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SPIN STRUCTURE OF THE OCTET BARYONS

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We analyze the semileptonic weak decays of the octet baryons in a *model independent* approach, based on the algebraic structure of the Chiral Quark-Soliton Model. We argue that this analysis is in fact more general than the model itself. While the symmetry breaking for the semileptonic decays themselves is not strong, other quantities like Δs and $\Delta\Sigma$ are much more affected. We calculate $\Delta\Sigma$ and Δq for all octet baryons. Unfortunately, large experimental errors of Ξ^- decays propagate in our analysis, in particular, in the case of $\Delta\Sigma$ and Δs. Only if the errors for these decays are reduced, the accurate theoretical predictions for $\Delta\Sigma$ and Δs will be possible.

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1. Introduction

The experimental results on the first moment of the proton spin structure function g_1^p [1–5] are usually interpreted in terms of the exact SU(3) symmetry. Then, in contrast to the Ellis–Jaffe sum rule [6], the strange quark contribution to the nucleon spin deviates from zero. The global fit performed by Ellis and Karliner [7] gives $\Delta s = -0.11 \pm 0.03$. For more recent analysis, see Refs. [8, 9].

Polarized structure functions have been studied within perturbative QCD (see for review [10] and recent papers [8, 9]). However, in order to get full information on the moments of the polarized quark distributions, an extra input from the low energy semileptonic decays is needed. It is precisely here, where the SU(3) symmetry is assumed. In this paper we shall study the influence of the SU(3) symmetry breaking in this low energy sector on the Δq's and spin content of the octet baryons.

One piece of information comes from the first moment of the spin structure function $g_1^p(x)$ of the proton:

$$I_p = \int\limits_0^1 dx\, g_1^p(x) = \frac{1}{18}\left(4\Delta u_p + \Delta d_p + \Delta s_p\right)\left(1 - \frac{\alpha_{\rm s}}{\pi} + \ldots\right). \tag{1}$$

The analysis of Karliner and Lipkin [11] implies $I_p = 0.124 \pm 0.011$ which can be translated into:

$$\Gamma_p \equiv 4\Delta u_p + \Delta d_p + \Delta s_p = 2.56 \pm 0.23 \tag{2}$$

if $\alpha_s(Q^2 = 3\ ({\rm GeV}/c)^2) = 0.4$ is assumed. Let us for completeness quote also the result for the neutron:

$$\Gamma_n \equiv 4\Delta d_p + \Delta u_p + \Delta s_p = -0.928 \pm 0.186\,, \tag{3}$$

where the isospin symmetry (Bjorken sum rule) has been assumed.

Another piece of information comes from the semileptonic decays, which in the case of the exact SU(3) symmetry can be parametrized by two reduced matrix elements F and D. Taking for $F = 0.46$ and for $D = 0.80$ together with Eq. (2), one gets for the proton: $\Delta u_p = 0.79$, $\Delta d_p = -0.47$ and $\Delta s_p = -0.13$, which implies $\Delta\Sigma_p = 0.19$, quite a small number as compared with the naive expectation from the quark model: $\Delta\Sigma_p = 1$.

It is important to realize that $\Delta\Sigma_p$ *is not directly measured*; it is extracted from the data through some theoretical model. The standard way to calculate $\Delta\Sigma_p$ is to assume the SU(3) symmetry for the semileptonic decays. In this case it is enough to take *any* two decays and Γ_p of Eq. (2) as an input. Normally, as in the example above, one uses neutron beta decay and Σ^- decay as an input. However, if the SU(3) symmetry breaking was not important, any pair out of six known semileptonic decays should give roughly the same number for $\Delta\Sigma_p$. This is, however, *not* the case. As we shall see in the next section, $\Delta\Sigma_p$ can be any number between 0.02 and 0.30. These numbers do not take into account the experimental errors, therefore, as shown in figure 1, the uncertainty of $\Delta\Sigma_p$ due to the SU(3) symmetry breaking in the semileptonic decays is even larger. This is the key observation which motivated this work.

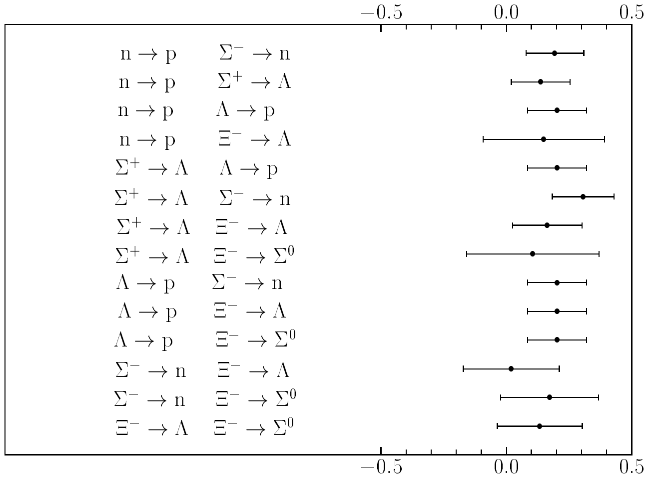

Fig. 1. $\Delta\Sigma_p$ with Γ_p and different semileptonic decays taken as an input in the SU(3) symmetry limit.

It is practically impossible to analyze the SU(3) symmetry breaking in weak decays without resorting to some specific model [11]. In this paper, following Refs. [12, 13], we will use the Chiral Quark-Soliton Model (χQSM) [14,15] (see Ref. [16] for review) to implement the symmetry breaking due to the non-zero strange quark mass. This model satisfactorily describes the axial-vector properties of the hyperons [17–20]. Since the symmetry breaking pattern of the χQSM is identical to the one derived in large N_c QCD [21], our analysis is in fact much more general than the model itself.

However, since $g_A^{(0)}(B)$ does not correspond to the SU(3) octet axial-vector current, it is an independent quantity in QCD and it cannot be expressed in terms of F and D without some further assumptions. The χQSM (as most of the hedgehog models [22]) has a remarkable virtue of connecting the singlet axial-vector constant with $g_A^{(3)}$ and $g_A^{(8)}$, and the semileptonic decay constants in a direct manner. This connection introduces a model dependence into our analysis. However, as we discussed in our previous paper on the proton spin structure [12] and on the Λ spin [13], and as will be shown in Section 5.1, there is no significant numerical difference between the results obtained with and without this model dependent ingredient. It cannot be checked whether this remains true for other baryons because of the lack of the data which could be additionally used if the model formula for $g_A^{(0)}(B)$ is abandoned.

