Vol. 32 (2001) *ACTA PHYSICA POLONICA B* No 4

THERMOSTATISTICAL PROPERTIES OF NUCLEAR MATTER AND THE NUCLEAR LIQUID–GAS PHASE TRANSITION

S. Haddad

Physics Department, Atomic Energy Commission of Syria
P.O.BOX 6091, Damascus, Syria

(Received October 26, 2000)

Thermostatistical properties of symmetric and asymmetric nuclear matter are studied in the framework of the relativistic mean field theory at a finite temperature. The statistical description via the grandcanonical potential produces an equation of state, which describes the nuclear liquid-gas phase transition as first order. The transition occurs at an excitation energy of 15–16 MeV per nucleon, and a density of 0.3–0.4 symmetric matter saturation density. This result is in accordance with the results of experimental observations of fragment distributions in heavy-ion collisions.

PACS numbers: 21.65.+f, 05.70.Ce, 25.70.Pq, 05.70.Fh

1. Introduction

Intensive experimental and theoretical investigations in heavy-ion reactions at energies ranging from a few tens to a few hundreds of MeV per nucleon revealed that the breakup of nuclear systems into complex fragments ($Z \geq 3$) is the most important reaction mechanism [1]. The basic feature is the possibility of a liquid-gas phase transition in nuclear matter [2].

Considerations based on an appreciable amount of experimental data indicate that the mechanisms responsible for the breakup of nuclear matter into complex fragments are of statistical nature, *i.e.*, the relative fragments multiplicities depend only on the excitation energy of the system, being independent on the colliding nuclei and bombarding energy [1].

In this work thermostatistical properties of symmetric and asymmetric nuclear matter are studied in the scheme of the Relativistic Mean Field theory (RMF) at a finite temperature. With a very limited number of parameters, RMF theory is able to give a quantitative description of ground-state properties of spherical and deformed nuclei at and away from the stability line [3].

A brief review of the relativistic mean field theory is provided in Section 2. In section 3 the statistical description of the thermal properties of nuclear matter via the grandcanonical potential is used to derive the Equation of State (EOS), and to describe the nuclear liquid-gas phase transition. The main results are summarized in Section 4.

2. Relativistic mean field theory

In the relativistic mean field theory (RMF) with σ, ω, and ρ mesons [4] the self-energy Σ has the form:

$$\Sigma = \Sigma_s + \gamma^0 \Sigma_0 \,, \tag{1}$$

and is given by the sum of the contributions of the three mesons:

$$\Sigma_s^{(\sigma)} = -\frac{g_\sigma^2}{m_\sigma^2}\rho_s - \frac{g_2}{m_\sigma^2 g_\sigma}\left(\Sigma_s^{(\sigma)}\right)^2 - \frac{g_3}{m_\sigma^2 g_\sigma^2}\left(\Sigma_s^{(\sigma)}\right)^3 \,, \tag{2}$$

$$\Sigma_0^{(\omega)} = +\frac{g_\omega^2}{m_\omega^2}\rho_B \,, \tag{3}$$

and:

$$\Sigma_0^{(\rho)} = \tau_3 \frac{g_\rho^2}{m_\rho^2}\left(\rho_p - \rho_n\right) \,. \tag{4}$$

$\tau_3 = +1$ for protons and -1 for neutrons. g_i and m_i $(i = \sigma, \omega, \rho)$ are the coupling constant and the mass of the i-meson. g_2 and g_3 are the constants of the non-linear coupling.

At a finite temperature T $(\beta = 1/T)$ the scalar density ρ_{s} and the baryon density ρ_B are given by [5]:

$$\rho_{\mathrm{s}} = \frac{1}{\pi^2}\int_0^\infty p^2 dp \frac{M}{\varepsilon(p)}\left(f_-(p) + f_+(p)\right) \,, \tag{5}$$

$$\rho_B = \frac{1}{\pi^2}\int_0^\infty p^2 dp \left(f_-(p) - f_+(p)\right) \,, \tag{6}$$

with:

$$f_-(p) = \left(\mathrm{e}^{\beta(\varepsilon(p)+\Sigma_0-\mu)} + 1\right)^{-1} \,, \tag{7}$$

$$f_+(p) = \left(\mathrm{e}^{\beta(\varepsilon(p)-\Sigma_0+\mu)} + 1\right)^{-1} \,, \tag{8}$$

$$\varepsilon(p) = \left(p^2 + M^2\right)^{1/2} \,, \tag{9}$$

$$M = m_N + \Sigma_{\mathrm{s}} \,, \tag{10}$$

m_N is the mass of the nucleon.

