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STOCHASTIC RESONANCE IN A CHAIN OF THRESHOLD ELEMENTS WITH UNIDIRECTIONAL COUPLING AND SPATIOTEMPORAL SIGNAL*

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Stochastic resonance is investigated in a chain of unidirectionally coupled threshold elements driven by independent noises and a plane travelling wave. Both stochastic resonance in an individual element embedded in the chain, characterized by a maximum of the signal-to-noise ratio for nonzero noise intensity, and stochastic resonance with spatiotemporal signal, characterized by a maximum of a spatiotemporal input–output correlation function, are observed. Both kinds of stochastic resonance can be enhanced due to proper coupling, although this effect is weaker than for bidirectional coupling and occurs for a smaller range of wavelengths of the plane wave. The enhancement is related to a maximum spatiotemporal synchronization among elements with the same phase of the periodic signal at input.

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1. Introduction

Stochastic Resonance (SR) [1] is a phenomenon in which noise plays a constructive role by increasing the degree of periodicity of a properly defined output signal in a system driven by a combination of a periodic input signal and noise (for review see [2, 3]). A commonly used measure of SR is the output Signal-to-Noise Ratio (SNR) which shows a maximum as a function of the input noise intensity. Most models of SR are based on

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bistable dynamical systems [4-6] and both dynamical [7] and non-dynamical [8-11] threshold-crossing systems. SR has been also investigated in spatially extended systems [12-26] (for review see [12]), usually in systems of coupled elements with noise uncorrelated in space and time and the input signal uniform in space and periodic in time, *e.g.* chains of diffusively coupled stochastic bistable oscillators [13,14], coupled map lattices [15] and the Ising model [19-22]. In such systems it was found that an optimum value of coupling and optimum noise strength exist such that the maximum of the SNR in every element is significantly enhanced over that in an uncoupled element. This phenomenon is called array enhanced SR [13] and it is related to maximum spatiotemporal synchronization of the elements with the input periodic signal and among themselves.

Recently, SR in spatially extended systems has been also studied with the input signal periodic in space [22, 23] and both in space and time [23-26]. In particular, we investigated a chain of coupled threshold elements with bidirectional symmetric coupling, driven by a plane travelling wave and spatiotemporal noise [26]. Such elements are known to exhibit SR [8-11] and can be used for qualitative simulations of SR in biological neuron models [11]. Apart from the enhancement of SR in an individual element embedded in the chain we demonstrated SR with spatiotemporal signal, characterized by a maximum of a spatiotemporal input–output correlation function, and its enhancement due to proper coupling for a wide range of the spatial wavelengths of the input signal. We also showed that for large wavelengths the enhancement of both kinds of SR coincides with the maximum of spatiotemporal synchronization among elements with the same phase of the periodic signal at input. In this paper we report on similar phenomena in a chain of threshold elements with unidirectional coupling.

2. The system and methods of analysis

We investigate a chain of N coupled threshold elements denoted as i, $i = 0, 1, 2 \ldots N-1$ with two-state output 0 or 1. The coupling is typical of artificial neural networks, unidirectional and constrained to nearest neighbours. The time steps $n = 0, 1, 2 \ldots$ are discrete. The chain is driven by a plane travelling wave with amplitude A, frequency ω_s, period $T_s = 2\pi/\omega_s$, wave vector k and wavelength $\lambda = 2\pi/k$. Besides, the elements are driven by independent white Gaussian noises $\eta_n^{(i)}$ with variance D. The system dynamics is given by

$$\begin{aligned} x_{n+1}^{(i)} &= \Theta\left[A\sin\left(\omega_s n - ki + \phi\right) + \eta_n^{(i)} + w x_n^{(i-1)} - b\right], \\ x_n^{(0)} &= x_n^{(N-1)}, \end{aligned} \tag{1}$$

where $x_n^{(i)}$ is the output of the element i at time n, $\Theta(\cdot)$ is the Heaviside step function, ϕ is the initial phase, w is the coupling strength and b is the threshold. The periodic signal is assumed as subthreshold with $A < b$, and the length of the chain N is an integer multiple of the wavelength, *i.e.* $N = N'\lambda$.