In Section 2.5 we give an additional theoretical argument in favor of the model prediction for $g_A^{(0)}(B)$.

In the previous papers [12,13] we have shown how the symmetry breaking influences the determination of $\Delta\Sigma_{p,\Lambda}$ from the existing data on the weak semileptonic baryon decays. The main goal of the present paper is to extend this analysis to the other members of the octet using the same *model independent* method and to give a self-contained description of this method presenting the details omitted in the previous publications. In the χQSM semileptonic decays are effectively parametrized by 6 constants which are in principle calculable within the model [17]. However, in the present *model independent* analysis they are treated as free parameters. By adjusting them to the experimentally known semileptonic decays we allow not only for maximal phenomenological input but also for minimal model dependence. In Refs. [20, 24–26] magnetic moments of the octet and decuplet have been studied in this way. Model calculations for the vector-axial properties of baryons have been presented in Ref. [20]. There also exist direct model calculations of the spin polarization function itself [27, 28].

Although the spin content of the hyperons will be most probably not directly measured (with an exception of Λ where spin structure function can be related to the measured fragmentation function [29, 30]), there is a substantial theoretical interest in the spin properties of the hyperons. We find that despite the fact that the symmetry breaking for the semileptonic decays themselves is not strong, other quantities like Δs and $\Delta\Sigma$ are much more affected. We observe splitting of $\Delta\Sigma$ for different baryons. Unfortunately our analysis suffers from large errors which are mainly due to the experimental errors of the Ξ^- decays. It is therefore of utmost importance to measure these two decays with higher precision.

The paper is organized as follows: In Section 2 we recall the SU(3) symmetry results and discuss various ways of determining $\Delta\Sigma$ and separately Δq's. In Section 3, following Ref. [23], we recall the main properties of the χQSM with special emphasis on the mass splittings, which we subsequently use in Section 4 to parametrize the SU(3) breaking of the semileptonic weak decays. In Section 5 numerical analysis is carried out and the conclusions are given in Section 6.

2. SU(3) symmetry at work

Let us first briefly recall how the standard analysis is carried out. Three diagonal axial-vector coupling constants define the integrated polarized quark densities for a given baryon B:

$$\begin{aligned} g_{\mathrm{A}}^{(3)}(B) &= \Delta u_B - \Delta d_B\,, \\ \sqrt{3}g_{\mathrm{A}}^{(8)}(B) &= \Delta u_B + \Delta d_B - 2\Delta s_B\,, \\ g_{\mathrm{A}}^{(0)}(B) &= \Delta u_B + \Delta d_B + \Delta s_B\,. \end{aligned} \tag{4}$$

Note that in our normalization $g_{\rm A}^{(0)}(B) = \Delta\Sigma_B$.

Assuming the SU(3) symmetry, one can calculate $g_{\rm A}^{(3,8)}(B)$ in terms of the reduced matrix elements F and D:[1]

$$\begin{aligned} g_{\rm A}^{(3)}(p) &= F+D\,, & \sqrt{3}g_{\rm A}^{(8)}(p) &= 3F-D\,,\\ g_{\rm A}^{(3)}(\Lambda) &= 0\,, & \sqrt{3}g_{\rm A}^{(8)}(\Lambda) &= -2D,\\ g_{\rm A}^{(3)}(\Sigma^+) &= 2F\,, & \sqrt{3}g_{\rm A}^{(8)}(\Sigma^+) &= 2D\,,\\ g_{\rm A}^{(3)}(\Xi^0) &= F-D\,, & \sqrt{3}g_{\rm A}^{(8)}(\Xi^0) &= -3F-D\,. \end{aligned} \tag{5}$$

At this stage $g_{\rm A}^{(0)} = \Delta\Sigma$ is an independent quantity and it is identical for all octet states. These equations together with (4) allow one to express Δq's in terms of D, F and $\Delta\Sigma$:

$$\begin{aligned} \Delta u_p &= \tfrac{1}{3}\ (D+3F+\Delta\Sigma)\ ,\\ \Delta d_p &= \tfrac{1}{3}\ (-2D+\Delta\Sigma)\ ,\\ \Delta s_p &= \tfrac{1}{3}\ (D-3F+\Delta\Sigma)\ ,\\ \Delta u_\Lambda &= \tfrac{1}{3}\ (-D+\Delta\Sigma)\ ,\\ \Delta s_\Lambda &= \tfrac{1}{3}\ (2D+\Delta\Sigma)\ ,\\ \Delta u_{\Sigma^0} &= \tfrac{1}{3}\ (D+\Delta\Sigma)\ . \end{aligned} \tag{6}$$

The SU(3) symmetry imposes certain relations between Δq's of different flavor for different baryons:

$$\begin{aligned} \Delta u_p &= \Delta u_{\Sigma^+} = \Delta s_{\Xi^0}\ ,\\ \Delta d_p &= \Delta s_{\Sigma^+} = \Delta u_{\Xi^0}\ ,\\ \Delta s_p &= \Delta d_{\Sigma^+} = \Delta d_{\Xi^0}\ , \end{aligned} \tag{7}$$

so that Δq's given in Eq. (6) are the only independent ones in the SU(3) symmetry limit. In addition we have the isospin relations

$$\begin{aligned} \Delta u_p &= \Delta d_n\,, & \Delta d_p &= \Delta u_n\,, & \Delta s_p &= \Delta s_n\,,\\ \Delta u_{\Sigma^+} &= \Delta d_{\Sigma^-}\,, & \Delta d_{\Sigma^+} &= \Delta u_{\Sigma^-}\,, & \Delta u_{\Sigma^0} &= \Delta d_{\Sigma^0}\\ \Delta u_\Lambda &= \Delta d_\Lambda\,, & \Delta s_{\Sigma^+} &= \Delta s_{\Sigma^-} = \Delta s_{\Sigma^0}\,, & &\\ \Delta u_{\Xi^0} &= \Delta d_{\Xi^-}\,, & \Delta d_{\Xi^0} &= \Delta u_{\Xi^-}\,, & \Delta s_{\Xi^0} &= \Delta s_{\Xi^-} \end{aligned} \tag{8}$$

which remain still valid after the inclusion of the SU(3) symmetry breaking.