Given the baryon density ρ_B, the chemical potential μ is determined from Eq. (6).

The proton density ρ_p and the neutron density ρ_n are the positive and the negative isospin components of the baryon density $\rho_B = \rho_p + \rho_n$, and the asymmetry parameter δ is defined by:

$$\delta = \frac{(\rho_n - \rho_p)}{\rho_B}\,. \tag{11}$$

The energy density is given by [6]:

$$\begin{aligned} e &= \rho_v + \left(m_N + \tfrac{1}{2}\Sigma_{\rm s}\right)\rho_{\rm s} + \tfrac{1}{2}\Sigma_0\rho_B \\ &- \frac{1}{6}g_2\left(\frac{\Sigma_{\rm s}}{g_\sigma}\right)^3 - \frac{1}{4}g_3\left(\frac{\Sigma_{\rm s}}{g_\sigma}\right)^4 , \end{aligned} \tag{12}$$

with:

$$\rho_v = \frac{1}{\pi^2}\int_0^\infty p^4 dp \frac{1}{\varepsilon(p)}\left(f_-(p) + f_+(p)\right)\,. \tag{13}$$

The energy per nucleon is given by:

$$\left(\frac{E}{A}\right)(\rho_B, \delta, T) = \frac{e}{\rho_B}\,. \tag{14}$$

The entropy per particle is given by [5]:

$$\begin{aligned} \sigma &= \frac{-1}{\pi^2\rho_B}\int_0^\infty p^2 dp\,[f_-(p)\ln f_-(p) + f_+(p)\ln f_+(p) \\ &+ (1 - f_-(p))\ln(1 - f_-(p)) \\ &+ (1 - f_+(p))\ln(1 - f_+(p))]\,. \end{aligned} \tag{15}$$

TABLE I

Parameter set NL3 and nuclear matter saturation properties. ρ_0 is the saturation density, a_v the saturation energy per particle (volume energy), a_4 the symmetry energy, and M the reduced mass at saturation density.

Meson	σ	ω	ρ
m_i (MeV)	508.194	782.501	763
g_i	10.217	12.868	4.474
$g_2(\mathrm{fm}^{-1})$	–10.431		
g_3	–28.885		
$\rho_0(\mathrm{fm}^{-3})$	a_v (MeV)	a_4 (MeV)	M/m_N
0.148	–16.299	37.4	0.60

In this work, relativistic mean field calculations are carried out using the parameter set NL3 of Ref. [3], which is given in Table I, together with the saturation properties of nuclear matter it produces.

3. Liquid-gas phase transition in the nuclear equation of state

Figure 1 shows the change of nucleon energy with increasing temperature for symmetric matter. The one-nucleon excitation energy is defined as the difference between the nucleon energy at T and its ground-state energy:

$$\frac{E^*}{A}(\rho_B, \delta, T) = \frac{E}{A}(\rho_B, \delta, T) - \frac{E}{A}(\rho_B, \delta, T = 0)\,. \tag{16}$$

It can be inferred from figure 1 that at a given temperature T, the one-nucleon excitation energy is higher at lower densities. This is in accordance with the non-relativistic results of Ref. [7].