In this paper SR in an individual element embedded in the chain, SR with spatiotemporal signal and the spatiotemporal synchronization between equivalent elements are investigated. As a measure of SR in an individual element we take the SNR in the middle element of the chain, obtained from the power spectral density $S(\omega)$ of its output signal and defined as $\mathrm{SNR} = 10\log\left[S_{\mathrm{P}}(\omega_s)/S_{\mathrm{N}}(\omega_s)\right]$. Here $S_{\mathrm{P}}(\omega_s) = S(\omega_s) - S_{\mathrm{N}}(\omega_s)$ is the height of the peak at $\omega = \omega_s$ and $S_{\mathrm{N}}(\omega_s)$ is the noise background in the vicinity of ω_s. In our numerical simulations the SNR is normalized to the frequency bandwidth $\Delta f = 2^{-12}$Hz.

As a measure of SR with spatiotemporal signal we take the correlation function between the spatiotemporal periodic input signal and the output signal

$$C = \frac{1}{N}\sum_{i=0}^{N-1} C^{(i)}, \quad C^{(i)} = \frac{\left\langle x_n^{(i)} A\sin(\omega_s n - ki + \phi)\right\rangle}{\sqrt{\left(\frac{A^2}{2}\right)\left[\left\langle\left(x_n^{(i)}\right)^2\right\rangle - \left\langle x_n^{(i)}\right\rangle^2\right]}}, \tag{2}$$

where the angular brackets denote the time average. The functions $C^{(i)}$ are obtained under the assumption that the mean value of the periodic signal at the input of every element is zero and the mean value of the square of this signal is $A^2/2$.

As a measure of spatiotemporal synchronization in the chain we take the mutual correlation function between elements, averaged over all pairs of elements with the same phase of the periodic signal at inputs

$$\begin{aligned} C_{\mathrm{mut}} &= \frac{1}{NN'}\sum_{\{i,j\}} C_{\mathrm{mut}}^{(i,j)}, \\ C_{\mathrm{mut}}^{(i,j)} &= \frac{\left\langle x_n^{(i)} x_n^{(j)}\right\rangle - \left\langle x_n^{(i)}\right\rangle\left\langle x_n^{(j)}\right\rangle}{\sqrt{\left[\left\langle\left(x_n^{(i)}\right)^2\right\rangle - \left\langle x_n^{(i)}\right\rangle^2\right]\left[\left\langle\left(x_n^{(j)}\right)^2\right\rangle - \left\langle x_n^{(j)}\right\rangle^2\right]}}, \end{aligned} \tag{3}$$

where in the case $k \neq 0$ the sum extends over all pairs of elements such that $|i-j| = m\lambda$, $m = 0, 1, 2 \ldots N'$, and in the case $k = 0$ — over all pairs. C_{mut} is at a maximum when the character of the plane travelling wave is best reflected in the activity of the elements of the chain [26].

3. Simplified adiabatic theory

In this section, in a manner analogous to our previous studies [24-26], an extension of the theory of SR in threshold elements with discrete time [10] to the case of a chain of unidirectionally coupled elements is presented. From this theory, the quantities SNR and C can be evaluated semi-analytically provided the time-dependent probability that $x_n^{(i)} = 1$, denoted as $\Pr\left(x_n^{(i)} = 1\right)$, is known. This probability is obtained here under certain simplifying assumptions.

The starting point is the equation for the complete probability that $x_n^{(i)}=1$. Then:

$$\begin{aligned}\Pr\left(x_{n+1}^{(i)} = 1\right) &= \Pr\left(x_{n+1}^{(i)} = 1 \,\middle|\, x_n^{(i-1)} = 1\right) \Pr\left(x_n^{(i-1)} = 1\right) \\ &\quad + \Pr\left(x_{n+1}^{(i)} = 1 \,\middle|\, x_n^{(i-1)} = 0\right) \Pr\left(x_n^{(i-1)} = 0\right) . \end{aligned} \tag{4}$$

