mu$ corrections to the quark–gluon interactions are

$$\begin{aligned}\mathcal{L}_1 &= -\frac{1}{2\mu}\sum_{\vec{v}_{\rm F}}\psi^\dagger(\vec{v}_{\rm F},x)\left(\gamma_\perp\cdot D\right)^2\psi(\vec{v}_{\rm F},x)\\ &= -\sum_{\vec{v}_{\rm F}}\left[\psi^\dagger\frac{D_\perp^2}{2\mu}\psi+g_s\psi^\dagger\frac{\sigma_{\mu\nu}F^{\mu\nu}}{4\mu}\psi\right].\end{aligned} \tag{18}$$

In the leading order in the HDL approximation, the loop correction to the vertex is neglected and the quark–gluon vertex is shifted by the $1/\mu$ correction as

$$-ig_s v_{\rm F}^i \mapsto -ig_s v_{\rm F}^i - ig_s\frac{l_\perp^i}{\mu}, \tag{19}$$

where l_i is the momentum carried away from quarks by gluons. We note that since the $1/\mu$ correction to the quark–gluon vertex does not depend on the Fermi velocity of the quark, it generates a repulsion for quark pairs, bound by magnetic forces. For a constant gap approximation, $\Delta(p_{||})\simeq\Delta$, the gap equation becomes in the leading order, as $p_{||}\to 0$,

$$1=\frac{g_s^2}{9\pi}\int\frac{d^2l_{||}}{(2\pi)^2}\left[\ln\left(\frac{\bar{\Lambda}}{|l_0|}\right)-\frac{3}{2}\right]\frac{1}{l_{||}^2+\Delta^2}=\frac{g_s^2}{36\pi^2}\ln\left(\frac{\bar{\Lambda}}{\Delta}\right)\left[\ln\left(\frac{\bar{\Lambda}}{\Delta}\right)-3\right]. \tag{20}$$

When $\bar{\Lambda}\le {\rm e}^3\Delta$, the gap due to the long-range color magnetic interaction disappears. Since the phase transition for color superconducting phase is believed to be of first order [24, 25], we may assume that the gap has the same dependence on the chemical potential μ as the leading order. Then, the critical density for the color superconducting phase transition is given by

$$1={\rm e}^3\exp\left[-\frac{3\pi^2}{\sqrt{2}g_s(\mu_{\rm c})}\right]. \tag{21}$$

Therefore, if the strong interaction coupling is too strong at the scale of the chemical potential, the gap does not form. In other words, the chemical potential has to be bigger than a critical value, $0.13\,{\rm GeV}<\mu_{\rm c}<0.31\,{\rm GeV}$, which is about the same as the one estimated in the literature [24–26].

3. The Color–Flavor-Locking phase

When $N_f = 3$, the spin-zero component of the condensate becomes (flavor) anti-triplet,

$$\left\langle \psi_{\mathrm{L}}{}^{a}_{i\alpha}(\vec{v}_{\mathrm{F}}, x)\psi_{\mathrm{L}}{}^{b}_{j\beta}(-\vec{v}_{\mathrm{F}}, x)\right\rangle = -\left\langle \psi_{\mathrm{R}}{}^{a}_{i\alpha}(\vec{v}_{\mathrm{F}}, x)\psi_{\mathrm{R}}{}^{b}_{j\beta}(-\vec{v}_{\mathrm{F}}, x)\right\rangle \quad (22)$$

$$= \varepsilon_{ij}\varepsilon^{abc}\varepsilon_{\alpha\beta\gamma}K_c^{\gamma}(p_{\mathrm{F}}), \quad (23)$$

where $\psi(\vec{v}_{\mathrm{F}}, x)$ is the quark near the Fermi surface with Fermi velocity $\vec{v}_{\mathrm{F}}$ [10,13]. Using the global color and flavor symmetry, one can always diagonalize the spin-zero condensate as $K_c^{\gamma} = \delta_c^{\gamma} K_{\gamma}$. To determine the parameters, K_u, K_d, and K_s, we need to minimize the vacuum energy for the condensate. By the Cornwall–Jackiw–Tomboulis formalism [27], the vacuum energy in the HDL approximation is given as

$$\begin{aligned} V(\Delta) &= -\mathrm{Tr}\ \ln S^{-1} + \mathrm{Tr}\ \ln \not\partial + \mathrm{Tr}\ (S^{-1} - \not\partial)S + (\text{2PI diagrams}) \\ &= \frac{\mu^2}{4\pi}\sum_{i=1}^{9}\int\frac{d^2 l_{\parallel}}{(2\pi)^2}\left[\ln\left(\frac{l_{\parallel}^2}{l_{\parallel}^2 + \Delta_i^2(l_{\parallel})}\right) + \frac{1}{2}\frac{\Delta_i^2(l_{\parallel})}{l_{\parallel}^2 + \Delta_i^2(l_{\parallel})}\right] + h.o., \end{aligned} \quad (24)$$

where h.o. are the higher order terms in the HDL approximation, containing more powers of coupling g_s, and Δ_i's are the eigenvalues of color anti-symmetric and flavor anti-symmetric 9×9 gap, $\Delta_{\alpha\beta}^{ab}$. The 2PI diagrams are two-particle-irreducible vacuum diagrams. There is only one such diagram (see Fig. 2) in the leading order HDL approximation.