In order to find the numerical values of Δq's one considers different scenarios which we shortly discuss in the following.

[1] Note that $g_{\rm A}^{(3)}$ is proportional to I_3 (third component of the isospin which we assume to take the highest value).

2.1. Naive quark model

In the naive quark model there exist two relations between the constants F and D :

$$\frac{F}{D} = \frac{2}{3}\,, \quad F + D = \frac{5}{3} \longrightarrow F = \frac{2}{3}\,, \quad D = 1\,. \tag{9}$$

Moreover, one assumes that the total spin is carried by the quarks, *i.e.*:

$$\Delta\Sigma = 1\,. \tag{10}$$

With these parameters one gets $\Delta s_p = 0$. Values for all Δq's and Γ_p are presented in Table I. The prediction for Γ_p is, however, very bad, about twice the experimental value.

TABLE I

The results for Δq's, $\Delta\Sigma$ and Γ_p for various phenomenological inputs (denoted by a $*$) in the case of the exact SU(3) symmetry.

	NRQM	Ellis & Jaffe		$\Gamma_{\rm p} = 2.56$		χQSM	
		A_1, A_4	average	A_1, A_4	average	A_1, A_4	average
D	^{*1}	$^*0.80$	$^*0.77$	$^*0.80$	$^*0.77$	$^*0.80$	$^*0.77$
F	$^*2/3$	$^*0.46$	$^*0.50$	$^*0.46$	$^*0.50$	$^*0.46$	$^*0.50$
Δu_p	4/3	0.92	1.00	0.79	0.81	0.77	0.98
Δd_p	−1/3	−0.34	−0.27	−0.47	−0.47	−0.49	−0.29
Δs_p	0	^{*0}	^{*0}	−0.13	−0.20	−0.15	−0.02
Δu_Λ	0	−0.07	−0.01	−0.20	−0.21	−0.22	−0.03
Δs_Λ	1	0.76	0.76	0.60	0.56	0.58	0.74
Δu_{Σ^0}	2/3	0.50	0.50	0.33	0.30	0.31	0.48
$\Delta\Sigma$	^{*1}	0.58	0.74	0.19	0.14	0.13	0.68
Γ_p	5	3.34	3.75	$^*2.56$	$^*2.56$	2.44	3.63

2.2. Extracting F and D from the semileptonic weak decays

Certainly these naive quark model values (9) are not realistic. One can do better by extracting F and D from experiment. For example, assuming the exact SU(3) symmetry, one has

$$A_1 = \left(\frac{g_1}{f_1}\right)^{(n\to p)} = F + D\,, \quad A_4 = \left(\frac{g_1}{f_1}\right)^{(\Sigma^-\to n)} = F - D\,. \tag{11}$$

For convenience, we denote the ratios of axial-vector to vector decay constants by A_i (see Table III). Taking for these decays the experimental values, one obtains

$$F = 0.46 \quad \text{and} \quad D = 0.80\,, \tag{12}$$

as displayed in the column (A_1, A_4) in Table I.

One could, however, use any two A_i's out of six known weak semileptonic decays to extract F and D. The number of combinations is fourteen (actually fifteen, but two conditions are linearly dependent). Taking these fourteen combinations into account, one gets:

$$F = 0.40 \div 0.55, \quad D = 0.70 \div 0.89\,. \tag{13}$$

These are the uncertainties of the *central values* due to the theoretical error caused by using the exact SU(3) symmetry to describe the weak semileptonic decays. These uncertainties are further increased by the experimental errors of all individual decays.

Looking at Eq. (13), one might get an impression that a typical error associated with the use of the SU(3) symmetry in analyzing the hyperon decays is of the order of 15% or so. While this is true for the hyperon decays themselves, the values of Δq and $\Delta\Sigma$ for various baryons might be much more affected by the symmetry breaking. Indeed, since

$$\Delta\Sigma = \tfrac{1}{2}\left(\Gamma_p - 3F - D\right) \tag{14}$$

in the SU(3) symmetry limit we get

$$\Delta\Sigma = 0.02 \div 0.30 \tag{15}$$

for F and D corresponding to Eq. (13) and Γ_p as given by Eq. (2). This large uncertainty of the central value of $\Delta\Sigma$ is *entirely* due to the SU(3) symmetry breaking in the hyperon decays. In Fig. 1 we plot $\Delta\Sigma$ together with experimental errors for each pair of the semileptonic decays.

Anticipating the results of Section 4 let us mention that there exist two linear combinations of A_i's which are free of the linear $m_{\rm s}$ corrections in the χQSM (and large $N_{\rm c}$ QCD [21]), namely:

$$\begin{aligned} F &= \frac{1}{12}(4A_1 - 4A_2 - 3A_3 + 3A_4 + 3A_5 + 5A_6)\,, \\ D &= \frac{1}{12}(4A_2 + 3A_3 - 3A_4 - 3A_5 + 3A_6) \end{aligned} \tag{16}$$

which give numerically

$$F = 0.50 \pm 0.07 \quad \text{and} \quad D = 0.77 \pm 0.04\,, \tag{17}$$

as displayed in Table I in the column "average". It is important to note that by adopting this way of extracting F and D in the symmetry limit, no refitting of F and D is required when m_s corrections are added.

In what follows we shall use these two sets — Eqs. (12),(17) — of values for F and D while discussing the predictions for Δq's.

In order to extract all Δq's separately, one needs some additional information. Either another experimental input is needed, or a model which predicts $g_{\mathrm{A}}^{(0)}$ (B) in terms of F and D.

2.3. Conjecture of Ellis and Jaffe

In 1974 Ellis and Jaffe [6] made an assumption, based on the naive quark model that

$$\Delta s_p = 0\,. \tag{18}$$

From our SU(3) formula (6), we see that this amounts to

$$\Delta\Sigma = 3F - D \tag{19}$$

which indeed gives 1 for the naive quark model values (9). For the experimental values of F and D discussed in the previous section we get $\Delta\Sigma$ around 0.6 as displayed in Table I. Unfortunately, the value of Γ_p is much larger than the experimental value.