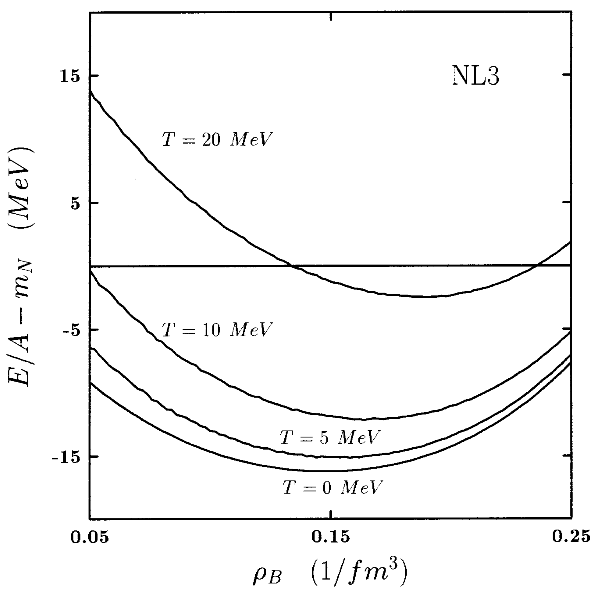

Fig. 1. Change of nucleon energy with increasing temperature for symmetric matter.

The thermodynamical properties of a system can be obtained by means of the grandcanonical potential density:

$$\omega(T, \mu) = e - \mu\rho_B - T\sigma\rho_B\,. \tag{17}$$

The last term includes ρ_B, since σ was defined in Eq. (15) as the entropy per particle. $\sigma\rho_B$ is therefor the entropy density.

Since pressure is given by [8]:

$$p = -\frac{\partial \Omega}{\partial V} = -\omega\,, \tag{18}$$

where Ω is the grandcanonical potential, and since at a given temperature T the chemical potential μ, the energy density e, and the entropy density $\sigma\rho_B$ are all given as a function of ρ_B and δ, we receive the nuclear matter equation of state:

$$p(\rho_B,\delta,T) = T\rho_B\sigma(\rho_B,\delta,T) + \rho_B\mu(\rho_B,\delta,T) - e(\rho_B,\delta,T)\,. \tag{19}$$

The basic feature observed in heavy-ion reactions at energies ranging from a few tens to a few hundreds of MeV per nucleon is the possibility of a liquid-gas phase transition in nuclear matter [2]. A clear signal of liquid-gas phase transition in nuclei is hinted at from the experimental caloric curve obtained in Au + Au collisions at 600 A MeV [9]. In the excitation energy range of 4–10 MeV per particle, the temperature T is found to be almost constant at a value of $T \sim 5$ MeV. The excitation energy range, over which T remains constant, could be termed as the latent heat of vaporization.

Investigations of the nuclear liquid-gas phase transition through heavy-ion reactions are based on three assumptions, which are not confirmed. The first assumption is that equilibrium thermodynamics is applicable for such a small system of a few hundred nucleons. The second is that a thermalized uniform system is formed before multifragmentation takes place. And the third is that the fragment distribution is directly related to the state of the thermalized uniform system before it breaks up.

Based on these assumptions, canonical ensemble models may be used in order to describe nuclear multifragmentation phenomena [10]. The main ingredient in the present analysis is the nuclear equation of state, based on the relativistic mean field theory. The mean field approximation is thermodynamically consistent, *i.e.*, it satisfies the relevant thermodynamic identities and the virial theorem [2], and a possible relation of the liquid-gas phase transition in the nuclear equation of state to the actual phase transition observed in heavy-ion reactions may be assumed. Although such analysis oversimplifies the problem, it provides a concrete description of the phase transition process in terms of a critical point, at which the transition occurs [2, 10, 11].

Two remarks should be added at this stage. The first concerns the use of more advanced approaches than the RMF for the study of the nuclear liquid-gas phase transition. For example, the use of the Relativistic Brueckner–Hartree–Fock approach [11]. Calculating RBHF at a finite temperature is numerically not an easy task. Furthermore, correlation effects disappear beyond $T \sim 3$ MeV. There is no need to do RBHF calculations in order to study the liquid-gas phase transition, which occurs at a much higher temperature. RMF is suitable for the study of phase transitions in nuclear matter.

The second remark concerns relativistic dynamical calculations, where the transition is described as two-dimensional in the case of asymmetric nuclear matter [2]. Ref. [2] assigns each phase a different asymmetry due to different dynamics, adding an extra degree of freedom. In the statistical calculations presented, the asymmetry is fixed, *i.e.*, has the same value for both phases. The effects of allowing gas and liquid phases to have different asymmetry values on the transition will be discussed at the end of this section.