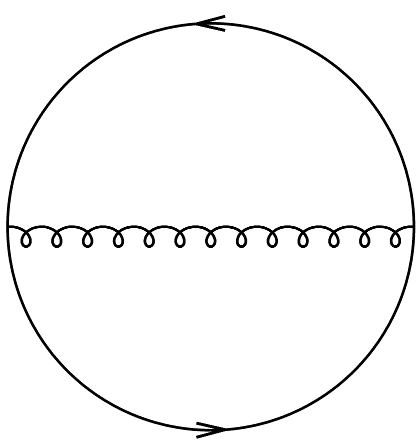

Fig. 2. The 2PI vacuum energy diagram.

Since the gap depends only on energy in the leading order, one can easily perform the momentum integration in (24) to get[3],

$$\begin{aligned} V(\Delta) &= \frac{\mu^2}{4\pi^2}\int_0^\infty dl_0 \left(-\frac{\Delta_i^2}{\sqrt{l_0^2}+\sqrt{l_0^2+\Delta_i^2}}+\frac{1}{4}\frac{\Delta_i^2}{\sqrt{l_0^2+\Delta_i^2}}\right) \\ &\simeq -0.43\frac{\mu^2}{4\pi^2}\sum_i \left|\Delta_i(0)\right|^2 , \end{aligned} \tag{25}$$

where in the second line we used an approximation that

$$\Delta_i(l_0) \simeq \begin{cases} \Delta_i(0) & \text{if } |l_0| < |\Delta_i(0)| , \\ 0 & \text{otherwise.} \end{cases} \tag{26}$$

Were Δ_i independent of each other, the global minimum should occur at $\Delta_i(0) = \text{const.}$ for all $i = 1, \cdots, 9$. But, due to the global color and flavor symmetry, only three of them are independent. Similarly to the condensate, the gap can be also diagonalized by the color and flavor symmetry as

$$\Delta_{ab}^{\alpha\beta} = \varepsilon_{\alpha\beta\gamma}\varepsilon^{abc}\Delta_\gamma\delta_c^\gamma . \tag{27}$$

Without loss of generality, we can take $|\Delta_u| \geq |\Delta_d| \geq |\Delta_s|$. Let $\Delta_d/\Delta_u = x$ and $\Delta_s/\Delta_u = y$. Then, the vacuum energy becomes

$$V(\Delta) \simeq -0.43\frac{\mu^2}{4\pi^2}\left|\Delta_u\right|^2 f(x,y) , \tag{28}$$

where $f(x,y)$ is a complicated function of $-1 \leq x, y \leq 1$ that has a maximum at $x = 1 = y$, $f(x,y) \leq 13.4$. Therefore, the vacuum energy has a global minimum when $\Delta_u = \Delta_d = \Delta_s$, or in terms of the eigenvalues of the gap

$$\Delta_i = \Delta_u \cdot (1,1,1,-1,1,-1,1,-1,-2) . \tag{29}$$

Among nine quarks, ψ_a^α, eight of them have (Majorana) mass Δ_u, forming an octet under SU(3), and one of them, a singlet under SU(3), has $2\Delta_u$.

Since the condensate is related to the off-diagonal component of the quark propagator at high momentum as, suppressing the color and flavor indices,

$$\begin{aligned} \langle\psi(\vec{v}_{\rm F},x)\psi(-\vec{v}_{\rm F},x)\rangle &\sim \lim_{y\to x}\int\frac{d^4l}{(2\pi)^4}{\rm e}^{il\cdot(x-y)}\frac{\Delta(l_\|)}{l_\|^2-\Delta^2(l_\|)} \\ &= \lim_{y\to x}\left[\delta^2(\vec{x}_\perp-\vec{y}_\perp)\frac{\Delta(0)}{4\pi^2|x_\|-y_\||^{\gamma_m}}+\cdots\right] , \end{aligned} \tag{30}$$

[3] If the condensate forms, the vacuum energy due to the gluons also depends on the gap due to the Meisner effect. But, it turns out to be subleading, compared to the quark vacuum energy; $V_g(\Delta) \sim M^2\Delta^2\ln(\Delta/\mu) \sim g_s\mu^2\Delta^2$ [9].

where γ_m is the anomalous dimension of the condensate and the ellipsis are less singular terms. Being proportional to the gap, the condensate is diagonalized in the basis where the gap is diagonalized. Thus, we have shown that in the HDL approximation the true ground state of QCD with three massless flavors at high density is the color-flavor locking phase, $K_\gamma = K$ for all $\gamma = u, d, s$. The condensate takes

$$\begin{aligned}\left\langle \psi_{\mathrm{L}}{}^{a}_{i\alpha}(\vec{v}_{\mathrm{F}}, x)\psi_{\mathrm{L}}{}^{b}_{j\beta}(-\vec{v}_{\mathrm{F}}, x)\right\rangle &= -\left\langle \psi_{\mathrm{R}}{}^{a}_{i\alpha}(\vec{v}_{\mathrm{F}}, x)\psi_{\mathrm{R}}{}^{b}_{j\beta}(-\vec{v}_{\mathrm{F}}, x)\right\rangle \\ &= \varepsilon_{ij}\varepsilon^{abI}\varepsilon_{\alpha\beta I}K(p_{\mathrm{F}}),\end{aligned} \tag{31}$$

breaking the color symmetry, $U(1)_{\mathrm{em}}$, the chiral symmetry, and the baryon number symmetry. The symmetry breaking pattern of the CFL phase is therefore

$$\mathrm{SU}(3)_{\mathrm{c}} \times \mathrm{SU}(3)_{\mathrm{L}} \times \mathrm{SU}(3)_{\mathrm{R}} \times \mathrm{U}(1)_{\mathrm{em}} \times \mathrm{U}(1)_B \mapsto \mathrm{SU}(3)_V \times \mathrm{U}(1)_{\tilde{Q}} \times Z_2, \tag{32}$$

where $\mathrm{SU}(3)_V$ is the diagonal subgroup of three SU(3) groups and the generator of $\mathrm{U}(1)_{\tilde{Q}}$ is a linear combination of the color hypercharge and $\mathrm{U}(1)_{\mathrm{em}}$ generator,

$$\tilde{Q} = \cos\theta Q_{\mathrm{em}} + \sin\theta Y_8\,, \tag{33}$$

where $\tan\theta = e/g_s$.