2.4. Linking hyperon decays with the high energy data

Instead of using the low energy data alone, one can also use the high energy data on the first moment of the polarized structure function of the proton (1) with $\Gamma_p = 2.56$. The results of such fits for two choices of F and D constants are presented in columns 5 and 6 of Table I. A striking feature of these fits is that the resulting $\Delta\Sigma$ is very small. This fact is often referred to as a *spin crisis.*

2.5. Chiral Quark Soliton Model

As will be shown in the following, the χQSM predicts in the SU(3) symmetry limit [18]:

$$\Delta\Sigma = 9F - 5D \tag{20}$$

for all octet baryons. This formula has a remarkable feature: It interpolates between the naive quark model and the Skyrme model. Indeed, for (9) $\Delta\Sigma = 1$, whereas in the case of the simplest Skyrme model for which $F/D = 5/9$, $\Delta\Sigma = 0$, as observed for the first time in Ref. [32].

Here $\Delta\Sigma$ is very sensitive to small variations of F and D, since it is a difference of the two, with relatively large coefficients. Indeed, for the 14 fits mentioned before Eq. (13) the central value for $\Delta\Sigma$ varies between -0.25 to approximately 1. Thus, despite the fact that the hyperon semileptonic decays are relatively well described by the model in the SU(3) symmetry limit, the singlet axial-vector constant is basically undetermined. This is a clear signal of the importance of the symmetry breaking for this quantity.

In fact, conclusions similar to ours have been obtained in chiral perturbation theory in Ref. [33].

3. Mass splittings in the χQSM

In this section we shall briefly recall how the model parameters are fixed. Because of the SU(3) symmetry breaking due to the strange quark mass m_s the collective baryon Hamiltonian is no longer SU(3)-symmetric. Indeed [34]:

$$\hat{H} = \hat{H}_0 + \hat{H}' \,, \tag{21}$$

where

$$\hat{H}_0 = M_{\rm sol} + \frac{1}{2I_1} S(S+1) + \frac{1}{2I_2}\left(C_2({\rm SU}(3)) - S(S+1) - \frac{N_c^2}{12}\right) \tag{22}$$

and

$$\hat{H}' = m_s \left(\alpha D_{88}^{(8)} + \beta \hat{Y} + \frac{\gamma}{\sqrt{3}} \sum_{A=1}^{3} D_{8A}^{(8)} \hat{S}_A\right) . \tag{23}$$

Here $\hat{S}_A$ denotes baryon spin, $C_2({\rm SU}(3))$ the Casimir operator and $D_{B\,S}^{(\mathcal{R})}$ are the SU(3) Wigner matrices in representation $\mathcal{R}$. Constants α, β and γ are given by Ref. [34]:

$$\alpha = -\sigma + \frac{K_2}{I_2} \,, \quad \beta = -\frac{K_2}{I_2} \,, \quad \gamma = 2\left(\frac{K_1}{I_1} - \frac{K_2}{I_2}\right) . \tag{24}$$

Here K_i and I_i are the "moments of inertia" and σ is related to the nucleon sigma term: $3\sigma = \Sigma/\overline{m}$, $\overline{m}$ being the average mass of the up and down quarks.

The collective splitting Hamiltonian (23) mixes the states in various SU(3) representations. The octet states are mixed with the higher representations such as antidecuplet $\overline{\mathbf{10}}$ and eikosiheptaplet $\mathbf{27}$. In the linear order in m_s the wave function of a state $B = (Y, I, I_3)$ of spin S_3 is given as:

$$\psi_{B,S_3} = (-)^{\frac{1}{2}-S_3}\left(\sqrt{8}\, D_{B\,S}^{(8)\,*} + c_B^{(\overline{10})} \sqrt{10}\, D_{B\,S}^{(\overline{10})\,*} + c_B^{(27)} \sqrt{27}\, D_{B\,S}^{(27)\,*}\right), \tag{25}$$

where $S = (-1, \frac{1}{2}, S_3)$. Mixing parameters $c_B^{(\mathcal{R})}$ can be found for example in Ref. [17]. They are given as products of m_s (which we assume to be 180 MeV) times a known numerical constant $N_B^{(\mathcal{R})}$ depending on the baryonic state B and a dynamical parameter $c_{\mathcal{R}}$. Since $c_{\mathcal{R}}$ depends on the model parameter I_2, which is responsible for the splitting between the octet and higher exotic multiplets [35] and is not constrained from the data we will take them as free parameters in our fits.

4. Semileptonic weak decays in the chiral quark-soliton model

The transition matrix elements of the octet hadronic axial-vector current $\langle B_2|A_\mu^X|B_1\rangle$ can be expressed in terms of three independent form factors:

$$\langle B_2|A_\mu^X|B_1\rangle = \bar{u}_{B_2}(p_2)\left[g_1(q^2)\gamma_\mu - \frac{ig_2(q^2)}{M_1}\sigma_{\mu\nu}q^\nu + \frac{g_3(q^2)}{M_1}q_\mu\right]\gamma_5 u_{B_1}(p_1), \tag{26}$$

where the axial-vector current is defined as

$$A_\mu^X = \bar{\psi}(x)\gamma_\mu\gamma_5\lambda_X\psi(x) \tag{27}$$

with $X = \frac{1}{2}(1 \pm i2)$ for strangeness conserving $\Delta S = 0$ currents and $X = \frac{1}{2}(4 \pm i5)$ for $|\Delta S| = 1$.

The $q^2 = -Q^2$ stands for the square of the momentum transfer $q = p_2 - p_1$. The form factors g_i are real and depend only on Q^2 in the case of the CP-invariant processes. We will neglect g_3 because it is supressed by the ratio $m_l^2/M_1^2 \ll 1$, where m_l is the lepton (e or μ) mass. Similarly we shall neglect g_2. In principle this form factor is proportional to m_s and therefore should be included in the consistent analysis of the weak decays data. Unfortunately, such an analysis is still missing and all experimental results on g_1 assume $g_2 \equiv 0$.

Another possible small m_s corrections come from the evolution of g_1 with Q^2, due to the non-conservation of the axial-vector currents caused by the SU(3) symmetry breaking. These corrections are also neglected in our approach.