The conditional probabilities can be evaluated analytically as

$$\Pr\left(x_{n+1}^{(i)}=1 \,\middle|\, x_n^{(i-1)}=\beta\right)=0.5\left\{1-\mathrm{erf}\left(\frac{b-w\delta_{\beta,1}-A\sin\left(\omega_s n-ki+\phi\right)}{\sqrt{2D^2}}\right)\right\} , \tag{5}$$

where $\beta \in \{0,1\}$ and $\delta_{\beta,1}$ is the Kronecker delta. In order to solve Eq. (4) for $\Pr\left(x_n^{(i)} = 1\right)$ the following assumptions are made. First, only the adiabatic limit $\omega_s \longrightarrow 0$ is considered. Then it is possible to assume on the lhs of Eq. (4) $\Pr\left(x_{n+1}^{(i)} = 1\right) = \Pr\left(x_n^{(i)} = 1\right)$. Second, since the input signal is periodic both in space and time it is also possible to assume that $\Pr\left(x_n^{(i-1)} = 1\right) = \Pr\left(x_{n+k/\omega_s}^{(i)} = 1\right)$. Taking also into account that $\Pr\left(x_n^{(i)} = 0\right) = 1 - \Pr\left(x_n^{(i)} = 1\right)$ Eq. (4) can be then rewritten as:

$$\begin{aligned}\Pr\left(x_n^{(i)} = 1\right) &= \sum_\beta \Pr\left(x_{n+1}^{(i)} = 1 \,\middle|\, x_n^{(i-1)} = \beta\right) \\ &\quad \times \left[\delta_{\beta,0} - (-1)^\beta \Pr\left(x_{n+k/\omega_s}^{(i)} = 1\right)\right] . \end{aligned} \tag{6}$$

The above difference equation can be efficiently solved numerically using the iterative method of Ref. [26]. At a first step the solution is assumed as for an uncoupled element,

$$\Pr\left(x_n^{(i)} = 1\right) = 0.5\left\{1 - \mathrm{erf}\left(\frac{b - A\sin\left(\omega_s n - ki + \phi\right)}{\sqrt{2D^2}}\right)\right\} ,$$

then it is inserted on the rhs of Eq. (6) which yields a new approximate solution and a whole procedure is repeated up to a moment when the consecutive iterated solutions do not change significantly. The solution, and what follows the SNR and $C^{(i)}$, do not depend on i.

According to Ref. [10] the SNR can be evaluated from $\Pr\left(x_n^{(i)}=1\right)$ as

$$\mathrm{SNR} = 10\log\frac{|P_1|^2}{\left(\overline{\Pr\left(x_n^{(i)}=1\right)} - \overline{\Pr^2\left(x_n^{(i)}=1\right)}\right)\Delta f}\,, \tag{7}$$

where P_1 is the first Fourier coefficient of $\Pr\left(x_n^{(i)}=1\right)$ with respect to time

$$P_1 = T_s^{-1}\sum_{n=0}^{T_s-1}\Pr\left(x_n^{(i)}=1\right)\exp\left(-i\omega_s n\right)\,, \tag{8}$$

and the bar denotes the time average over T_s. However, it should be pointed out that Eq. (7) is exact only in the case of an uncoupled threshold element driven by a sum of a periodic signal and white noise [10]. Thus in our case Eq. (7) is only approximate since the total random input to element i in Eq. (1) consists of a sum of white noise $\eta_n^{(i)}$ and non-white noise $wx_n^{(i-1)}$.

The correlation function $C=C^{(i)}$ can be also evaluated using $\Pr\left(x_n^{(i)}=1\right)$ since

$$\begin{aligned}\langle x_n^{(i)}\rangle &= \langle\left(x_n^{(i)}\right)^2\rangle = \overline{\Pr\left(x_n^{(i)}=1\right)}\,,\\ \langle x_n^{(i)}A\sin\left(\omega_s n - ki+\phi\right)\rangle &= \overline{A\sin\left(\omega_s n-ki+\phi\right)\Pr\left(x_n^{(i)}=1\right)}\,.\end{aligned} \tag{9}$$

4. Results and discussion

The numerical (obtained for the $n=63$ element) and theoretical results for the SNR in an individual element embedded in the chain are shown in Fig. 1. The numerical results show that for $0\le k\le\pi/4$ an optimum value of coupling $w_{\mathrm{opt}}>0$ exists for which the maximum of the SNR reaches its highest possible value, *i.e.* SR in an individual element is enhanced due to proper coupling (Fig. 1(a),(b)). For $k=\pi/2$ the SNR decreases for any coupling (Fig. 1(c)). For $k=\pi$ the SNR increases for $w\longrightarrow-\infty$ and $D\longrightarrow\infty$, however, without the increase of the maximum of the SNR (Fig. 1(d)). These results qualitatively resemble the ones obtained for bidirectionally coupled elements and can be understood using similar arguments concerning the increase of probability of the two coupled elements being in the same state for

