CP, T AND CPT IN B-FACTORIES *

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I discuss a novel method to study the indirect violation of the discrete symmetries CP, T and CPT in the B_d-system. It is based on the opportunity offered by B-factories to tag the CP eigenstate of B_d by means of the CP Conserving Direction. Transitions between both flavour and CP tagged states are used to construct genuine and non-genuine CP-odd, T-odd and CPT-odd asymmetries.

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1. Introduction

The study of the violation of CP, T and CPT symmetries in the time evolution of the $K^0-\bar{K}^0$system has been performed by the CP-LEAR experiment [1]. The construction of appropriate observables is based on flavour-to-flavour transitions. These T-odd and CPT-odd observables would be zero, even in presence of T and CPT fundamental violation, if there were no absorptive components in the effective Hamiltonian. For the kaon system, the different lifetimes of physical states, $K_{\rm S}$ and $K_{\rm L}$, ensures this is not the case. On the contrary, in the case of the B_d-system, the width difference $\Delta\Gamma$ between the physical states is expected [2] to be negligible. Then the T-odd and CPT-odd observables proposed for kaons, which are based on flavour tag, vanish for a B_d-meson system with $\Delta\Gamma=0$.

In this presentation I discuss alternative observables [3] that can be constructed from the entangled states of B_d mesons in a B-factory, in order to study indirect violation of CP, T and CPT. These are based on CP tag [4] and do not need the support of a non-vanishing $\Delta\Gamma$. In Section 2, I introduce the rephasing invariant parameters ε and δ for the neutral B_d meson system to describe the violation of CP, T and CP, CPT symmetries, respectively,

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in the time evolution Hamiltonian. Section 3 establishes the requirements imposed by CPT, T and CP invariances, separately, as well as the remaining non-vanishing parameters for symmetry violation when $\Delta\Gamma = 0$. In Section 4, I use the well known experimental hierarchy among the mixings of quark flavours to find a well defined CP operator, without the freedom of CP phases, and a unique separation between CP conserving and CP violating parts of the effective Hamiltonian. Section 5 then discusses, on the basis of the CP conserving direction, the CP tag of B_d from the entangled states in a B-factory. In Section 6, I review how to construct the flavour asymmetries, the analogous in the B-system to those observables measured for kaons. In Section 7, I construct alternative asymmetries based on a CP tag. These CP, T- and CPT-odd time dependent asymmetries not only do not cancel for $\Delta\Gamma = 0$ but they are "genuine" in the sense that cannot be mimic by the presence of absorptive parts. The limit of small $\Delta\Gamma$ causes the time-reversal operation and the exchange of decay products of the two B's entangled state to be equivalent. I exploit this result in Section 8 to build "non-genuine" asymmetries that involve only the $J/\Psi\ K_{\rm S}$ final state. Finally, in Section 9, I summarise the conclusions.

2. The parameters ε, δ

Starting from the flavour state $|B^0\rangle$ the corresponding CP eigenstates are

$$|B\pm\rangle \equiv \frac{1}{\sqrt{2}}(I \pm CP)|B^0\rangle \tag{1}$$

which are physical and well defined if the CP operator is unique. One should realize that the identification of $|\bar{B}^0\rangle$ in terms of $CP|B^0\rangle$ is rephasing variant, but Eq. (1) is free from this variance: it only depends on the CP operator.

Writing the effective Hamiltonian responsible of the time evolution of the $B^0 - \bar{B}^0$ system as

$$H = M - \frac{i}{2}\Gamma \tag{2}$$

its non-invariance under CP, $[H,\ CP] \neq 0$, leads to physical eigenstates of mass and width

$$\begin{aligned} |B_1\rangle &= \frac{1}{\sqrt{1+|\varepsilon_1|^2}}\left[|B_+\rangle + \varepsilon_1|B_-\rangle\right] \\ |B_2\rangle &= \frac{1}{\sqrt{1+|\varepsilon_2|^2}}\left[|B_-\rangle + \varepsilon_2|B_+\rangle\right] , \end{aligned} \tag{3}$$

where $\varepsilon_{1,2}$ are complex parameters describing CP-violation in the system.