Hadronic matrix elements such as $\langle B_2|A_\mu^X|B_1\rangle$ have been throughly studied in the χQSM (see for example [16] and references therein). Taking into account the $1/N_c$ rotational and m_s corrections, we can write the resulting axial-vector constants $g_1^{B_1\to B_2}(0)$ in the following form:

$$\begin{aligned} g_1^{(B_1\to B_2)} = {} & a_1\langle B_2|D_{X3}^{(8)}|B_1\rangle + a_2 d_{pq3}\langle B_2|D_{Xp}^{(8)}\hat{S}_q|B_1\rangle + \frac{a_3}{\sqrt{3}}\langle B_2|D_{X8}^{(8)}\hat{S}_3|B_1\rangle \\ & + m_s\frac{a_4}{\sqrt{3}}d_{pq3}\langle B_2|D_{Xp}^{(8)}D_{8q}^{(8)}|B_1\rangle \\ & + m_s a_5\langle B_2|\left(D_{X3}^{(8)}D_{88}^{(8)} + D_{X8}^{(8)}D_{83}^{(8)}\right)|B_1\rangle \\ & + m_s a_6\langle B_2|\left(D_{X3}^{(8)}D_{88}^{(8)} - D_{X8}^{(8)}D_{83}^{(8)}\right)|B_1\rangle\,, \end{aligned} \tag{28}$$

where a_i denote parameters depending on the specific dynamics of the chiral soliton model. Their explicit form in the χQSM can be found in Ref. [17].

Analogously to Eq. (28) one defines the diagonal axial-vector couplings. In that case X can take two values: $X=3$ and $X=8$. For $X=0$ (singlet axial-vector current) we have the following expression [17, 18]:

$$g_B^{(0)}\,\hat{S}_3 = a_3\,\hat{S}_3 + \sqrt{3}m_s(a_5-a_6)\,\langle B|D_{83}^{(8)}|B\rangle. \tag{29}$$

From theoretical point of view it is interesting to anlyze the N_c dependence of the a_i's [36, 37]. Constants a_2 and a_3 are both subleading in $1/N_\mathrm{c}$ and come from the anomalous part of the effective Euclidean action. In the Skyrme model they are related to the Wess–Zumino term. However in the simplest version of the Skyrme model (which is based on the pseudo-scalar mesons only) $a_3=0$ identically [32]. In the case of the χQSM $a_3\neq 0$ and it provides a link between the SU(3) octet of axial-vector currents and the singlet current of Eq. (29). It was shown in Ref. [31] that in the limit of the artificially large soliton, which corresponds to the "Skyrme limit" of the present model, $a_3/a_1\to 0$ in agreement with [32]. On the contrary, for the small solitons $g_p^{(0)}\to 1$ reproducing the result of the non-relativistic quark model.

Instead of calculating 7 dynamical parameters a_i and I_2 (or $c_{\overline{10}}$ and c_{27}) within the χQSM (what was done in Ref. [20]), we shall fit them from the weak semileptonic decay data. It is convenient to introduce the following set of new parameters:

$$r=\frac{1}{30}\left(a_1-\frac{1}{2}a_2\right),\qquad s=\frac{a_3}{60},\quad x=\frac{m_s\,a_4}{540},\quad y=\frac{m_s\,a_5}{90},\quad z=\frac{m_s\,a_6}{30},$$

$$p=\frac{1}{6}m_s\,c_{\overline{10}}\left(a_1+a_2+\frac{1}{2}a_3\right),\quad q=-\frac{1}{90}m_s\,c_{27}\left(a_1+2a_2-\frac{3}{2}a_3\right). \tag{30}$$

Employing this new set of parameters, we can immediately express all possible semileptonic decay constants between the octet baryons:

$$\begin{aligned}
\left(\frac{g_1}{f_1}\right)^{(n\to p)} &= -14r+2s-44x-20y-4z-4p+8q,\\
\left(\frac{g_1}{f_1}\right)^{(\Sigma^+\to\Lambda)} &= -9r-3s-42x-6y-3p+15q,\\
\left(\frac{g_1}{f_1}\right)^{(\Lambda\to p)} &= -8r+4s+24x-2z+2p-6q,\\
\left(\frac{g_1}{f_1}\right)^{(\Sigma^-\to n)} &= 4r+8s-4x-4y+2z+4q,\\
\left(\frac{g_1}{f_1}\right)^{(\Xi^-\to\Lambda)} &= -2r+6s-6x+6y-2z+6q,
\end{aligned}$$

$$\begin{aligned}
\left(\frac{g_1}{f_1}\right)^{(\Xi^-\to\Sigma^0)} &= -14r+2s+22x+10y+2z+2p-4q,\\
\left(\frac{g_1}{f_1}\right)^{(\Sigma^-\to\Lambda)} &= -9r-3s-42x-6y-3p+15q,\\
\left(\frac{g_1}{f_1}\right)^{(\Sigma^-\to\Sigma^0)} &= -5r+5s-18x-6y+2z-2p,\\
\left(\frac{g_1}{f_1}\right)^{(\Xi^-\to\Xi^0)} &= 4r+8s+8x+8y-4z-8q,\\
\left(\frac{g_1}{f_1}\right)^{(\Xi^0\to\Sigma^+)} &= -14r+2s+22x+10y+2z+2p-4q\,. \qquad (31)
\end{aligned}$$

The U(3) axial-vector constants $g_A^{(0,3,8)}$ can be also expressed in terms of the new set of parameters Eq. (30). For the triplet ones[2] we have:

$$\begin{aligned}
g_{\rm A}^{(3)}(p) &= -14r+2s-44x-20y-4z-4p+8q,\\
g_{\rm A}^{(3)}(\Lambda) &= 0,\\
g_{\rm A}^{(3)}(\Sigma^+) &= -10r+10s-36x-12y+4z-4p,\\
g_{\rm A}^{(3)}(\Xi^0) &= 4r+8s+8x+8y-4z-8q, \qquad (32)
\end{aligned}$$

and for the octet ones, we get:

$$\begin{aligned}
g_{\rm A}^{(8)}(p) &= \sqrt{3}(-2r+6s+12x+4p+24q),\\
g_{\rm A}^{(8)}(\Lambda) &= \sqrt{3}(6r+2s-36x+36q),\\
g_{\rm A}^{(8)}(\Sigma^+) &= \sqrt{3}(-6r-2s+20x+8y+4p+16q),\\
g_{\rm A}^{(8)}(\Xi^0) &= \sqrt{3}(8r-4s-24x-12y+24q). \qquad (33)
\end{aligned}$$