$w > 0$ and in the opposite states for $w < 0$ [24-26]. In particular, the effect of array enhanced SR is present in the model (1), although a significant increase of the SNR due to proper coupling is obtained only for $k = 0$ and in other cases this effect is weak. Taking into account the approximate character of Eq. (7) the quantitative agreement between theoretical and numerical results in Fig. 1 is good, and for all k the theory qualitatively predicts the dependence of the SNR on w. The agreement is better than in the case of bidirectional coupling [25, 26]. *E.g.*, for $D \longrightarrow 0$, contrary to the latter case, the theoretical curves never diverge so that the presence of maxima of the curves SNR *vs* D is correctly predicted for any k and w. Hence if for some k the effect of array enhanced SR occurs it is also correctly predicted. This is because for unidirectional coupling Eq. (6) is strict, apart from the adiabatic approximation, while the corresponding equation for a chain with bidirectional coupling is only approximate [26]. Also the deterministic dynamics of Eq. (1), which becomes important for small noise, is simpler than for a chain with bidirectional coupling and does not cause a disagreement with the theory of Sec. 3 based only on the stochastic approach.

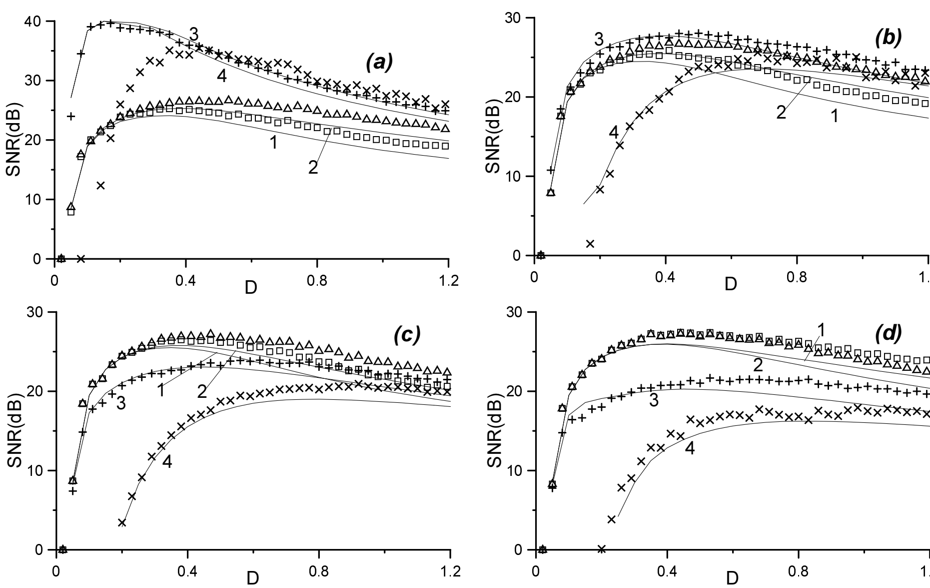

Fig. 1. The SNR *vs* D for various wave vectors k and coupling constants w, and for the length of the chain $N = 128$ and period $T_s = 128$: (a) $k = 0$, (b) $k = \pi/4$, (c) $k = \pi/2$, (d) $k = \pi$. Numerical results are shown with symbols: ($\square$) $w = -1.5$, ($\triangle$) $w = -0.1$, (+) $w = 1.0$, ($\times$) $w = 1.5$. Theoretical results are shown with numbered solid lines: (1) $w = -1.5$, (2) $w = -0.1$, (3) $w = 1.0$, (4) $w = 1.5$.

The numerical and theoretical results for C are shown in Fig. 2. The numerical results show that for any k and w SR with spatiotemporal signal occurs, characterized by the presence of the maximum of the curve C *vs* D. For $k = 0$ this kind of SR can be significantly enhanced due to proper coupling $w_{\rm opt} > 0$ (Fig. 1(a)), while for $k = \pi/4$ and $k = \pi/2$ the quality of SR is deteriorated by any coupling (Fig. 1(b),(c)) and for $k = \pi$ SR is most enhanced for $w \longrightarrow -\infty$. These results are similar to the ones obtained in a chain with bidirectional coupling apart from the fact that SR with spatiotemporal signal is not enhanced due to proper positive coupling for $0 < k \leq \pi/2$ [26]. Comparison of Fig. 1(b) and Fig. 2(b) shows that in the system (1) the enhancement of C due to coupling need not occur although the SNR is improved. The agreement between theoretical and numerical results is very good and again better than in Ref. [25, 26]. In particular, the theoretical curves C *vs* D for any k and w do not diverge for $D \longrightarrow 0$ but show maxima, hence if the enhancement of SR with spatiotemporal signal for $w = w_{\rm opt}$ occurs it is correctly predicted (Fig. 2(a)).