These rephasing invariant parameters are better interpreted in terms of

$$\begin{aligned}\varepsilon &\equiv \frac{\varepsilon_1+\varepsilon_2}{2}=\frac{\sqrt{H_{12}CP_{12}^*}-\sqrt{H_{21}CP_{12}}}{\sqrt{H_{12}CP_{12}^*}+\sqrt{H_{21}CP_{12}}}\,,\\ \delta &\equiv \varepsilon_1-\varepsilon_2=\frac{2\,(H_{11}-H_{22})}{\left(\sqrt{H_{12}CP_{12}^*}+\sqrt{H_{21}CP_{12}}\right)^2}\,.\end{aligned} \tag{4}$$

The rephasing invariance of ε and δ is apparent from Eq. (4). The matrix elements of H and CP are understood in the flavour basis: $H_{12}\equiv\langle B^0|H|\bar{B}^0\rangle$, *etc.* The observable character of the parameters is apparent [5] in the time evolution of the state prepared as $|B\pm\,\rangle$ at $t=0$.

3. Symmetry restrictions

— CPT invariance requires $H_{11}=H_{22}$, so that $\delta=0$, irrespective of the value of ε.

— T invariance imposes $\mathrm{Im}(M_{12}CP_{12}^*)=\mathrm{Im}\,(\Gamma_{12}CP_{12}^*)=0$, so that $\varepsilon=0$, independently of the value of δ.

— CP invariance leads to both $\varepsilon=\delta=0$.

Therefore we have four real parameters which carry information on the symmetries of the effective Hamiltonian. An analysis of Eq. (4) shows that Re (ε) and Im (δ) need not only symmetry violation but $\Delta\Gamma\neq 0$.

For the B_d system, one expects that the width matrix $\Gamma\propto I$ or, equivalently, $\Delta\Gamma=0$ is an excellent approximation, so that

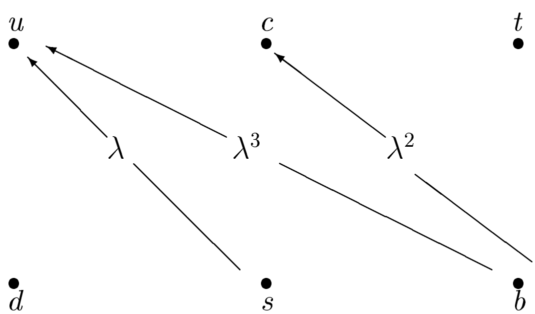
$$\begin{aligned}\mathrm{Re}(\varepsilon) &= 0, \frac{\mathrm{Im}\,(\varepsilon)}{1+|\varepsilon|^2}=\frac{\mathrm{Im}\,(M_{12}CP_{12}^*)}{\Delta m}\,,\\ \mathrm{Im}(\delta) &= 0, \frac{\mathrm{Re}\,(\delta)}{1+|\varepsilon|^2}=\frac{M_{22}-M_{11}}{\Delta m}\,.\end{aligned} \tag{5}$$

The result is that $\mathrm{Im}(\varepsilon)\neq 0$ is a proof of CP- and T-violation and $\mathrm{Re}(\delta)\neq 0$ is a proof of CP- and CPT-violation, whereas $\mathrm{Re}(\varepsilon)=0$ and $\mathrm{Im}(\delta)=0$ are not a proof of T- and CPT-invariances, respectively.

4. The *CP* conserving direction

The Langrangian of the Standard Model, when written in terms of the fermion fields with definite mass, gets the contribution for both Flavour Mixing and CP-violation from the same ingredient: the Cabibbo–Kobayashi–Maskawa (CKM) mixing matrix [6] for quarks. There is no phase choice for the CP-transformed fields which leaves the Langrangian invariant. Different choices of phases in the CP operator would yield to non-unique separation between the CP-conserving and CP-violating components of the Langrangian.