As already explained in the Introduction the model provides a link between the octet currents and the singlet axial current. For the singlet axial-vector constants, we have:

$$\begin{aligned}
g_{\rm A}^{(0)}(p) &= 60s-18y+6z,\\
g_{\rm A}^{(0)}(\Lambda) &= 60s+54y-18z\,,\\
g_{\rm A}^{(0)}(\Sigma) &= 60s-54y+18z\,,\\
g_{\rm A}^{(0)}(\Xi) &= 60s+72y-24z\,. \qquad (34)
\end{aligned}$$

[2] Triplet $g^{(3)}$'s are proportional to I_3, formulae in Eq. (32) correspond to the highest isospin state.

Let us note that by redefinition of q and x we can get rid of the variable p:

$$x' = x - \frac{1}{9}p\,, \qquad q' = q - \frac{1}{9}p\,. \tag{35}$$

In the chiral limit parameters x, y, z, p and q vanish and we recover the SU(3) symmetric relations from Section 2 with

$$D = -3\,s - 9\,r, \quad F = 5\,s - 5\,r\,, \tag{36}$$

from which Eq. (20) follows.

5. The SU(3) symmetry breaking

We fix the newly-defined set of parameters from the experimental data on the semileptonic decays. Their numerical values are given in Table III. We do not quote the experimental errors on these parameters, since they are highly correlated and cannot be used directly to calculate the errors of the physical quantities of interest. Instead, we expressed all observables directly in terms of the A_i's. This is, however, not enough since, as in the chiral limit, the extra input is needed.

TABLE II

Δs_p, $\Delta\Sigma_p$ and $\Gamma_{p,n}$ for various phenomenological inputs (denoted by a $*$) in the case of the broken SU(3) symmetry.

	NRQM	Ellis & Jaffe	$\Gamma_p = 2.56$	χQSM
$\Delta\Sigma_p$	^{*1}	-0.47	0.56	0.51
Δs_p	0.49	^{*0}	0.31	0.32
Γ_p	3.65	0.71	$^*2.56$	2.67
Γ_n	-0.12	-3.06	-1.21	-1.10

At this point a necessity of a complete description of the symmetry breaking is clearly seen. The strange quark mass causes all SU(3) symmetry relations (7) to break. So in principle one needs one extra experimental input for each isospin multiplet. Let us first discuss the case of the nucleon.

5.1. Spin content of the nucleon

We shall repeat here the analysis of Section 2, however, with the symmetry breaking taken into account. Again four different choices for an additional input will be considered: (1) $\Delta\Sigma_p = 1$, (2) $\Delta s_p = 0$, (3) $\Gamma_p = 2.56$

and (4) the χQSM formulae (34) for $g_{\rm A}^{(0)}$. The results are summarized in Table II. It can be immediately seen that the first two possibilities are in contradiction with experimental data on Γ_p and $\Gamma_{\rm n}$. On the other hand, if we use the experimental value of Γ_p as an additional input (but no model formula (34) for $g_{\rm A}^{(0)}$), or alternatively the χQSM prediction for $g_{\rm A}^{(0)}$, the results are almost indistinguishable. This gives a numerical support for the correctness of the χQSM formula for the axial-vector singlet current with the SU(3) symmetry breaking.

TABLE III

Model parameters $r,\ldots,q'$ extracted from the data together with the predictions for the semileptonic decays and $\Gamma_{p,n}$ in the case of the exact SU(3) and broken SU(3). Results for A_i's with m_s corrections correspond to the experimental data [39].

		exact SU(3)	broken SU(3)
	r	-0.0892	-0.0892
	s	0.0113	0.0113
	x'	0	-0.0055
	y	0	0.0080
	z	0	-0.0038
	q'	0	-0.0140
A_1	$(g_1/f_1)^{n\to p}$	1.271 ± 0.11	1.2573 ± 0.0028
A_2	$(g_1/f_1)^{\Sigma^+\to\Lambda}$	0.769 ± 0.04	0.742 ± 0.018
A_3	$(g_1/f_1)^{\Lambda\to p}$	0.758 ± 0.08	0.718 ± 0.015
A_4	$(g_1/f_1)^{\Sigma^-\to n}$	-0.267 ± 0.04	-0.340 ± 0.017
A_5	$(g_1/f_1)^{\Xi^-\to\Lambda}$	0.246 ± 0.07	0.25 ± 0.05
A_6	$(g_1/f_1)^{\Xi^-\to\Sigma^0}$	1.271 ± 0.11	1.278 ± 0.158
	Γ_p	3.63 ± 1.12	2.67 ± 0.33
	Γ_n	-0.19 ± 0.84	-1.10 ± 0.33

Of course the results of Table II have to be taken with a bit of care because of large experimental errors which are not displayed. As we have argued in Ref. [12], one could still accommodate $\Delta s_p = 0$ due to the large errors of Ξ decays. We shall come back to this point in the following.

5.2. Numerical results

It the present section we shall present the numerical results of our analysis based on the Chiral Quark Soliton Model with the SU(3) symmetry breaking. Our strategy is very simple: using model parametrization (31) we expressed Δq's and $\Delta\Sigma$'s in terms of the six known weak semileptonic decays. Errors are added in quadrature. The numerical results are summarized in Table IV and in figures 2–9. To guide an eye it is convenient to restore the linear $m_{\rm s}$ dependence for the quark densities in the following way:

$$\Delta q = \Delta q^{(0)} + \frac{m_{\rm s}}{180\,{\rm MeV}}\left(\Delta q - \Delta q^{(0)}\right)\,, \tag{37}$$

and similarly for $\Delta\Sigma$. This is possible because our chiral parameters r and s do not need to be refitted as the symmetry breaking corrections are included. In order to display the errors which come from the experimental errors of the weak decays, at both ends of each figure we also plot the theoretical predictions as black dots together with the error bars.