4. Meson mass

In the CFL phase of color superconductors, there are 8 pseudo Nambu–Goldstone (NG) bosons and one genuine NG boson. Since the (pseudo) NG bosons are very light, they constitute the low-lying excitations of the CFL phase, together with the modified photon, which are relevant in the low energy phenomena like the cooling process of color superconductors.

The pseudo NG bosons will get mass due to interactions, that break SU(3) chiral symmetry, such as Dirac mass terms [28–32] electromagnetic interactions [33, 34], and instantons [31]. It is important to note that Dirac mass term and instanton effects are suppressed by powers of $1/\mu$ at high density since they involve anti-quarks, while the electromagnetic interaction is not. In this section we derive the meson mass due to the Dirac mass term and the electromagnetic interaction, by matching the vacuum energy shift in microscopic theory (QCD) and the effective theory of mesons, which was used in [28, 29]. But, we present the calculation, using the effective theory constructed in [10, 13], as was done by Beane *et al.* [32]. As in [10, 13], if we introduce the charge conjugated field $\psi_{\mathrm{c}} = C\bar{\psi}^T$ with $C = i\gamma^0\gamma^2$ and decompose the quark field into states (ψ_+) near the Fermi surface and the states (ψ_-) deep in the Dirac sea, the Dirac mass term can be rewritten as

$$m_q\bar{\psi}\psi = \tfrac{1}{2}m_q\left(\bar{\psi}_+\psi_- + \bar{\psi}_-\psi_+\right) + \tfrac{1}{2}m_q^T\left(\bar{\psi}_{c+}\psi_{c-} + \bar{\psi}_{c-}\psi_{c+}\right), \tag{34}$$

which becomes, if one integrates out the antiquarks (ψ_- or ψ_{c-}),

$$m_q\bar{\psi}\psi = \frac{m_q^2}{2\mu}\bar{\psi}_+\widehat{\psi_-\bar{\psi}_-}\psi_+ + \frac{m_q m_q^T}{2\mu}\bar{\psi}_+\widehat{\psi_-\bar{\psi}_{c-}}\psi_{c+} + \cdots, \tag{35}$$

where $\widehat{\psi_-\bar{\psi}_-}$ and $\widehat{\psi_-\bar{\psi}_{c-}}$ are the antiquark propagators, propagating into the antiquark field itself or its charge-conjugated field, respectively. Antiquark fields propagate into their charge conjugated fields only if they have a Majorana mass and thus the meson mass due to Dirac mass is zero if the antiquark Majorana mass is zero.

At first one may think that the Majorana mass of antiquarks is zero, since it is energetically not preferred for them to develop a condensate due to the gap ($\sim\mu$) to create an antiquark. But, it is shown [33] that the antiquark fields get a radiative Majorana mass, which is equal to the Cooper-pair gap of quarks near the Fermi surface, since all the symmetries that forbid the Majorana mass term for antiquarks are broken once the Cooper gap is open for the quarks near the Fermi surface.

Having shown that the antiquarks have same Majorana mass as quarks near the Fermi surface, we may write the inverse propagator of the Nambu–Gorkov antiquark field, $(\psi_-, \psi_{c-})^T$, as

$$S_c^{-1}(p) = -i\frac{1-\vec{\alpha}\cdot\vec{v}_F}{2}\begin{pmatrix} p_0\gamma_0 - \vec{p}\cdot\vec{\gamma} + 2\mu\gamma_0 & -\Delta^\dagger \\ -\Delta & p_0\gamma_0 + \vec{p}\cdot\vec{\gamma} - 2\mu\gamma_0 \end{pmatrix} \tag{36}$$

$$= -i\frac{1-\vec{\alpha}\cdot\vec{v}_F}{2}\gamma_0\begin{pmatrix} l\cdot V + 2\mu & -\Delta^\dagger \\ -\Delta & l\cdot\bar{V} - 2\mu \end{pmatrix}, \tag{37}$$

where the projector $(1-\vec{\alpha}\cdot\vec{v}_F)/2$ is to project out the states in the Dirac sea, $V^\mu = (1, \vec{v}_F)$, $\bar{V} = (1, -\vec{v}_F)$, and we decompose $p^\mu = \mu v^\mu + l^\mu$ with $v^\mu = (0, \vec{v}_F)$ in the second line. Since the states in the Dirac sea can propagate into their charge conjugated states via the radiatively generated Majorana mass term, Eq. (35) becomes

$$m_q\bar{\psi}\psi = \frac{m_q^2}{2\mu}\psi_+^\dagger\left(1 - \frac{i\partial\cdot V}{2\mu}\right)\psi_+ + \frac{m_q m_q^T}{4\mu^2}\psi_+^\dagger\Delta\psi_{c+} + \cdots, \tag{38}$$

where V^μ=$(1, \vec{v}_F)$ and the ellipsis denotes the terms higher order in $1/\mu$. Then, the vacuum energy shift due to the Dirac mass term is $\sim m_q^2\Delta^2\ln(\mu^2/\Delta^2)$ in the leading order, which has to be matched with the vacuum energy in the meson Lagrangian, $m_\pi^2 F^2$ with the pion decay constant $F \sim \mu$ [28]. Therefore, one finds the meson mass due to the Dirac mass $m_\pi^2 \sim m_q^2 \Delta^2/\mu^2 \cdot \ln(\mu^2/\Delta^2)$ [28–32]. The electromagnetic interaction also contributes to the meson mass, since it breaks the SU(3) flavor symmetry. Among 8 pseudo Nambu–Goldstone bosons, four of them have the unbroken U(1) charge and receive a correction, [33, 34] $\delta m_\pi \simeq 12.7 \sin\theta\Delta \left[\ln(\mu^2/\Delta^2)\right]^{1/2}$, where $\theta = \tan^{-1}(e/g_s)$[4].