The determination of the CP operator is, however, possible and unambiguous [4] when the experimental hierarchy in the CKM mixing matrix is considered. In terms of the Cabibbo mixing parameter λ, this hierarchy among the three families of quarks

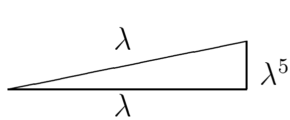


allows a perturbative separation of the Langrangian as

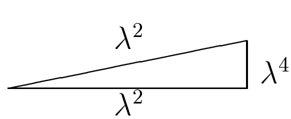
$$L_{\rm SM} = L^{(0)} + \Delta L_{\rm SM}\,, \tag{6}$$

where $L^{(0)}$ is $L_{\rm SM}$ up to order $0(\lambda^3)$ included.

With the use of $L^{(0)}$, the effective Hamiltonian (2) $H(s,d)$ for the (sd) neutral meson system is CP conserving, because the corresponding unitarity triangle [7]



collapses to a line. Similarly, the effective Hamiltonian $H(b,s)$ for the (bs) system is CP conserving too at $0(\lambda^3)$, because the associated unitarity triangle



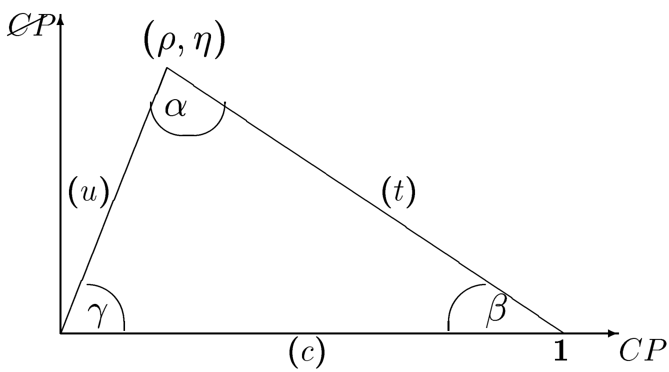
again collapses to a line. On the contrary, $H(b,d)$ is CP-violating, because the three sides of the unitarity triangle are of the same order $0(\lambda^3)$. The three CP phases associated to the three "down" triangles are not independent, due to the cyclic relation

$$e^{i(\theta_b-\theta_d)} = e^{i(\theta_b-\theta_s)} e^{i(\theta_s-\theta_d)} . \tag{7}$$

To order $0(\lambda^3)$, the invariances of $H(s,d)$ and $H(b,s)$ determine the phases of the right-hand side of Eq. (7). As a consequence, the CP-phase for $H(b,d)$, even if it is CP-violating, is determined. The result is [4]

$$e^{i(\theta_b-\theta_d)} = \left.\frac{V_{cd}V_{cb}^*}{|V_{cd}V_{cb}^*|}\right|_{0(\lambda^3)} \tag{8}$$

and the CP-conserving direction matches the charm side of the (bd) unitarity triangle



With this fixing, the separation

$$H(b,d) = H_{CP} + H_{\not{C}\not{P}} \tag{9}$$

in the Hamiltonian induced by $L^{(0)}$ is unique. The CP-operator becomes well defined. The conceptual scheme should thus be

$$L = L^{(0)} + \Delta L_{\rm SM} + L_{\rm NP} , \tag{10}$$

where the last two terms should be treated in perturbation theory. Possible CPT-violation due to the new physics Lagrangian $L_{\rm NP}$ can then be incorporated.

5. CP tag from entangled states

In a B-factory operating at the $\Upsilon(4S)$ peak, correlated pairs of neutral B-mesons are produced through the reaction

$$e^+e^- \to \Upsilon(4S) \to B\bar{B}\,. \tag{11}$$

Bose statistics and charge conjugation symmetry require that this initial state is

$$|\, i\rangle = \frac{1}{\sqrt{2}}\left[\left|B^0\left(\vec{k}\right)\bar{B}^0\left(-\vec{k}\right)\right\rangle - \left|\bar{B}^0\left(\vec{k}\right)B^0\left(-\vec{k}\right)\right\rangle\right] \tag{12}$$

when written in the CM frame. This permits the performance of a flavour tag: if at time t_0 one of the mesons decays to $X = l$ (or a channel only allowed for one flavour), the other meson must have the opposite flavour at t_0. Its later evolution during $\Delta t = t - t_0$ will lead to its decay to Y.