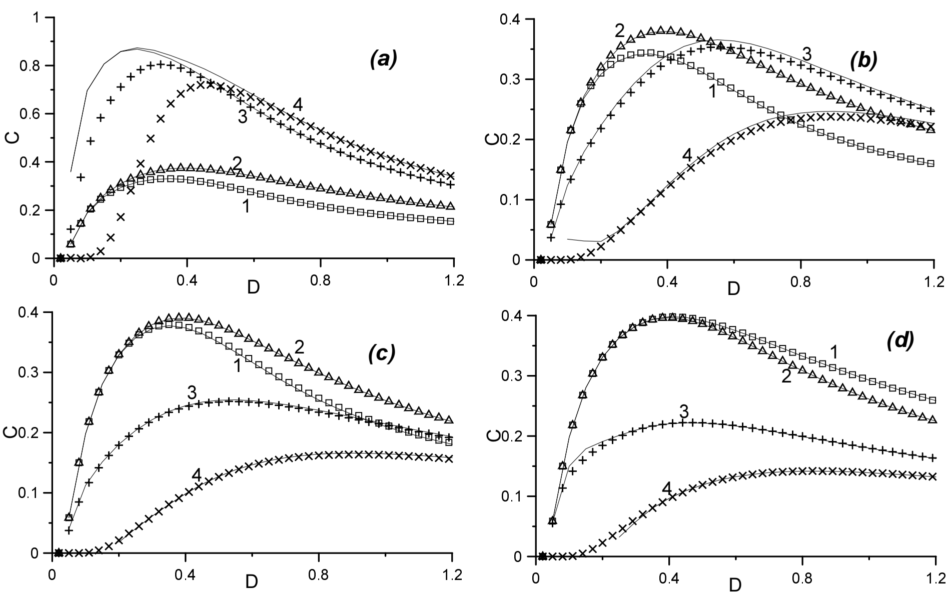

Fig. 2. C *vs* D for various wave vectors k and coupling constants w, and for the length of the chain $N = 128$ and period $T_s = 128$: (a) $k = 0$, (b) $k = \pi/4$, (c) $k = \pi/2$, (d) $k = \pi$. Numerical results are shown with symbols: ($\Box$) $w = -1.5$, ($\triangle$) $w = -0.1$, ($+$) $w = 1.0$, ($\times$) $w = 1.5$. Theoretical results are shown with numbered solid lines: (1) $w = -1.5$, (2) $w = -0.1$, (3) $w = 1.0$, (4) $w = 1.5$.

The numerical curves C_{mut} *vs* D are shown in Fig. 3. It can be seen that these curves for all k exhibit maxima for nonzero noise intensity. The presence of these maxima provides evidence for the noise-induced order which emerges in the system due to the cooperative influence of the spatiotemporal subthreshold periodic signal and noise. This order results in the maximum spatiotemporal synchronization among elements with the same phase of the periodic signal at inputs. In the most ordered state the character of the plane travelling wave is best reflected in the activity of the elements of the chain [26]. For $0 \leq k \leq \pi/4$ an optimum value of coupling exists for which the maximum of C_{mut} reaches the highest possible value (Fig. 3(a),(b)), while for $k = \pi/2$ the maximum of C_{mut} decreases with any coupling (Fig. 3(c)) and for $k = \pi$ the values of C_{mut} increase for $w \longrightarrow 0$, but with no visible increase of the maximum (Fig. 3(d)). For longwave input signals with