Finally, the instanton breaks the chiral symmetry and contributes to the meson mass. But, its effect at high density is suppressed by μ^{-14} for three flavors [11] and thus negligible.

5. Bosonization of the CFL dense QCD

In this CFL phase, the particle spectrum can be precisely mapped into that of the hadronic phase at low density. Observing this map, Schäfer and Wilczek [35,36] further conjectured that two phases are in fact continuously connected to each other. The CFL phase at high density is complementary to the hadronic phase at low density. This conjecture was subsequently supported [37,38] by showing that quarks in the CFL phase are realized as Skyrmions, called superqualitons, just like baryons are realized as Skyrmions in the hadronic phase.

This phase continuity can be explained heuristically in the following thought experiment. Suppose we inject a hydrogen atom into a CFL color superconductor as in Fig. 3. In the color superconductor, being bombarded by energetic (gapped) quarks, the atom will be ionized and the quarks in the proton will get deconfined to form, for example, a Cooper pair of u and d, leaving the up quark alone. From this thought experiment we find two things: The baryon number of up quark is same as that of proton, since the CFL vacuum provides the missing two thirds. As the hydrogen atom is neutral in the vacuum, the charge of up quark has to be opposite to that of electron with respect to whatever unbroken charges in the color superconductor. Thereby, the gapped quarks in the CFL phase correspond to baryons in the hadronic phase.

Furthermore, it is possible to bosonize the CFL color superconductor [38], realizing Skyrme's original motivation for the Skyrmion [39]. We introduce a bosonic variable,

$$U_{\mathrm{L}ai}(x) \equiv \lim_{y\to x} \frac{|x-y|^{\gamma_m}}{\Delta(p_{\mathrm{F}})}\, \varepsilon_{abc}\varepsilon_{ijk}\psi_{\mathrm{L}}^{bj}(-\vec{v}_{\mathrm{F}}, x)\psi_{\mathrm{L}}^{ck}(\vec{v}_{\mathrm{F}}, y), \tag{39}$$

[4] In [34], the result is a little different from the one obtained in [33].

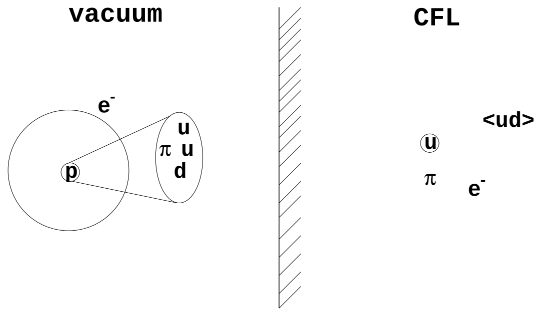

Fig. 3. The phase continuity.

where γ_m ($\sim \alpha_s$) is the anomalous dimension of the diquark field. Similarly, we define $U_{\rm R}$ in terms of right-handed quarks to describe the small fluctuations of the condensate of right-handed quarks. Since the bosonic fields, $U_{\rm L,R}$, are colored, they will interact with gluons. In fact, the colored massless excitations will constitute the longitudinal components of gluons through Higgs mechanism. Among the small fluctuations of condensates, the colorless excitations correspond to genuine Nambu–Goldstone (NG) bosons, which can be described by a color singlet combination of $\mathrm{U}_{\rm L,R}$ [29,40], given as

$$\Sigma_i^j \equiv \mathrm{U}_{{\rm L}ai} U_{\rm R}^{*aj}. \tag{40}$$

The NG bosons transform under the $\mathrm{SU}(3)_{\rm L} \times \mathrm{SU}(3)_{\rm R}$ chiral symmetry as

$$\Sigma \mapsto g_{\rm L} \Sigma g_{\rm R}^\dagger, \quad \text{with} \quad g_{\rm L,R} \in \mathrm{SU}(3)_{\rm L,R}. \tag{41}$$

Since the chiral symmetry is explicitly broken by the current quark mass, the instanton effects, and the electromagnetic interaction, the NG bosons will get mass, which has been calculated by various groups [28–30,33]. Here we focus on the meson mass due to the current strange quark mass (m_s), since it will be dominant for the intermediate density. Then, the meson mass term is simplified as

$$\mathcal{L}_m = C\,\mathrm{tr}(M^T\Sigma)\mathrm{tr}(M^*\Sigma^\dagger) + O(M^4), \tag{42}$$

where $M = \mathrm{diag}(0,0,m_s)$ and $C \sim \Delta^4/\mu^2\,\ln(\mu^2/\Delta^2)$. (Note that in general there will be two more mass terms quadratic in M. But, they all vanish if we neglect the current mass of up and down quarks and also the small color-sextet component of the Cooper pair [29].)