Figure 2 shows the pressure as a function of baryon density at different temperatures for symmetric matter. Figure 2 indicates the occurrence of a phase transition at a temperature close to 15 MeV.

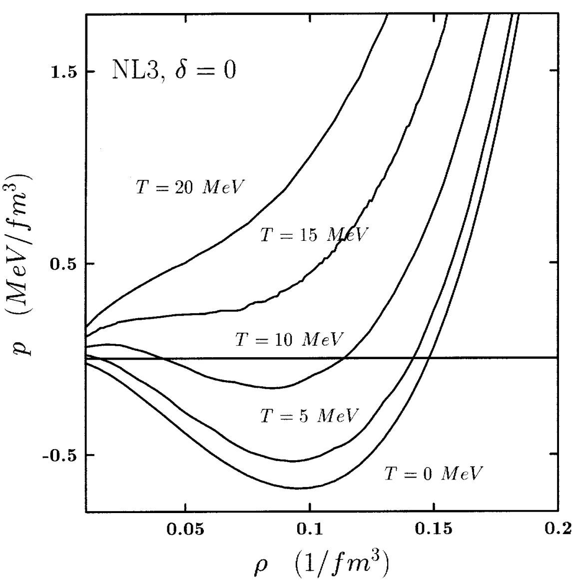

Fig. 2. Pressure as a function of density at different temperatures for symmetric matter.

The critical temperature T_c of the nuclear liquid-gas phase transition is determined within ± 0.5 MeV. The critical density ρ_c is the density, where the function $p(\rho_B, T_c)$ has its turning point, and is determined within $\pm 0.01 \mathrm{fm}^{-3}$. The critical pressure p_c is the pressure at ρ_c and T_c, and is determined within $\pm 0.05 \mathrm{MeV/fm}^3$. The one-nucleon critical excitation energy E_c^*/A is the one-nucleon excitation energy at the critical point, and is determined within

± 0.5 MeV. Table II lists the critical properties observed for different values of the asymmetry parameter. Notice the decrease of all critical quantities with increasing asymmetry. This is in accordance with non-relativistic [7] and relativistic dynamical [2] results.

TABLE II

Critical properties observed using the parameter set NL3 of Table I for different values of the asymmetry parameter δ. $T_{\rm c}$ is the critical temperature, $\rho_{\rm c}$ the critical density, $p_{\rm c}$ the critical pressure, and $E^*_{\rm c}/A$ the critical excitation energy per particle.

	$\delta = 0.0$	$\delta = 0.2$	$\delta = 0.5$
$T_{\rm c}$ (MeV)	15.0	14.0	11.5
$\rho_{\rm c}(1/{\rm fm}^3)$	0.05	0.05	0.04
$p_{\rm c}({\rm MeV/fm}^3)$	0.20	0.20	0.15
$E^*_{\rm c}/A$ (MeV)	15.5	14.5	11.0

The phase transition occurs at an excitation energy of 15–16 MeV per nucleon for symmetric matter, and a density of 0.3–0.4 nuclear matter saturation density. To compare this results with the experimental values of Ref. [9], *i.e.*, excitation energy of 10 MeV per nucleon and a density of 0.15–0.30 normal nuclear density, one should notice that finite nuclei are composed of a limited number of nucleons. This has a broadening effect on the phase transition, which results in a reduction of the critical temperature. Furthermore, finite nuclei are not surrounded by an external pressure field, and will expand prior to their disassembly. This results in a reduction of critical density.

Beyond the critical temperature, nucleon–nucleon forces are no more able to affect the thermal behavior of the nucleons. The pressure grows monotonically with density. For neutron matter this is the case even at $T = 0$. The liquid-gas phase transition cannot be observed in the case of neutron matter, *i.e.*, neutron matter is stable against the transition. This result is identical with the result of the dynamical calculations of Ref. [2]. A maximum value of the asymmetry parameter $\delta_{\rm max}$ exists, ca. 0.9, beyond which nuclear matter is stable against the transition.