The same entangled $B - \bar{B}$ state can also be expressed in terms of CP eigenstates as

$$|\, i\rangle = \frac{1}{\sqrt{2}}\left[\left|B_-\left(\vec{k}\right)B_+\left(-\vec{k}\right)\right\rangle - \left|B_+\left(\vec{k}\right)B_-\left(-\vec{k}\right)\right\rangle\right]\,. \tag{13}$$

Thus, if the CP operator is well defined, it is also possible to carry out a CP tag. We need a CP-conserving decay into a definite CP final state X at time t_0, so that its detection allows us to identify the decaying meson as a B_+ or a B_-. This provides the tag in the opposite side as a B_- or a B_+, respectively, and we can study its decay to Y after a time Δt. In terms of the decays of the two-particle system,

$$|\, i\rangle \to\ (X(t_0), Y(t)) \tag{14}$$

it is possible to establish a dictionary for the corresponding single particle mesonic transitions $B_+ \to B^0$, $\bar{B}^0 \to B_-$, *etc.*, governed by the effective $H(b,d)$ Hamiltonian.

6. Flavour tag

The final configuration denoted by (l, l) is equivalent to a flavour→flavour evolution, at the meson level. The associated dictionary is shown in the Table:

Exp.(X, Y)	**Meson transition**	
(l^+, l^+)	$\bar{B}^0 \to B^0$	CP, T
(l^-, l^-)	$B^0 \to \bar{B}^0$	
(l^+, l^-)	$\bar{B}^0 \to \bar{B}^0$	CP, CPT
(l^-, l^+)	$B^0 \to B^0$	

The first two transitions are self-conjugated under CPT and they are connected by the CP- or T- transformation. The corresponding Kabir asymmetry [8] is

$$A(T) \equiv \frac{I(l^+, l^+) - I(l^-, l^-)}{I(l^+, l^+) + I(l^-, l^-)} \simeq \frac{4\frac{\mathrm{Re}(\varepsilon)}{1+|\varepsilon|^2}}{1 + 4\left(\frac{\mathrm{Re}(\varepsilon)}{1+|\varepsilon|^2}\right)^2}\,, \tag{15}$$

where only linear terms in δ (absent) have been kept, since CPT violation is treated in perturbation theory. We observe that this time-reversal asymmetry is time independent! In the limit $\Delta\Gamma = 0$ (5), one has $\mathrm{Re}(\varepsilon) = 0$. The asymmetry $A(T)$ then vanishes even if CP and T violation exist. For the B_d-system, experimental limits for $\mathrm{Re}(\varepsilon)$ are at the level of few percent [9].

The second asymmetry to be considered is

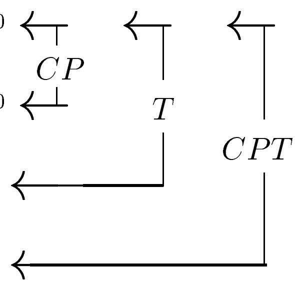
$$\begin{aligned} A(CPT) &\equiv \frac{I(l^+, l^-) - I(l^-, l^+)}{I(l^+, l^-) + I(l^-, l^+)} \\ &\simeq -2\frac{\mathrm{Re}\left(\frac{\delta}{1-\varepsilon^2}\right)\sinh\frac{\Delta\Gamma\Delta t}{2} - \mathrm{Im}\left(\frac{\delta}{1-\varepsilon^2}\right)\sin\left(\Delta m\Delta t\right)}{\cosh\frac{\Delta\Gamma\Delta t}{2} + \cos\left(\Delta m\Delta t\right)} \end{aligned} \tag{16}$$

also to linear order in δ. This CPT-odd asymmetry, contrary to (15), is an odd function of time. In the limit $\Delta\Gamma \to 0$, both terms of the numerator vanish, the first explicitly, the second from (5) $\mathrm{Im}(\delta) \to 0$. Present limits on $\mathrm{Im}(\delta)$ [9] are again at the level of few percent for the B_d-system.

