FACTORIZATION IN TWO-BODY NONLEPTONIC *B*-DECAYS*

C.T. Sachrajda

Department of Physics and Astronomy
University of Southampton
Southampton SO17 1BJ, UK

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I review the theoretical foundations of *QCD Factorization* in nonleptonic B-Decays. This approach, developed in collaboration with Martin Beneke, Gerhard Buchalla and Matthias Neubert, provides a rigorous framework for the analysis of these decays in the heavy-quark limit. The significance of *power corrections*, terms which are formally suppressed by powers of $\Lambda_{\mathrm{QCD}}/m_b$ and which are not calculable in perturbation theory but which may have a significant impact on phenomenologically important decays, is discussed.

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1. Introduction

Exclusive B-decays are a particularly important source of information about CP-violation and the parameters of the standard model (for an introduction to B-Physics see the excellent textbook by Branco, Lavoura and Silva [1] or the *BaBar Physics Book* [2], where the reader can also find references to the original literature). However, apart from the *golden mode* $B \to K_{\mathrm{S}} J/\Psi$ from which we get $\sin(2\beta)$, attempts to obtain fundamental information from experimental data are made difficult by our inability to quantify the non-perturbative strong-interaction effects. For example, in the decay $\bar{B}_d \to \pi^+\pi^-$ there are both *tree* and *penguin* contributions, with different CKM phases

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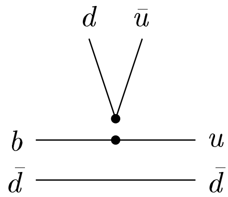
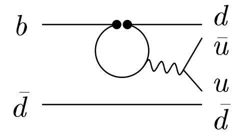


Tree Diagram



Penguin Diagram

and it is necessary to determine the relative contributions to the amplitude from these two sources.

In this talk I will report on the conceptual framework developed in collaboration with Beneke, Buchalla and Neubert for the evaluation of strong interaction effects on non-leptonic two-body B-decays [3–6]. The framework is based on a detailed analysis of Feynman diagrams and our observation that it is possible to separate long and short-distance effects in these B-decays. This allows us to derive a factorization formula, in an analogous way to those used in other applications of QCD to hard processes, both inclusive, such as deep inelastic scattering or the Drell-Yan process, and exclusive, such as the electromagnetic form-factors of hadrons at large momentum transfers. The factorization formula leads to a model-independent treatment of exclusive hadronic B-decays in the heavy-quark limit. The corrections to our results are of $O(\Lambda_{\rm QCD}/m_b)$, and for these factorization does not hold. As will be discussed below, improved estimates of the power corrections will be necessary for a successful phenomenology, at least for some important decays.

The starting point for the study of hadronic B-decays is the Operator Product Expansion (OPE). Strong interaction effects which involve virtualities above m_b can be summed using perturbation theory and renormalization group techniques, leading to an amplitude for the decay of the B into two mesons $M_{1,2}$ of the form:

$$\mathcal{A}(B\to M_1M_2)=\frac{G_F}{\sqrt{2}}\sum_i\lambda_i\,C_i(\mu)\,\langle\,M_1M_2\,|\,O_i(0)\,|\,B\,\rangle_\mu\,, \tag{1}$$

where λ_i is the CKM-matrix element; $C_i(\mu)$ is the Wilson coefficient function; μ is the renormalization scale and the hadronic matrix elements $\langle\,M_1M_2\,|\,O_i(0)\,|\,B\,\rangle$ contain all the non-perturbative QCD-effects. These matrix elements are the object of our study. We study the long-distance properties of these decays in the limit $m_b\to\infty$ and find that there is a considerable simplification leading to a **Generalized Factorization Formula**.

Note that standard approaches to the evaluation of non-perturbative QCD effects, such as Lattice QCD, QCD Sum-Rules and large N expansions, have so far made little progress in quantifying these effects in B-decays.