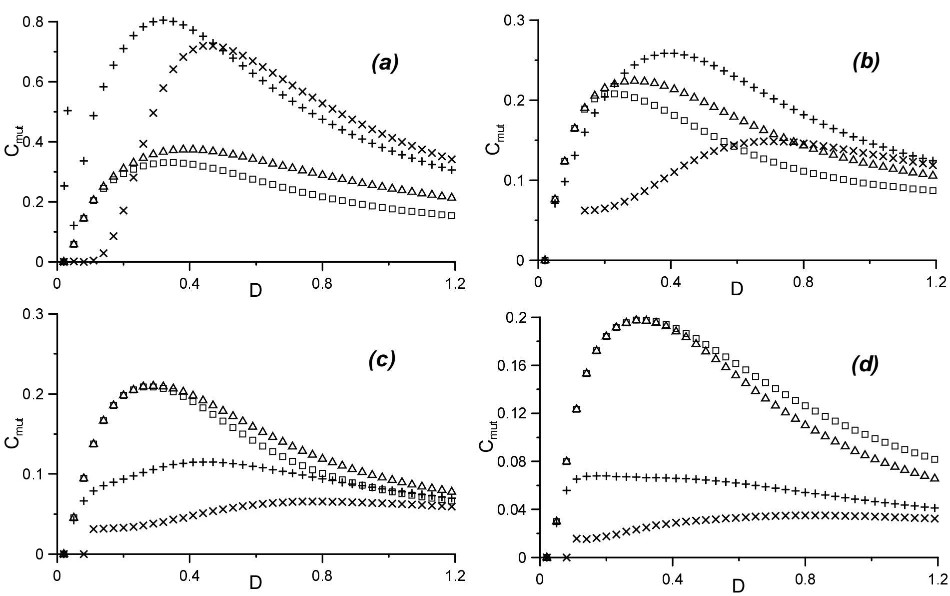

Fig. 3. Numerical curves C_{mut} *vs* D for various wave vectors k and coupling constants w, and for the length of the chain $N = 128$ and period $T_s = 128$: (a) $k = 0$, (b) $k = \pi/4$, (c) $k = \pi/2$, (d) $k = \pi$; ($\square$) $w = -1.5$, ($\triangle$) $w = -0.1$, ($+$) $w = 1.0$, ($\times$) $w = 1.5$.

$0 \leq k \leq \pi/4$ the values of the optimum coupling coincide with the ones for which SR in an individual element is most pronounced, but the locations of the maxima of the curves C_{mut} *vs* D and SNR *vs* D for $w = w_{\mathrm{opt}}$ do not coincide (*cf.* Fig. 3(a),(b) and Fig. 1(a),(b)).This situation is slightly different from what is observed in the case of array enhanced SR with the input signal uniform in space [13-15] or longwave signal periodic in space and time in a chain of coupled threshold elements with bidirectional coupling [26]. In these cases the enhancement of SR in an individual element is always connected with the maximum spatiotemporal synchronization in the system and thus also the locations of the maxima of the two above-mentioned curves coincide. Also for shortwave input signals with $k = \pi/2$ the locations of the maxima of the curves C *vs* D and $C_{\rm mut}$ *vs* D do not perfectly coincide (*cf.* Fig. 3(c) and Fig. 2(c)), while for a chain with bidirectional coupling the maximum spatiotemporal synchronization in such a case is connected with the enhancement of SR with spatiotemporal signal [26].

5. Summary and conclusions

In this paper we investigated SR in a chain of unidirectionally coupled threshold elements driven by a plane travelling wave and independent noises. Two kinds of SR were studied: SR in an individual element embedded in the chain and SR with spatiotemporal signal, characterized by a local and global measure of periodicity of the output signal, respectively. It was shown that both kinds of SR can be enhanced due to proper coupling, *i.e.* a counterpart of the array enhanced SR effect for spatiotemporal periodic signals was observed. However, this enhancement is in general weaker and occurs for a smaller range of wavelengths of the input signal than in a chain with bidirectional coupling. It was also shown that the enhancement of SR in an individual element is related to maximum synchronization among elements with the same phase of the periodic signal at inputs. Although the differences between the results of this paper and the ones of Refs [13-15,26] seem to be of minor importance they suggest that in the systems with asymmetric coupling the relationship between the concept of spatiotemporal synchronization and the two kinds of SR can be more complex than in the systems with symmetric coupling.

The results of this paper and our recent works [23-26] prove that chains of coupled threshold elements can be used efficiently for investigation of various aspects of SR in spatially extended systems. The present results show that the possibility of the ocurrence of SR with spatiotemporal signal or array enhanced SR is independent of the symmetry of connections between the neighbouring elements in the chain, though quantitative differences in these phenomena can appear depending on this symmetry. This independence opens a way to investigate these phenomena in systems with the structure of connections closer to that in typical artificial neural networks. Besides, chains of unidirectionally coupled elements are more often employed for the signal transmission along the chain than for the signal detection. In a chain of coupled bistable stochastic oscillators it was shown that the signal transmission can also benefit from external noise; this phenomenon is called array enhanced signal transmission [27]. The results of this paper show that a chain of threshold elements with unidirectional coupling can be also used for the signal detection, as a typical spatially extended stochastic resonator.