7. Genuine asymmetries

We may construct alternative asymmetries making use of the CP eigenstates of the B_d-system, which can be identified in this system by means of a CP tag. Starting from the decay channel $(J/\Psi K_{\rm S},\, l^+)$ detected at times $(t_0,\, t)$, we can consider the dictionary of the Table:

Exp.(X,Y)	**Meson transition**
$(J/\Psi, K_{\rm S}, l^+)$	$B_+ \to B^0$
$(J/\Psi, K_{\rm S}, l^-)$	$B_+ \to \bar{B}^0$
$(l^-, J/\Psi K_{\rm L})$	$B^0 \to B_+$
$(l^+, J/\Psi K_{\rm L})$	$\bar{B}^0 \to B_+$



From these four decays of the entangled state, we build three genuine asymmetries under CP, T and CPT operations. To linear order in δ and in the limit $\Delta\Gamma = 0$, we get [3]

— The CP-odd asymmetry

$$\begin{aligned} A(CP) &\equiv \frac{I\left(J/\Psi K_{\rm S}, l^-\right) - I\left(J/\Psi K_{\rm S}, l^+\right)}{I\left(J/\Psi K_{\rm S}, l^-\right) + I\left(J/\Psi K_{\rm S}, l^+\right)} \\ &= -2\frac{{\rm Im}\,(\varepsilon)}{1+|\varepsilon|^2}\sin\left(\Delta m \Delta t\right) + \frac{1-|\varepsilon|^2}{1+|\varepsilon|^2}\frac{2{\rm Re}\,(\delta)}{1+|\varepsilon|^2}\sin^2\left(\frac{\Delta m \Delta t}{2}\right) \end{aligned} \tag{17}$$

which has contributions from T-violating and CPT-violating terms. These are, respectively, odd and even functions of time Δt. The CP asymmetry corresponds to the well known "gold plate" decay [10] and it has been measured recently [11]. The inclusion of a non-vanishing ${\rm Re}(\delta)$ into this CP-odd asymmetry was considered in Refs. [12]. The separation of T-odd and CPT-odd terms can be done [3] by constructing different asymmetries.

— The T-odd asymmetry

$$A(T) \equiv \frac{I\left(l^-, J/\Psi K_{\rm L}\right) - I\left(J/\Psi K_{\rm S}, l^+\right)}{I\left(l^-, J/\Psi K_{\rm L}\right) + I\left(J/\Psi K_{\rm S}, l^+\right)} = -2\frac{{\rm Im}\,(\varepsilon)}{1+|\varepsilon|^2}\sin\left(\Delta m\Delta t\right)\left[1 - \frac{1-|\varepsilon|^2}{1+|\varepsilon|^2}\frac{2{\rm Re}\,(\delta)}{1+|\varepsilon|^2}\sin^2\left(\frac{\Delta m\Delta t}{2}\right)\right] \tag{18}$$

which is purely odd in Δt and needs ${\rm Im}(\varepsilon) \neq 0$.

— The CPT-odd asymmetry

$$A(CPT) \equiv \frac{I\left(l^+, J/\Psi K_{\rm L}\right) - I\left(J/\Psi K_{\rm S}, l^+\right)}{I\left(l^+, J/\Psi K_{\rm L}\right) + I\left(J/\Psi K_{\rm S}, l^+\right)} = \frac{1-|\varepsilon|^2}{1+|\varepsilon|^2}\frac{2{\rm Re}\,(\delta)}{1+|\varepsilon|^2}\frac{1}{1-2\frac{{\rm Im}(\varepsilon)}{1+|\varepsilon|^2}\sin\left(\Delta m\Delta t\right)}\sin^2\left(\frac{\Delta m\Delta t}{2}\right) \tag{19}$$

which needs ${\rm Re}(\delta)\neq 0$, with both even and odd time dependences.

These three asymmetries are genuine. The inclusion of a possible absorptive part, like $\Delta\Gamma \neq 0$, cannot induce by itself a non-vanishing effect. Linear $\Delta\Gamma$ corrections to these asymmetries can be considered [3]. They affect $A(CP)$ and $A(T)$, but no fake manifestation is generated. The decay channels involved here are required to tag both B_+ and B_- CP-eigenstates. This needs a good reconstruction of the decay $B \to J/\Psi K_{\rm L}$ too, besides the $J/\Psi K_{\rm S}$ channel. There is still the possibility to stay with the $J/\Psi K_{\rm S}$ only, but at the expense of considering non-genuine observables.