TABLE IV

Integrated polarized quark densities for various baryons.

	exact SU(3)	broken SU(3)
$\Delta u_{\rm p} = \Delta d_{\rm n}$	0.98 ± 0.23	0.72 ± 0.07
$\Delta d_{\rm p} = \Delta u_{\rm n}$	-0.29 ± 0.13	-0.54 ± 0.07
$\Delta s_{\rm p} = \Delta s_{\rm n}$	-0.02 ± 0.09	0.33 ± 0.51
$\Delta u_{\Lambda} = \Delta d_{\Lambda}$	-0.03 ± 0.14	-0.02 ± 0.17
Δs_{Λ}	0.74 ± 0.17	1.21 ± 0.54
$\Delta u_{\Sigma^+} = \Delta d_{\Sigma^-}$	0.98 ± 0.23	0.73 ± 0.17
$\Delta d_{\Sigma^+} = \Delta u_{\Sigma^-}$	-0.02 ± 0.09	-0.37 ± 0.19
$\Delta s_{\Sigma^+} = \Delta s_{\Sigma^-} = \Delta s_{\Sigma^0}$	-0.29 ± 0.13	-0.18 ± 0.39
$\Delta u_{\Sigma^0} = \Delta d_{\Sigma^0}$	0.48 ± 0.16	0.18 ± 0.08
$\Delta u_{\Xi^0} = \Delta d_{\Xi^-}$	-0.29 ± 0.13	-0.14 ± 0.21
$\Delta d_{\Xi^0} = \Delta u_{\Xi^-}$	-0.02 ± 0.09	0.02 ± 0.16
$\Delta s_{\Xi^0} = \Delta s_{\Xi^-}$	0.98 ± 0.23	1.50 ± 0.60

Let us first comment on the results on Γ_p and $\Gamma_{\rm n}$. We see from Table III that the experimental values are quite well reproduced by the model, provided the $m_{\rm s}$ corrections are included. In the symmetry limit their values are way off from the experimental data.

Next, let us observe that the singlet axial-vector current couplings $g_{\rm A}^{(0)}$ split when the symmetry breaking is switched on. This is due to the term proportional to $D_{83}^{(8)}$ in Eq. (29). This splitting is depicted in Fig. 2. We see that $\Delta\Sigma_p$ shows the weakest $m_{\rm s}$ dependence, whereas $\Delta\Sigma_{\Lambda}$ and $\Delta\Sigma_{\Xi}$ depend quite strongly on $m_{\rm s}$. Large error bars for these quantities are due almost entirely to the large errors of Ξ decays A_5 and A_6. It is however evident from Fig. 2 that Λ and Ξ are much closer to the nonrelativistic limit than p and Σ.

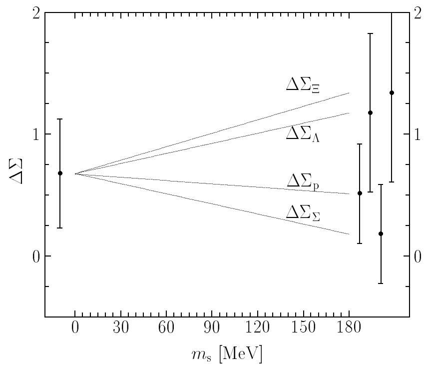

Fig. 2. $\Delta\Sigma_B$ with and without SU(3) symmetry breaking. In the χQSM with m_s dependence restored according to Eq. (37). Black dots denote model predictions (same as lines) with errors coming from the experimental errors of the semileptonic weak decays.

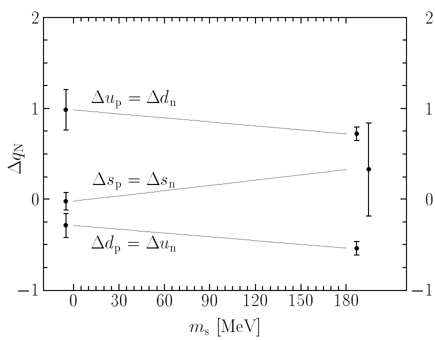

Fig. 3. Δq' for the nucleon; dots and error bars have the same meaning as in Fig. 2.

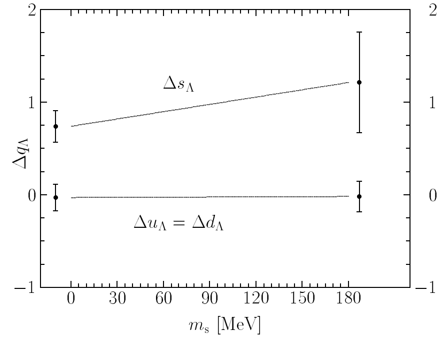

Fig. 4. Δq' for the Λ; dots and error bars have the same meaning as in Fig. 2.

In Figs. 3–6 we plot Δq for the nucleon, Λ, Σ and Ξ, respectively. We see that in all 4 cases Δs rises relatively strongly with m_{s}. It is therefore not justified to extract the strange quark polarization assuming the exact SU(3) symmetry. Unfortunately, Δs's have also the largest errors coming, as in the case of $\Delta\Sigma$, almost entirely from the errors of Ξ decays.

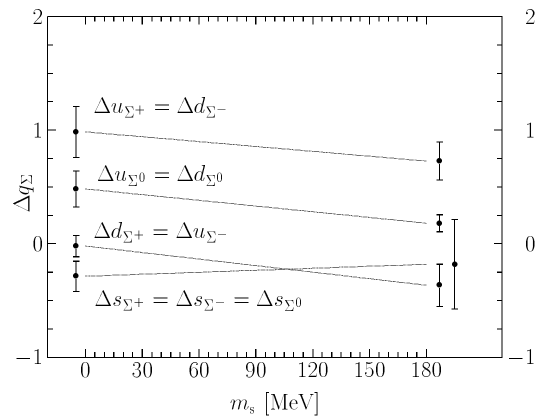

Fig. 5. Δq' for the Σ ; dots and error bars have the same meaning as in Fig. 2.