Thus, the low-energy effective Lagrangian density for the bosonic fields in the CFL phase can be written as

$$\mathcal{L}_{\rm eff} \;=\; \mathcal{L}_g + \left[\frac{1}{4}F^2\mathrm{tr}(\partial_\mu U_{\rm L}^\dagger \partial^\mu U_{\rm L}) + n_{\rm L}\mathcal{L}_{\rm WZW} + ({\rm L} \leftrightarrow {\rm R})\right]$$

$$+\mathcal{L}_m + g_s G_\mu^A J^{\mu A} + \cdots\,, \qquad (43)$$

where $\mathcal{L}_g$ is the Lagrangian of Higgsed gluons, G_μ^A, and the ellipsis denotes the higher order terms in the derivative expansion, including mixing terms between $U_{\rm L}$ and $U_{\rm R}$. The gluons couple to the bosonic fields through a minimal coupling with a conserved current, given as

$$\begin{aligned} J^{A\mu} &= \frac{i}{2}F^2 {\rm Tr}\; U_{\rm L}^{-1}T^A\partial^\mu U_{\rm L} + \frac{1}{24\pi^2}\varepsilon^{\mu\nu\rho\sigma}{\rm Tr}\; T^A U_{\rm L}^{-1}\partial_\nu U_{\rm L}U_{\rm L}^{-1}\partial_\rho U_{\rm L}U_{\rm L}^{-1}\partial_\sigma U_{\rm L} \\ &\quad +({\rm L}\leftrightarrow{\rm R}) + \cdots\,, \end{aligned} \qquad (44)$$

where the ellipsis denotes the currents from the higher order derivative terms in Eq. (43). F is a quantity analogous to the pion decay constant, calculated to be $F \sim \mu$ in the CFL color superconductor [28]. The Wess–Zumino–Witten (WZW) term [41,42] is described by the action

$$\Gamma_{\rm WZW} \equiv \int d^4x\, \mathcal{L}_{\rm WZW} = -\frac{i}{240\pi^2}\int_{\mathsf{M}} d^5r\varepsilon^{\mu\nu\alpha\beta\gamma}{\rm tr}(l_\mu l_\nu l_\alpha l_\beta l_\gamma)\,, \qquad (45)$$

where $l_\mu = U_{\rm L}^\dagger \partial_\mu U_{\rm L}$ and the integration is defined on a five-dimensional manifold $\mathsf{M} = V\otimes S^1\otimes I$ with the three dimensional space V, the compactified time S^1, and a unit interval I needed for the local form of WZW term. The coefficients of the WZW terms in the effective Lagrangian (43) have been shown to be $n_{\rm L,R} = 1$ by matching the flavor anomalies [37], which is later confirmed by an explicit calculation [43].

Now, let us try to describe the CFL color superconductor in terms of the bosonic variables. We start with the effective Lagrangian (43), which is good at low energy, without putting in the quark fields. As in the Skyrme model of baryons, we anticipate the gaped quarks come out as solitons, made of the bosonic degrees of freedom. That the Skyrme picture can be realized in the CFL color superconductor is already shown in [37], but there the mass of the soliton is not properly calculated. Here, by identifying the correct ground state of the CFL superconductor in the bosonic description, we find the superqualitons have same quantum numbers as quarks with mass of the order of gap, showing that they are really the gaped quarks in the CFL color superconductor. Furthermore, upon quantizing the zero modes of the soliton, we find that high spin excitations of the soliton have energy of order of μ, way beyond the scale where the effective bosonic description is applicable, which we interpret as the absence of high-spin quarks, in agreement with the fermionic description. It is interesting to note that, as we will see below, by calculating the soliton mass in the bosonic description, one finds the coupling and the chemical potential dependence of the Cooper-pair gap, at least numerically, which gives us a complementary way, if not better, of estimating the gap.

As the baryon number (or the quark number) is conserved, though spontaneously broken, the ground state in the bosonic description should have the same baryon (or quark) number as the ground state in the fermionic description. Under the $U(1)_Q$ quark number symmetry, the bosonic fields transform as

$$U_{\rm L,R} \mapsto {\rm e}^{i\theta Q} U_{\rm L,R} {\rm e}^{-i\theta Q} = {\rm e}^{2i\theta} U_{\rm L,R}\,, \tag{46}$$

where Q is the quark number operator, given in the bosonic description as

$$Q = i \int d^3x\ \frac{F^2}{4} {\rm Tr}\left[U_{\rm L}^\dagger \partial_t U_{\rm L} - \partial_t U_{\rm L}^\dagger U_{\rm L} + ({\rm L} \leftrightarrow {\rm R})\right]\,, \tag{47}$$

neglecting the quark number coming from the WZW term, since the ground state has no nontrivial topology. The energy in the bosonic description is

$$E = \int d^3x \frac{F^2}{4} {\rm Tr}\left[|\partial_t U_{\rm L}|^2 + \left|\vec{\nabla} U_{\rm L}\right|^2 + ({\rm L} \leftrightarrow {\rm R})\right] + E_m + \delta E\,, \tag{48}$$

where E_m is the energy due to meson mass and δE is the energy coming from the higher derivative terms. Assuming the meson mass energy is positive and $E_m + \delta E \geq 0$, which is reasonable because $\Delta/F \ll 1$, we can take, dropping the positive terms due to the spatial derivative,

$$E \geq \int d^3x \frac{F^2}{4} {\rm Tr}\left[|\partial_t U_{\rm L}|^2 + ({\rm L} \leftrightarrow {\rm R})\right] (\equiv E_Q)\,. \tag{49}$$

Since for any number α

$$\int d^3x\ {\rm Tr}\left[|U_{\rm L} + \alpha i \partial_t U_{\rm L}|^2 + ({\rm L} \leftrightarrow {\rm R})\right] \geq 0\,, \tag{50}$$

we get a following Schwartz inequality,

$$Q^2 \leq I\, E_Q\,, \tag{51}$$

where we defined

$$I = \frac{F^2}{4} \int d^3x\, {\rm Tr}\left[U_{\rm L} U_{\rm L}^\dagger + ({\rm L} \leftrightarrow {\rm R})\right]. \tag{52}$$