Figure 3 shows the one-nucleon entropy as a function of temperature under constant pressure. To clarify the results, σ/T is depicted. The transition is first order. Amount of latent heat:

$$Q_{\rm L} = T_{\rm c}\ \Delta\sigma \tag{20}$$

has to be transferred to the system during the transition, where $\Delta\sigma$ is the entropy change at the critical temperature. Figures 4 and 5 show that the transition is first order in the case of asymmetric matter too. Allowing gas and liquid phases to have different asymmetry values, as done in Ref. [2], smears the S-shaped curves seen in figures 4 and 5, leading to a second order transition in the case of asymmetric nuclear matter.

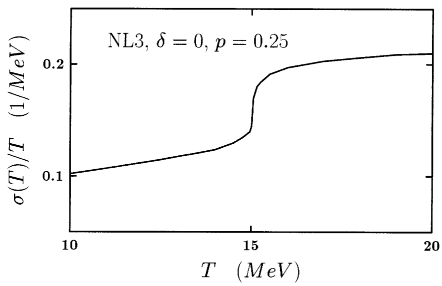

Fig. 3. Specific entropy as a function of temperature under constant pressure for symmetric matter. To clarify the results, σ/T is depicted.

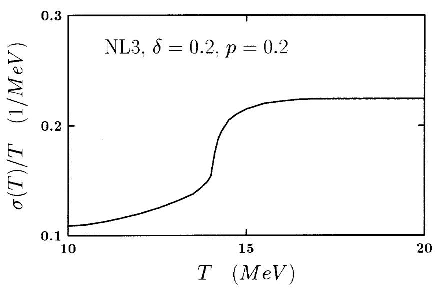

Fig. 4. Similar to figure 5, but for $\delta = 0.2$.

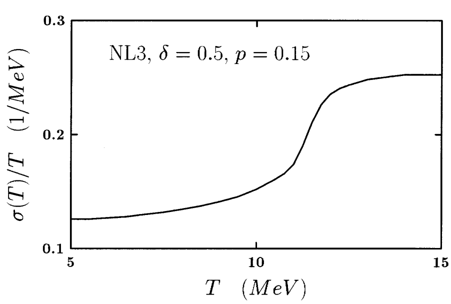

Fig. 5. Similar to figure 5, but for $\delta = 0.5$.

4. Summary

At a given temperature T the one-nucleon excitation energy is higher at lower densities.

The statistical description via the grandcanonical potential is used to derive the nuclear matter equation of state. The liquid-gas phase transition occurs at an excitation energy of 15–16 MeV per nucleon, and a density of 0.3–0.4 of symmetric matter saturation density, in accordance with experimental observations.

All critical quantities decrease with increasing asymmetry. Neutron matter is stable against the transition. A maximum asymmetry value $\delta_{\rm max}$ exists, beyond which nuclear matter is stable against the transition.

The transition is first order for both symmetric and asymmetric matter. Allowing gas and liquid phases to have different asymmetry values leads to a second order transition in the case of asymmetric matter.

The author acknowledges financial support by the Syrian Atomic Energy Commission and its director general Prof. I. Othman.

REFERENCES

[1] R. Donangelo, S.R. Souza, *Phys. Rev.* **C56**, 1504 (1997).
[2] H. Müller, B.D. Serot, *Phys. Rev.* **C52**, 2072 (1995).
[3] G.A. Lalazissis, J. König, P. Ring, *Phys. Rev.* **C55**, 540 (1997).
[4] B.D. Serot, J.D. Walecka, *Adv. Nucl. Phys.* **16**, 1 (1986).
[5] S. Haddad, M.K. Weigel, *J. Phys.* **G20**, 593 (1994).
[6] D. Von-Eiff, M.K. Weigel, *Phys. Rev.* **C46**, 1797 (1992).
[7] C. Das, R. Sahu, R.K. Tripathi, *Phys. Rev.* **C48**, 1056 (1993).
[8] L.D. Landau, E.M. Lifshitz, *Statistical Physics*, Pergamon: Oxford, 1969.
[9] J. Pochodzalla *et al.*, *Phys. Rev. Lett.* **75**, 1040 (1995).
[10] S.J. Lee, A.Z. Mekjian, *Phys. Rev.* **C45**, 1284 (1992); *Phys. Rev.* **C56**, 2621 (1997).
[11] H. Huber, F. Weber, M.K. Weigel, *Phys. Rev.* **C57**, 3484 (1998).