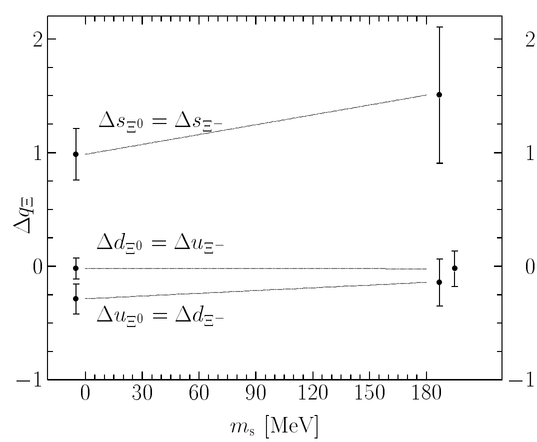

Fig. 6. Δq' for the Ξ ; dots and error bars have the same meaning as in Fig. 2.

In Figs. 7–9 we examine the breaking of the SU(3) relations given by Eqs. (7). Interestingly we find that there is an approximate equality between Δu_p and Δu_{Σ^+} for all values of m_s.

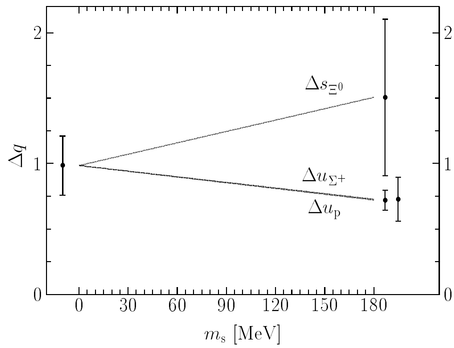

Fig. 7. Breaking of the first SU(3) relation of Eq. (7); dots and error bars have the same meaning as in Fig. 2.

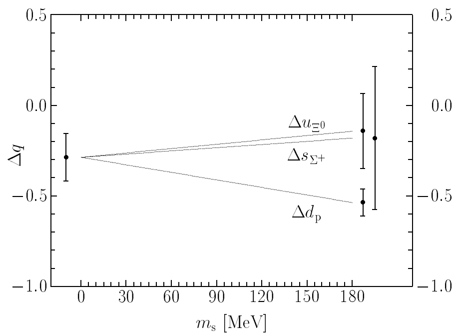

Fig. 8. Breaking of the second SU(3) relation of Eq. (7); dots and error bars have the same meaning as in Fig. 2.

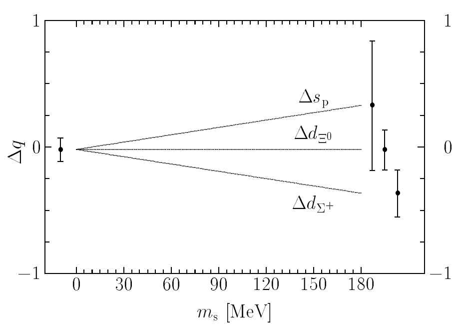

Fig. 9. Breaking of the third SU(3) relation of Eq. (7); dots and error bars have the same meaning as in Fig. 2.

6. Summary and conclusions

In the analysis of the polarized structure function g_1 of the proton and neutron one has to take an additional input from the low energy hyperon decays. Customarily the SU(3) symmetry for these decays is assumed. However, if one takes all possible combinations of the low energy decays the resulting $\Delta\Sigma$ can take any value between 0.02 and 0.30. As depicted in Fig. 1 this range is further increased if the errors coming from the experimental error bars of the semileptonic decays are properly included. This observation implies that the SU(3) symmetry breaking plays an essential role in extracting $\Delta\Sigma$ from the experimental data. It was therefore the aim of this paper to study the influence of the symmetry breaking on the determination of $\Delta\Sigma$ and Δs for the octet baryons in a consistent way.

For this purpose we have performed the "*model-independent*" analysis based on the algebraic structure of the Chiral Quark Soliton Model. In this approach one makes merely use of the algebraical structure of the model, treating the dynamical quantities, which are in principle calculable in the model, as free parameters. Model predictions of the axial-vector properties of the octet baryons have been already calculated elswhere [20]. There are two model ingredients which are of importance. The first one is the model formula for the octet axial-vector currents which have been derived in the linear order in m_s and $1/N_{\rm c}$. Our formulae here have the same algebraical structure as in the large $N_{\rm c}$ QCD [21], and therefore they are more general than the model itself. Secondly, unlike in QCD, the model provides a link between the octet axial-vector currents and the singlet axial-vector current. This connection is a truly model-dependent ingredient, however, we have given the arguments in favor of Eq. (29), based on the fact that apart from the general success of the χQSM in reproducing form factors and parton distributions, in the limit of the small soliton it properly reduces to the Nonrelativistic Quark Model prediction, and in the limit of the large soliton it reproduces the Skyrme Model prediction for $\Delta\Sigma$. Similarly, in Ref. [38] the argument has been given that Eq. (20) naturally emerges in the limit of the large m_s, where the SU(3) flavor symmetry reduces to the SU(2) one. The numerical analysis of Section 5.1 provides a further support for the model formula for $\Delta\Sigma$.

We have presented two parametrizations of all available semileptonic decays. The first one is obtained assuming the SU(3) symmetry, however the two reduced matrix elements F and D were extracted from the combinations of the semileptonic decays which are free of the m_s corrections (16), rather than from the neutron and Σ^- decays alone. The second one is obtained by fitting all 6 measured semileptonic decays in terms of 6 free parameters defined in Eqs. (30), (31). The difference between the two fits, as seen from

Table III, is rather small, except perhaps for the $\Sigma^- \to n$ decay. Despite the fact that the symmetry breaking for the semileptonic decays themselves is not strong, other quantities like Δs and $\Delta\Sigma$ are much more affected by taking into account the effects of the non-zero strange quark mass. This is clearly shown in Figs. 2–9.

Whether this sensitivity is a sign of the breakdown of the perturbative approach to the strangenes, as it was recently suggested in Ref. [38], is hard to say, since our anaysis suffers from large errors which are mainly due to the experimental errors of the Ξ^- decays. It is therefore of utmost importance to measure these two decays with the precision comparable to the other four decays. One should bare in mind that this is one of a few cases, where the low energy data have an important impact on our understanding of the high energy scattering. Given the theoretical implications of these experiments as far as the role of the axial anomaly and the gloun polarization is concerned [8–10], one should make it clear how important the new measurements of the Ξ^- decays would be. This is perhaps the most important message of our analysis.

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