REFERENCES

[1] R. Benzi, A. Sutera, A. Vulpiani, *J. Phys.* **A14**, L453 (1981).

[2] L. Gammaitoni, P. Hänggi, P. Jung, F. Marchesoni, *Rev. Mod. Phys.* **70**, 223 (1998).

[3] V.S. Anishchenko, A.B. Neiman, F. Moss, L. Schimansky-Geier, *Phys. Usp.* **42**, 7 (1999); [*Usp. Fiz. Nauk* **169**, 7 (1999)].

[4] B. McNamara, K. Wiesenfeld, *Phys. Rev.* **A39**, 4854 (1989).

[5] P. Jung, P. Hänggi, *Phys. Rev.* **A44**, 8032 (1991).

[6] L. Gammaitoni, M. Löcher, A. Bulsara, P. Hänggi, J. Neff, K. Wiesenfeld, W. Ditto, M. E. Inchiosa, *Phys. Rev. Lett.* **82**, 4574 (1999).

[7] K. Wiesenfeld, D. Pierson, E. Pantazelou, C. Dames, F. Moss, *Phys. Rev. Lett.* **72**, 2125 (1994).

[8] Z. Gingl, L.B. Kiss, F. Moss, *Europhys. Lett.* **29**, 191 (1995).

[9] K. Loerincz, Z. Gingl, L.B. Kiss, *Phys. Lett.* **A224**, 63 (1996).

[10] F. Chapeau-Blondeau, *Phys. Rev.* **E53**, 5469 (1996).

[11] F. Chapeau-Blondeau, X. Godivier, *Int. J. Bifurcation & Chaos* **6**, 2069 (1996).

[12] M. Löcher, D. Cigna, E.R. Hunt, G.A. Johnson, F. Marchesoni, L. Gammaitoni, M.E. Inchiosa, A.R. Bulsara, *Chaos* **8**, 604 (1998).

[13] J.F. Lindner, B.K. Meadows, W.L. Ditto, M.E. Inchiosa, A.R. Bulsara, *Phys. Rev. Lett.* **75**, 3 (1995).

[14] J.F. Lindner, B.K. Meadows, W.L. Ditto, M.E. Inchiosa, A.R. Bulsara, *Phys. Rev.* **E53**, 2081 (1996).

[15] P.M. Gade, R. Rai, H. Singh, *Phys. Rev.* **E56**, 2518 (1997).

[16] I. Dikshtein, A. Neiman, L. Schimansky-Geier, *J. Magn. Magn. Mater.* **188**, 301 (1998).

[17] B. von Haeften, R. Deza, H.S. Wio, *Phys. Rev. Lett.* **84**, 404 (2000).

[18] J.M.G. Vilar, J.M. Rubí, *Phys. Rev. Lett.* **78**, 2886 (1999).

[19] J. Javier Brey, A. Prados, *Phys. Lett.* **A216**, 241 (1996).

[20] U. Siewert, L. Schimansky-Geier, *Phys. Rev.* **E58**, 2843 (1998).

[21] K. Leung, Z. Néda, *Phys. Rev.* **E59**, 2730 (1999).

[22] Z. Néda, A. Rusz, E. Ravasz, P. Lakdawala, P.M. Gade, *Phys. Rev.* **E60**, R3463 (1999).

[23] J.M.G. Vilar, J.M. Rubí, *Physica* **A277**, 327 (2000).

[24] A. Krawiecki, A. Sukiennicki, R.A. Kosiński, *Acta Phys. Pol.* **B31**, 1015 (2000.)

[25] A. Krawiecki, A. Sukiennicki, R.A. Kosiński, *Int. J. Modern Phys.* **B14**, 837 (2000).

[26] A. Krawiecki, A. Sukiennicki, R.A. Kosiński, *Phys. Rev.* **E62**, 7683 (2000).

[27] J.F. Lindner, S. Chandramouli, A.R. Bulsara, M. Löcher, W.L. Ditto, *Phys. Rev. Lett.* **81**, 5048 (1998).