The plan for the remainder of this talk is as follows. In the following section I will introduce the factorization formula and explain its ingredients. Although I am unable in this talk to derive the formula in detail, in Section 3 I demonstrate its validity, at one-loop order, for $\bar{B} \to D\pi$ decays. The motivation for this is to show how the long-distance effects (*i.e.* the mass-singularities) factorize in this simple case, and how this leads to our generalized factorization formula. The implications of factorization for the phenomenology of two-body B-decays is briefly summarised in section 4 and applications to $B \to \pi K$ decays are outlined in Section 5. Finally in Sec. 6 I present the conclusions.

2. The generalized factorization formula

Before presenting our factorization formula I will remind you of what is meant by (naive) factorization in B-decays.

2.1. Naive factorization

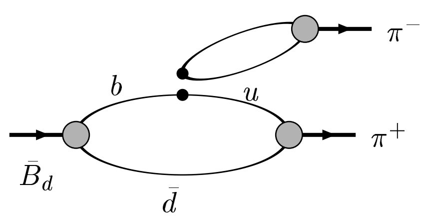


Fig. 1. Quark flow diagram contributing to the amplitude for the decay $\bar{B}_d \to \pi^+\pi^-$. The two black dots represent the product of two currents in the weak Hamiltonian.

As an example consider the decay $\bar{B}_d \to \pi^+\pi^-$. A quark flow diagram for this decay is shown in Fig. 1. The diagram has the appearance of being composed of two independent parts and the naive factorization assumption is to take this literally and to write

$$\langle\, \pi^+\pi^- \,|\, (\bar{u}b)_{V-A}\; (\bar{d}u)_{V-A} \,|\, \bar{B}_d \,\rangle \simeq \langle\, \pi^- \,|\, (\bar{d}u)_{V-A} \,|\, 0 \,\rangle\; \langle\, \pi^+ \,|\, (\bar{u}b)_{V-A} \,|\, \bar{B}_d \,\rangle. \tag{2}$$

The two-matrix elements on the right-hand side are known in principle, the first is simply proportional to the pion decay constant, f_π, and the second is proportional to $F_0^{B\to\pi}(m_\pi^2)$), one of the form-factors for $B \to \pi$ semileptonic decays. Thus naive factorization relates the amplitude for nonleptonic decays to simpler quantities, the decays constants and semileptonic form factors.

Clearly this simple picture cannot be correct. The diagram in Fig. 1 does not show the gluonic effects which are necessarily present. Naive factorization therefore has no rescattering in the final state and no strong interaction phase-shifts. The short-distance behaviour is also wrong, the renormalization scale dependence does not match on the two sides of (2). Various generalizations of naive factorization have been proposed leading to new parameters. Below we will demonstrate the rather surprising result that naive factorization is actually the leading term in QCD at large m_b, with corrections of $O(\alpha_s(m_b))$ and $O(\Lambda_{\mathrm{QCD}}/m_b)$. Thus in fact that it seems so simple and arbitrary, naive factorization is true in the large quark-mass limit of QCD.

2.2. The generalized factorization formula

From an analysis of mass-singularities in higher-order Feynman diagrams we have derived the generalized factorization formula for the decay $B \to M_1M_2$ (with M_2 being a light meson), represented by the diagrams in Fig. 2:

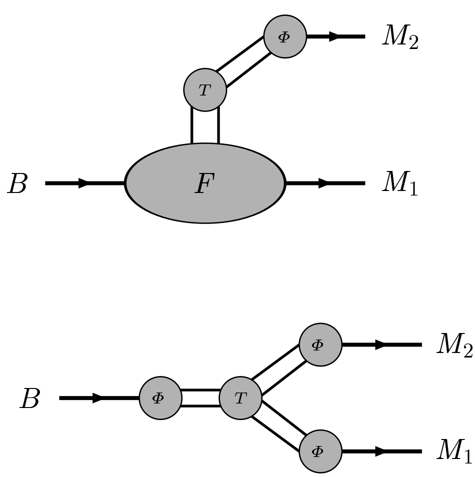


Fig. 2. Representation of the two contributions to the generalized factorization formula in Eq. (3).

$$\langle\, M_1, M_2\,|\, O_i\,|\, B\,\rangle \;=\; \sum_j F_j^{B\to M_1}(m_2^2)\int_0^1 du\, T_{ij}^I(u)\Phi_{M_2}(u) + (M_1 \leftrightarrow M_2)$$
$$+\int_0^1 d\xi\, du\, dv\, T_i^{II}(\xi,u,v)\,\Phi_B(\xi)\,\Phi_{M_1}(v)\,\Phi_{M_2}(u)\,. \tag{3}$$