8. Non-genuine asymmetries

There is a discrete transformation that cannot be associated to any of the fundamental symmetries. It consists of the exchange in the order of appearance of the decay products X and Y, *i.e.*, $\Delta t \to -\Delta t$. It transforms

$$\left(J/\Psi K_{\rm S}, l^+\right) \underrightarrow{\Delta t} \left(l^+, J/\Psi K_{\rm S}\right) . \tag{20}$$

Its effect at the meson level is shown in the following dictionary

		Exp.(X,Y)	**Meson transition**	
$CP\,\Delta t$	Δt	$(J/\Psi, K_{\rm S}, l^+)$	$B_+ \to B^0$	CP
		$(J/\Psi, K_{\rm S}, l^-)$	$B_+ \to \bar{B}^0$	
		$(l^+, J/\Psi K_{\rm S})$	$\bar{B}^0 \to B_-$	
		$(l^-, J/\Psi K_{\rm S})$	$B^0 \to B_-$	

If the limit $\Delta\Gamma = 0$ is valid, the temporal asymmetry from Eq. (20) satisfies

$$A(\Delta t) \equiv \frac{I\left(l^+, J/\Psi K_{\rm S}\right) - I\left(J/\Psi K_{\rm S}, l^+\right)}{I\left(l^+, J/\Psi K_{\rm S}\right) + I\left(J/\Psi K_{\rm S}, l^+\right)} = A(T)\,. \tag{21}$$

In general, the equivalence of T and Δt inversions is only valid for Hamiltonians with the property of hermiticity, up to a global (proportional to unity) absorptive part. The approximation $\Delta\Gamma \simeq 0$ is expected to be valid for the B_d-system, but not for B_s and even less for K.

In addition, under the same assumptions to find Eq. (21), one has

$$A(CP\Delta t) \equiv \frac{I\left(l^-, J/\Psi K_{\rm S}\right) - I\left(J/\Psi K_{\rm S}, l^+\right)}{I\left(l^-, J/\Psi K_{\rm S}\right) + I\left(J/\Psi K_{\rm S}, l^+\right)} = A(CPT)\,. \tag{22}$$

Eqs. (21) and (22) have thus access to $\mathrm{Im}(\varepsilon)$ and $\mathrm{Re}(\delta)$, respectively. Their separation is associated with resolving odd and even functions of Δt, respectively.

The asymmetries defined by the first equalities of Eqs. (21) and (22) are non-genuine, in the sense that linear $\Delta\Gamma$-corrections induce new terms which survive even in the limit $\varepsilon = \delta = 0$. These fake effects are, however, calculable and controllable.

9. Conclusions

I have advocated here a method to study the indirect violation of the discrete symmetries CP, T and CPT in the B_d-system. The fundamental steps are based on the following:

- The symmetry violation in the mixing Hamiltonian for $B^0 - \bar{B}^0$ is described by two rephasing invariant complex ε, δ parameters.
- In the limit $\Delta\Gamma = 0$, only the two real quantities $\mathrm{Im}(\varepsilon)$, $\mathrm{Re}(\delta)$ remain non-vanishing.
- The transitions leading to tag Flavour→Flavour are unable to find these parameters.
- B-factories offer the opportunity to tag the CP-eigenstate of B_d,by means of the entangled state and the use of the CP Conserving Direction.
- The transitions $CP \leftrightarrows$Flavour can be chosen to construct genuine CP-odd, T-odd and CPT-odd asymmetries, able to extract ε and δ. They need the identification of both $J/\Psi K_{\mathrm{S}}$ and $J/\Psi K_{\mathrm{L}}$ decay channels.
- With only $J/\Psi K_{\mathrm{S}}$ and flavour, one can construct a temporal asymmetry Δt which is equivalent to T-odd in the limit $\Delta\Gamma = 0$. The asymmetries based on Δt exchange are non-genuine, in the sense that $\Delta\Gamma$-corrections induce new terms independent of ε and δ. These fake effects are known explicitly.

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