Note that the lower bound in Eq. (51) is saturated for $E_Q = \omega Q$ or

$$U_{\rm L,R} = {\rm e}^{i\omega t} \quad \text{with} \quad \omega = \frac{Q}{I}. \tag{53}$$

The ground state of the color superconductor, which has the lowest energy for a given quark number Q, is nothing but a so-called Q-matter, or the interior of a very large Q-ball [44,45]. Since in the fermionic description the system has the quark number $Q = \mu^3/\pi^2 \int \mathrm{d}^3x = \mu^3/\pi^2 \cdot I/F^2$, we find, using $F \simeq 0.209\mu$ [28],

$$\omega = \frac{1}{\pi^2}\left(\frac{\mu}{F}\right)^3 F \simeq 2.32\mu\,. \tag{54}$$

In passing, we note that ω is numerically very close to $4\pi F$. The ground state of the system in the bosonic description is a Q-matter whose energy per unit quark number is ω. Now, let us suppose we consider creating a $Q = 1$ state out of the ground state. In the fermionic description, this corresponds that we excite a gaped quark in the Fermi sea into a free state, which costs energy at least 2Δ. In the bosonic description, this amounts to creating a superqualiton out of the Q-matter, while reducing the quark number of the Q-matter by one. Therefore, since we gain energy ω by reducing the quark number of the Q-matter by one, the energy cost to create a gapped quark from the ground state is

$$\delta\mathcal{E} = M_Q - \omega\,, \tag{55}$$

where M_Q is the energy of the superqualiton configuration. Since $M_Q \sim 4\pi F$, we see that the energy of the superqualiton configuration is almost canceled out by ω to give the gap much smaller than $4\pi F$ or μ. Varing the strange quark mass, we find numerically that the twice of u- and s-superqualiton masses are given as

$$\begin{array}{lll} \Delta_u = 0.079\times 4\pi F, & \Delta_s = 0.081\times 4\pi F, & \text{for } \frac{m_K}{F} = 0.1 \\ \Delta_u = 0.079\times 4\pi F, & \Delta_s = 0.089\times 4\pi F, & \text{for } \frac{m_K}{F} = 0.3 \\ \Delta_u = 0.079\times 4\pi F, & \Delta_s = 0.109\times 4\pi F, & \text{for } \frac{m_K}{F} = 0.8. \end{array} \tag{56}$$

From the relation that $2\Delta = M_Q - \omega$, we can estimate numerically the coupling and the chemical potential dependence of the Cooper gap [38]. This gives an alternative way of calculating the Cooper gap, if not better.

6. Neutrino interaction in CFL

To discuss the interaction of neutrinos in color superconductors [46], we first note that gluons mix with weak gauge bosons, since the diquark condensates in color superconductors carry not only a color but also a weak charge.

Consider the color-gluon annihilation into the lepton pair, $l\bar{l}$, as an example of weak neutral current interactions:

$$\tilde{V}^+ + \tilde{V}^- \to \tilde{Z} \to l\bar{l}\,, \tag{57}$$

$$\tilde{V}^+ + \tilde{V}^- \to \tilde{V}_0 \to l\bar{l}\,. \tag{58}$$

The coupling at the $\tilde{V}\tilde{V}\tilde{Z}$ vertex in the process mediated by $\tilde{Z}$, Eq. (57), is given by

$$f\cos^2\delta\,\frac{4}{\sqrt{3}}\frac{g_s g}{g^2+g'^2}\frac{\sigma^2}{v^2}\cos\theta_W \sim \frac{4}{\sqrt{3}}g\cos^3\theta_W\left(\frac{M_V}{M_W}\right)^2 \tag{59}$$

which gives a suppression factor

$$\sim\left(\frac{M_V}{M_W}\right)^2 \tag{60}$$

compared to the conventional $\nu\bar{\nu}$ production. The suppression factor in the process mediated by $\tilde{V}_0$, Eq. (58), due to the vertex $\tilde{V}_0\,l\bar{l}$ is given by

$$\sim\sin\beta\sim\frac{g}{g_s}\,. \tag{61}$$

The propagator in the low energy limit $Q^2 \ll M_V^2$ is greater than in Eq. (57), *i.e.*,

$$\frac{1}{Q^2-M_V^2}\sim\frac{1}{M_V^2}\,. \tag{62}$$

However the amplitude for fusion is enhanced at the strong interaction vertex, $\tilde{V}\tilde{V}\tilde{V}_0$, by a factor of f, and we get the factor for the amplitude

$$\sim\;Q_f g\frac{g}{g_s}\frac{1}{M_V^2}g_s = Q_f\frac{g^2}{M_V^2} \tag{63}$$

with the modified electric charge Q_f. One can now see that the gluon fusion into the charged flavor $l\bar{l}$ pair is greater than the weak neutral current by a factor of $\sim(M_V/M_W)^{-2}\sim 10^6$ and hence comparable to photon mediated processes [47]. However this enhancement does not apply to gluon-mediated $\nu\bar{\nu}$ processes because Q_f is zero for neutrino. In general, for the neutral current with neutrinos, the contribution from color-gluon mediated processes in the broken phase vanishes since the amplitude is proportional to Q_f(neutrino) which is $=0$. We arrive at the same conclusion for $q\bar{q}\to\nu\bar{\nu}$.