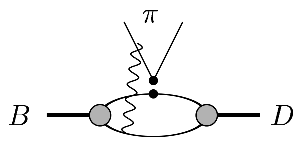
The ingredients in Eq. (3) are as follows: $F_j^{B\to M_1}(m^2)$ denotes one of the $B\to M_1$ form-factors and is a non-perturbative input into the calculation; $\Phi_X(u)$ is the light-cone distribution amplitude for the quark-antiquark Fock-state of meson X and is also a non-perturbative input; in contrast $T_{ij}^I(u)$ and $T_i^{II}(\xi,u,v)$ are perturbatively calculable hard-scattering functions. All the non-perturbative QCD effects are *factorized* into the light-cone distribution amplitudes and form-factors, quantities which are simpler than the original matrix elements and which can either be determined from experiment (in particular the form-factors) or computed using standard techniques. Eq. (3) is valid up to $O(\Lambda_{\rm QCD}/m_b)$ corrections.

There is a considerable simplification for decays in which the spectator quark (*i.e.* the light-quark or antiquark in the B-meson) goes into a heavy meson (*e.g.* $\bar B_d\to D^+\pi^-$ decays) since in these cases the last term in the factorization formula is absent. For such decays the hard interactions with the spectator quark are power-suppressed in the heavy quark limit.

It is not appropriate for me in this talk to give a detailed derivation of Eq. (3). Instead I will illustrate the basic ideas, by briefly discussing the factorization of mass singularities at one-loop order for the simpler case in which M_1 is a heavy meson, *i.e.* for $\bar B\to D\pi$ decays. Such a factorization of mass-singularities is necessary for the validity of Eq. (3).

3. $\bar B\to D\pi$ decays

In this section I outline how the mass-singularities in $B\to$ heavy-light decays, which are much simpler than $B\to$ light-light decays, factorize. For these decays, diagrams in which the *spectator* interacts with the emitted π-meson, such as



are suppressed by factors of $\Lambda_{\rm QCD}/m_b$. Thus for $B \to D\pi$ decays the T^{II} term in the factorization formula is absent. If there were remaining mass-singularities which were not absorbed into the decay constant or form factors, this would be a signal that the decay amplitude depends on other long-distance physics and is not calculable in perturbation theory. Fortunately this is not the case.

The standard classification of quark-level topologies for $B \to D\pi$ decays is presented in Fig 3. In the heavy quark limit only class-I decays contribute at leading order in the $\Lambda_{\rm QCD}/m_b$ expansion. Class-II and annihilation contributions ar suppressed by at least one power of $\Lambda_{\rm QCD}/m_b$. We now study the perturbative corrections in class-I decays.

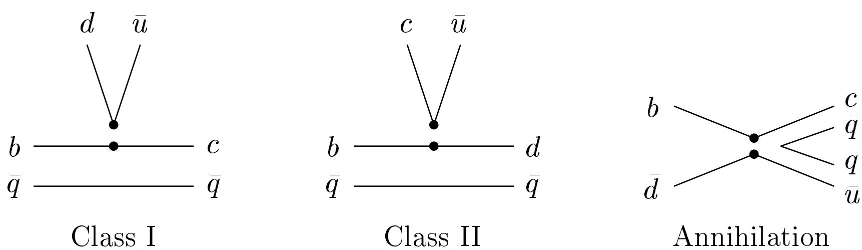


Fig. 3. Quark level topologies for $B \to D\pi$ decays.

3.1. Factorization at one-loop level

3.1.1. Cancellation of infra-red divergences

To start the discussion, consider diagram (a) in Fig. 4. For small momenta k the loop integral is divergent:

$$\int_{\text{small } k} d^4k \; \frac{1}{(up+k)^2\, k^2\, [(p_b+k)^2 - m_b^2]} \sim \int_{\text{small } k} d^4k \; \frac{1}{(2up\cdot k)\, k^2\, [2p_b\cdot k]}$$
$$\sim \int_{\text{small } k} d^4k \; \frac{1}{k^4} \quad \text{DIVERGENT!}$$

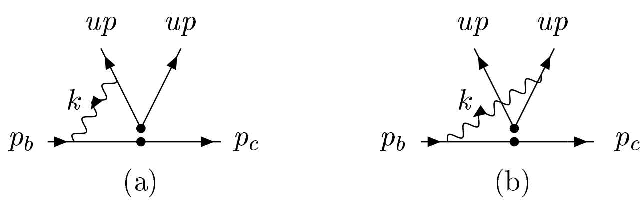


Fig. 4. Two one-loop diagrams contributing to class-I $B \to D\pi$ decays, where p is the momentum of the pion. Only the $b \to c\pi$ component is shown. The quark and antiquark pointing upwards are the constituents of the pion, with momenta up and $\bar{u}p$ respectively $(u + \bar{u} = 1)$.

and therefore we have to consider infra-red divergences. The divergence cancels against the one present in the diagram of Fig. 4(b). To see this note that only the light-quark propagator is different in the two diagrams. In diagram (a) this propagator is given by

$$\frac{\gamma^\mu (u\not{p}+\not{k})\,\Gamma}{(up+k)^2} \simeq \frac{2up^\mu\,\Gamma}{2u\,p\cdot k} = \frac{p^\mu}{p\cdot k}\,\Gamma\,,$$

where we have used the fact that k is small and that the constituents of the pion are on-shell (*i.e.* have momenta of $O(\Lambda_{\rm QCD}) << m_b$). The propagator in diagram (b) on the other hand is given by:

$$-\frac{\Gamma\,(\bar{u}\not{p}+\not{k})\gamma^\mu)}{(\bar{u}p+k)^2} \simeq -\frac{p^\mu}{p\cdot k}\,\Gamma\,.$$

Thus the infrared divergences in diagrams (a) and (b) cancel. There is a similar cancellation of infra-red divergences in diagrams (c) and (d) in Fig. 5.

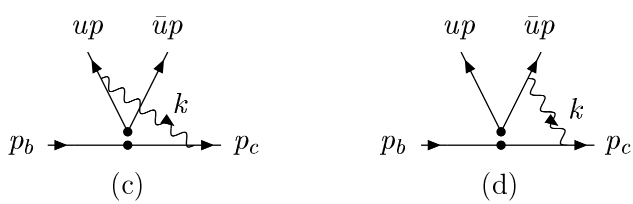


Fig. 5. Two more one-loop diagrams contributing to class-I $B \to D\pi$ decays. The notation is the same as that in Fig. 4.

This cancellation of infrared divergences is a technical manifestation of Bjorken's colour transparency argument: *soft-gluon interactions with the emitted $\bar{u}d$ pair are suppressed, because soft-gluons only interact with the colour dipole moment of the compact $\bar{u}d$ pair.*

3.1.2. Cancellation of collinear divergences

Infrared divergences are not the only source of long-distance effects in diagrams (a)–(d). Consider the region of phase space in which k is parallel to p (we write $k \simeq \alpha p$). Specifically let:

$$k^+ = O(1) \qquad \text{and} \qquad k^- = O(k_\perp^2) \ ,$$

where the pion's momentum $p = E_\pi(1,0,0,1)$ is taken to be in the + direction (in the rest-frame of the B-meson). Consider diagram (a) in which the gluon and light-quark propagators combine to give a factor:

$$\frac{1}{(up+k)^2\, k^2} \sim \frac{1}{k_\perp^4} \ .$$

The phase space is also of $O(k_\perp^4)$ and so we have a divergence and hence we have to study the collinear divergences.

The collinear divergences cancel in diagrams (a) and (c), specifically diagram (a) has a factor

$$\frac{(u+\alpha)\, 2p\cdot p_b}{2\alpha p\cdot p_b} = \frac{(u+\alpha)}{\alpha} \ ,$$

whereas the corresponding factor in diagram (c) is:

$$\frac{(u+\alpha)\, 2p\cdot p_c}{-2\alpha p\cdot p_c} = -\frac{(u+\alpha)}{\alpha} \ .$$

Therefore the collinear divergences in diags. (a) and (c) cancel. There is a similar cancellation of the collinear divergences in diags. (b) and (d).