The charged current weak interaction in the process mediated by $\tilde{V}_0$ is also comparable to the ordinary weak interaction strength for the neutrino-quark interaction in the low-energy limit. Consider the following processes in matter,

$$q+l\;\to\;q'+\nu(\bar{\nu})\,, \tag{64}$$

$$q\;\to\;q'+l+\nu(\bar{\nu})\,. \tag{65}$$

As in the gluon annihilation processes, there are two amplitudes that can be decomposed into three parts: quark gauge boson vertex, propagator, gauge boson–lepton–neutrino vertex,

$$qq'\tilde{W}^{\pm} \to \tilde{W}^{\pm} \to l\nu\tilde{W}^{\pm}, \tag{66}$$

$$qq'\tilde{V}^{\pm} \to \tilde{V}^{\pm} \to l\nu\tilde{V}^{\pm}. \tag{67}$$

In the low energy limit, Eq. (66) gives the ordinary weak amplitude

$$\sim \frac{g^2}{M_W^2}. \tag{68}$$

It is easy to see that the contribution of the color gauge-boson-mediated process, Eq. (67), also gives an amplitude comparable to that of the $W^{\pm}$ mediated process,

$$\sim g\frac{g}{g_s}\left(\frac{M_V}{M_W}\right)^2 \frac{1}{M_V^2} g_s \sim \frac{g^2}{M_W^2}. \tag{69}$$

It should be noted however that the quark decay mediated by $\tilde{V}_0$ in Eq. (65) cannot take place because of the energy conservation: the quarks with different colors but with same flavor have the same mass. Therefore the neutrino production mediated by the color-changing weak current is limited to the process in Eq. (64)

$$q_r + \mathrm{e}^- \to q_b + \nu, \tag{70}$$

$$q_b + \mathrm{e}^+ \to q_r + \bar{\nu}. \tag{71}$$

To keep the system in a color-singlet state in the cooling process, these processes should occur equally to compensate the color change in each process. It implies that these processes depend on the abundance of positrons in the system. At finite temperature in the cooling period, it is expected that there will be a substantial amount of positrons as well as electrons as long as the temperature is not far below $\sim$ MeV. Of course the additional enhancement of the neutrino production due to the CFL phase depends on the abundance of positrons in the system which depends mainly on the temperature. If confined colored gluons are present in the matter in the CFL phase, the same amplitude can be obtained in Eq. (67) when qq' is replaced with VV'.

The result obtained above can be summarized as predicting an enhancement of the effective four-point coupling constant for the neutrino production process in the low energy limit. The enhancement due to the neutrino-color interaction is suppressed by factors of $\mathrm{e}^{-\Delta/T}$ or $\mathrm{e}^{-M_V/T}$, since it depends on the unpaired excitations above gap which can participate into neutrino-color interaction. Hence for the cooling process at low temperatures as $\sim 10^9$K it is not so effective. However during the early stage of proto-neutron star the temperature is expected to be high enough $\sim$ 20–50 MeV [48] to see the effect of the enhancement due to color excitations.

Let us now consider the weak decay of light quasi-quarks, described by the four-Fermi interaction:

$$\mathcal{L}_{\mathrm{Fermi}} = \frac{G_{\mathrm{F}}}{\sqrt{2}} \sum_{\vec{v}_{\mathrm{F}}} \bar{\psi}_{\mathrm{L}}(\vec{v}_{\mathrm{F}}, x)\gamma^{\mu}\psi_{\mathrm{L}}(\vec{v}_{\mathrm{F}}, x)\bar{\nu}_{\mathrm{L}}(x)\gamma_{\mu}\nu_{\mathrm{L}}(x) \tag{72}$$

$$= \frac{G_{\mathrm{F}}}{\sqrt{2}} \sum_{\vec{v}_{\mathrm{F}}} \psi^{\dagger}_{\mathrm{L}}(\vec{v}_{\mathrm{F}}, x)\psi_{\mathrm{L}}(\vec{v}_{\mathrm{F}}, x)\bar{\nu}_{\mathrm{L}}(x)\not{v}\,\nu_{\mathrm{L}}(x)\,, \tag{73}$$

where $G_{\mathrm{F}} = 1.166 \times 10^{-5}$ GeV^{-2} is the Fermi constant and ψ denotes the quasi-quark near the Fermi surface, projected from the quark field Ψ as in [10],

$$\psi(\vec{v}_{\mathrm{F}}, x) = \frac{1 + \vec{\alpha}\cdot\vec{v}_{\mathrm{F}}}{2} \mathrm{e}^{-i\vec{v}_{\mathrm{F}}\cdot\vec{x}}\Psi(x)\,. \tag{74}$$

Since the four-Fermi interaction of quarks with opposite momenta are marginally relevant and gets substantially enhanced at low energy, it may have significant corrections to the couplings to quarks of those weakly interacting particles [10]:

$$\begin{aligned}\delta\mathcal{L}_{\nu q} &= \frac{G_{\mathrm{F}}}{\sqrt{2}}\psi^{\dagger}_{\mathrm{L}}(\vec{v}_{\mathrm{F}}, x)\psi_{\mathrm{L}}(\vec{v}_{\mathrm{F}}, x)\bar{\nu}_{\mathrm{L}}(x)\not{v}\ \nu_{\mathrm{L}}(x) \\ &\quad\times\frac{ig_{\bar{3}}}{2M_V^2}\delta^A_{tv;us}\int_y \left[\bar{\psi}_t(\vec{v}^{\prime}_{\mathrm{F}}, y)\gamma^0\psi_s(\vec{v}^{\prime}_{\mathrm{F}}, y)\bar{\psi}_v(-\vec{v}^{\prime}_{\mathrm{F}}, y)\gamma^0\psi_u(-\vec{v}^{\prime}_{\mathrm{F}}, y)\right] \\ &= \frac{4}{3}\frac{g_{\bar{3}}}{2\pi}\frac{G_{\mathrm{F}}}{\sqrt{2}}\psi^{\dagger}_{\mathrm{L}}(\vec{v}_{\mathrm{F}}, x)\psi_{\mathrm{L}}(\vec{v}_{\mathrm{F}}, x)\bar{\nu}_{\mathrm{L}}(x)\bar{v}\cdot\gamma\nu_{\mathrm{L}}(x)\,,\end{aligned} \tag{75}$$