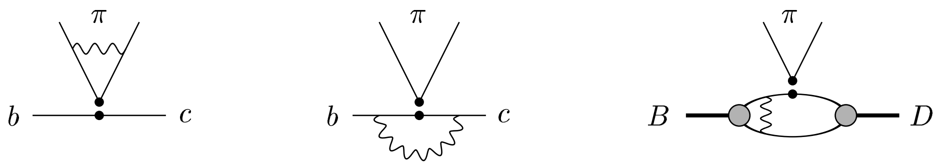
3.1.3. Summary

We have seen above that there are no mass singularities in the sum of the non-factorizing diagrams (a)–(d). These diagrams can therefore be evaluated in perturbation theory. The analysis above does not mean however, that there are no mass-singularities at one-loop order. Recall the factorization formula [1]:

$$\langle \pi\, D \,|\, O \,|\, \bar{B}\,\rangle = F^{B\to D}(m_\pi^2) \int_0^1 du\, T^I(u) \varPhi_\pi(u) \ . \tag{4}$$

The remaining mass-singularities come from diagrams such as

[1] Recall that the term proportional to T_i^{II} in Eq. (3) are suppressed for $B \to D\pi$ decays.



and are absorbed into the form-factor $F^{B\to D}(m_\pi^2)$ and the light-cone distribution amplitude $\Phi_\pi(u)$. The key feature is that **all** the mass singularities are factorized into these quantities and that the hard scattering amplitude T is calculable is perturbation theory.

In Ref. [5] these arguments were extended to two-loop order.

4. Implications of factorization

In this section I briefly summarise the implications of the factorization formula.

1. The significance and usefulness of the factorization formula (3) stems from the fact that the non-perturbative quantities which appear on the RHS are much simpler than the original matrix elements which appear on the LHS. They either reflect universal properties of a single meson state (the light-cone distribution amplitudes) or refer to a $B\to$ meson transition matrix element of a local current (form-factor). They can be determined using non-perturbative methods such as lattice QCD or QCD sum-rules, or even be partially determined from experimental measurements (particularly in the case of the form-factors).

2. Conventional (naive) factorization is recovered as a rigorous prediction in the infinite quark-mass limit (*i.e.* neglecting $O(\alpha_s(m_b))$ and $O(\Lambda_{\rm QCD}/m_b)$ corrections).

3. Perturbative corrections to naive factorization can be computed systematically. The results are, in general, non-universal (*i.e.* process dependent).

4. All strong interaction phases are generated perturbatively in the heavy quark limit (as form factors have no imaginary parts).

5. Many observables of interest for CP-violation become accessible. The precision of the calculations is limited by our knowledge of the wave-functions and of the power corrections.

6. The problem of scheme dependence in naive factorization is solved in the same way as in any other NLO computation of the weak effective Hamiltonian.

5. Applying the factorization formula to $B \to \pi K$ decays

As an example of the application of the factorization formula let us consider $B \to \pi K$ decays. These decays are discussed and analysed in great detail in Ref. [6]. The factorization formula leads to the following expression for the decay amplitudes.

$$\langle \pi K | \mathcal{H}_{\rm eff} | \bar{B} \rangle = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \langle \pi K | \mathcal{T}_p | \bar{B} \rangle \ , \tag{5}$$

where λ_p is the product of CKM matrix elements, $\lambda_p = V_{pb} V^*_{ps}$, and the transition operator is given by

$$\begin{aligned}
&\mathcal{T}_p = a_1(\pi K)\delta_{pu}\,(\bar{u}b)_{V-A} \otimes (\bar{s}u)_{V-A} \\
&+a_2(\pi K)\delta_{pu}\,(\bar{s}b)_{V-A} \otimes (\bar{u}u)_{V-A} + a_3(\pi K)\sum_q (\bar{s}b)_{V-A} \otimes (\bar{q}q)_{V-A} \\
&+a_4^p(\pi K)\sum_q (\bar{q}b)_{V-A} \otimes (\bar{s}q)_{V-A} + a_5(\pi K)\sum_q (\bar{s}b)_{V-A} \otimes (\bar{q}q)_{V+A} \\
&+a_6^p(\pi K)\sum_q (-2)(\bar{q}b)_{S-P} \otimes (\bar{s}q)_{S+P} + a_7(\pi K)\sum_q (\bar{s}b)_{V-A} \otimes \frac{3}{2}e_q(\bar{q}q)_{V+A} \\
&+a_8^p(\pi K)\sum_q (-2)(\bar{q}b)_{S-P} \otimes \frac{3}{2}e_q(\bar{s}q)_{S+P} + a_9(\pi K)\sum_q (\bar{s}b)_{V-A} \otimes \frac{3}{2}e_q(\bar{q}q)_{V-A} \\
&+a_{10}^p(\pi K)\sum_q (\bar{q}b)_{V-A} \otimes \frac{3}{2}e_q(\bar{s}q)_{V-A} \, .
\end{aligned} \tag{6}$$

In Eq. (6) the $\otimes$ product implies that the corresponding operator should be interpreted in the naive-factorization sense, *e.g.*

$$\begin{aligned}
\langle \pi K | (\bar{u}b)_{V-A} \otimes (\bar{s}u)_{V-A} | \bar{B} \rangle &\equiv \langle \pi | (\bar{u}b)_{V-A} | \bar{B} \rangle \times \langle K | (\bar{s}u)_{V-A} | 0 \rangle \\
&\simeq i m_B^2 F_0^{B\to\pi}(m_K^2) f_K \ .
\end{aligned} \tag{7}$$

The coefficients a_i in Eq. (6) are calculated in perturbation theory.

5.1. Chirally enhanced power corrections

The factorization formula is valid up to $O(\Lambda_{\rm QCD})/m_b$ corrections, which, depending on the process, can be significant. One important source of power corrections are the *chirally enhanced* ones. To illustrate what these are consider the penguin contribution proportional to a_6^p. This is proportional to

$$(-2)\langle \pi | (\bar{u}b)_S | \bar{B} \rangle \times \langle K | (\bar{s}u)_P | 0 \rangle = r_\chi^K(\mu) A_{\pi K} \, , \tag{8}$$

where

$$r_\chi^K(\mu) \equiv \frac{2m_K^2}{\overline{m}_b(\mu)(\overline{m}_s(\mu) + \overline{m}_u(\mu))} \ , \tag{9}$$

and

$$A_{\pi K} = i\frac{G_F}{\sqrt{2}}\, m_b^2\, F_0^{B\to\pi}(m_K^2)\, f_K \ . \tag{10}$$

Thus although this is formally a power correction, suppressed by $1/m_b$ compared to the leading one, it has a large coefficient

$$2\mu_K \equiv \frac{2m_K^2}{\overline{m}_s + \overline{m}_d} \approx 3\,\mathrm{GeV} \gg \Lambda_{\mathrm{QCD}} \ . \tag{11}$$

Thus the chiral enhanced power corrections cannot be neglected in interpreting nonleptonic B-decays.

As an example our final result for the amplitude for the decay $\overline{B}_d$ takes the form:

$$-\mathcal{A}(\bar{B}_d \to \pi^+ K^-) = \left\{\lambda_u a_1 + \lambda_p(a_4^p + a_{10}^p) + \lambda_p r_\chi^K(a_6^p + a_8^p)\right\} A_{\pi K} \ . \tag{12}$$

It should be remembered however, that the terms proportional to r_χ^K are formally power corrections, albeit chirally enhanced ones.

5.2. Power corrections and mass singularities

Power corrections (*i.e.* terms which are suppressed by the powers of $O(\Lambda_{\mathrm{QCD}}/m_b)$) would not be a problem if they could all be identified and the factorization formula applied to them. If this were the case higher-order perturbative corrections would also not contain any non-factorizing infrared logarithms for these higher-twist contributions. However this is not the case in general (although it is the case for a_6 above). This non-factorization of mass singularities represents a significant difficulty, particularly since for many decays of phenomenological importance the terms which are suppressed by powers of $1/m_b$ are enhanced by CKM-factors (and colour factors) and are likely to play a significant rôle (*e.g.* for $B \to \pi K$ decays we would expect annihilation topologies, which are formally suppressed by $1/m_b$ to give significant contributions). It is therefore important to develop our understanding of power corrections.