where $\vec{v}_{\mathrm{F}}$ and $\vec{v}^{\prime}_{\mathrm{F}}$ are summed over and $g_{\bar{3}}$ is the value of the marginal four-quark coupling at the screening mass scale M. In terms of the renormalization group (RG) equation at a scale E

$$\frac{dG_{\mathrm{F}}(t)}{dt} = \frac{4}{3}\frac{g_{\bar{3}}(t)}{2\pi}G_{\mathrm{F}}(t)\,, \tag{76}$$

where $t = \ln E$. The scale dependence of the marginal four-quark coupling in the color anti-triplet channel is calculated in [10, 13]. At $E \ll \mu$

$$\bar{g}_{\bar{3}}(t) \simeq \frac{4\pi}{11}\alpha_s(t)\,. \tag{77}$$

Since $\alpha_s(t) = 2\pi/(11t)$, we get

$$G_{\rm F}(E) \simeq G_{\rm F}(\mu)\left(\frac{\mu}{E}\right)^{\frac{16\pi}{363}} . \tag{78}$$

Since the RG evolution stops at scales lower than the gap, the low energy effective Fermi coupling in dense matter is therefore

$$G_{\rm F}^{\rm eff} = G_{\rm F}\left(\frac{\mu}{\Delta}\right)^{\frac{16\pi}{363}} . \tag{79}$$

We emphasize that this enhancement applies equally to the β decay of quarks and other neutrino production processes described in the previous section.

At asymptotic density and low temperature ($T \neq 0$), the relevant excitations are quasi-quarks that are not Cooper-paired, and 17 Nambu–Goldstone bosons. All other massive particles, Higgsed gluons and other massive excitations are expected to be out of thermal equilibrium and decoupled. Thus the main cooling processes must be the emission of weakly interacting light particles like neutrinos or other (weakly interacting) exotic light particles (*e.g.* axions) from the quasi-quarks and Nambu–Goldstone bosons in the thermal bath.

For the neutrino emissivity from quasi quarks, the so-called Urca process is relevant. The neutrino emissivity by the direct Urca process in quark matter, which is possible for most cases in quark matter, was calculated by Iwamoto [49]. For the CFL superconductor, we expect the calculation goes in parallel and the neutrino emissivity is

$$\varepsilon_{\rm direct} \propto \alpha_s \rho Y_{\rm e}^{1/3} T^6, \tag{80}$$

where ρ is the density, T is the temperature of the quark matter, and Y_e is the ratio between the electron and baryon density. On the other hand, the neutrino emissivity by the modified Urca process, which is the dominant process in the standard cooling of neutron stars [50], is suppressed by $(\Delta/\mu)^4$, since the pion coupling to quarks is given by $g_{qq\pi} \sim \Delta/\mu$ [29]. Thus, the neutrino emissivity by the modified Urca process in the CFL quark matter is greatly suppressed in the CFL quark matter, compared to normal quark matter. Furthermore, since the pion-pion interaction in the CFL quark matter are also suppressed by Δ/μ [28, 30], we note that all the low energy excitations in the CFL quark matter are extremely weakly coupled. But, since most excitations in the CFL quark matter are gapped and frozen out, the CFL quark matter has a quite small heat capacity and cools down very rapidly at temperatures lower than the gap [51].

Together with the general enhancement of the effective four-point coupling constant in RG analysis, the enhancement of the neutrino production implies that the cooling process speeds up as the CFL phase sets in dense hadronic matter near the critical temperature. But, at temperature much below the critical temperature, the interaction of quasi-quarks and pions and kaons is extremely weak, suppressed by Δ/μ, and the CFL quark matter cools down extremely rapidly.

For a realistic calculation of the cooling rate of compact stars, we need to also consider the neutrino propagation in the CFL matter before the neutrinos come out of the system. A recent study [52] suggests that the presence of the CFL phase can accelerate the cooling process because neutrino interactions with matter are reduced in the presence of a superconducting gap Δ. However this result is subject to modification by the effect of additional interactions — not taken into account in this work — mediated by the colored gluons on the quark polarization. It would be interesting to see how the enhancement of the neutrino production correlates with the neutrino-medium interaction. This is one of the physically relevant questions on how the enhanced neutrino interaction could affect neutron-star(neutron-proto star) cooling following supernova explosion. This issue is currently under investigation.

7. Conclusion

I have discussed some aspects of the exciting recent developments in color superconductivity in high density quark matter. I have calculated the Cooper pair gap and the critical points at high density where magnetic gluons are not screened. The ground state of high density QCD with three light flavors is shown to be a color-flavor locking state, which can be mapped into the low-density hadronic phase. The meson mass at the CFL superconductor is also calculated. The CFL color superconductor is bosonized, where the Fermi sea is identified as a Q-matter and the gapped quarks as topological excitations, called superqualitons, of mesons. Finally, as an application of color supercoductivity, I have discussed the neutrino interactions in the CFL color superconductor.

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