Since we are considering power corrections we have to consider the twist-three mesonic distribution amplitudes, and one of these (conventionally called $\Phi_P(u)$) does not vanish at the end-points ($u \to 0, 1$). This leads to logarithmically divergent integrals of the form

$$\int_0^1 \frac{du}{u} \ , \tag{13}$$

where u represents the light-cone fraction of a parton in a meson. This is not unexpected for power corrections (indeed such divergences already appear in semileptonic decay form-factors), but nevertheless can lead to a loss of precision in phenomenological predictions. We assume that the divergent integral can be replaced by a universal constant, and the uncertainty in this constant is the largest theoretical error in the analysis.

It has been argued that some of the higher-order perturbative corrections can be resummed and give a Sudakov form-factor which suppresses the end-point singularity [7, 8] in the power corrections (see also the talk by Ward at this conference [10]). From this the authors of Ref. [7,8] deduce that the these terms are calculable in perturbation theory, including, for example, the semileptonic $B \to \pi, \rho$ form-factors. We believe that this approach merits further investigation (in order to gain an understanding of the precision which can be achieved), but in our view it is likely that if m_b is not sufficiently large to suppress the chirally enhanced terms then it is also not large enough to make Sudakov suppression effective and reliable.

6. Conclusions

The Factorization Formulae derived here, provide a powerful and systematic framework for the computation of non-leptonic decay amplitudes in the large m_b limit. I must stress that the existence of such a framework in itself, represents a major development. The technical difficulties in calculating the decay amplitudes accurately are still considerable, but can be faced within the coherent framework described above. Among the outstanding issues still to be resolved are:

- A verification that the approach is valid in higher orders of perturbation theory is still needed, in particular a full two-loop study for $B \to$light-light decays is still to be completed.
- Perhaps the major difficulty is to understand the best way of dealing with power-corrections in general (*i.e.* corrections which are suppressed by powers of $\Lambda_{\rm QCD}/m_b$), and the chirally enhanced power corrections in particular. This is particularly true for processes in which the power corrections are enhanced by CKM factors, as happens for example in $B \to \pi K$ decays.
- We need a better understanding of the rôle of Sudakov form-factors, particularly in the evaluation of the power corrections.

We have presented a number of phenomenological applications of the generalised factorization formalism. Most recently this has included a detailed phenomenological analysis of $B \to \pi K$ and $B \to \pi\pi$ decays [6] in which we:

1. include the matrix elements of electroweak penguin operators, which play an important rôle in $b \to s\bar{q}q$ transitions;

2. present hard-scattering kernels for general, asymmetric meson light-cone distribution amplitudes. This is important for addressing the question of nonfactorizable SU(3)-breaking corrections, since the distribution amplitudes of strange mesons are, in general, not symmetric with respect to the quark and antiquark momenta;

3. estimate the leading power corrections to the heavy-quark limit (including those arising from annihilation topologies). This is essential for controlling the theoretical uncertainties of our approach.

In this talk I have concentrated on the conceptual aspects of factorization in nonleptonic B-decays and I cannot present the details of the phenomenological analysis. For illustration however, I present in table 6 the predicted ratios of CP-averaged branching ratios for four values of the angle γ of the Unitary Triangle. For comparison, the current experimental values for these ratios (which we obtain by averaging recent CLEO [11], BaBar [12] and Belle [13] data) are also presented.

TABLE

Predicted ratios of CP-averaged branching fractions for selected values of γ. The last column shows the experimental values obtained by averaging over data from the CLEO, BaBar and Belle collaborations (obtained ignoring correlations between the individual measurements).

Ratio	40°	70°	100°	130°	Experiment
$\frac{2\mathrm{Br}(\pi^0 K^\pm)}{\mathrm{Br}(\pi^\pm K^0)}$	0.94 ± 0.07	1.16 ± 0.07	1.44 ± 0.16	1.70 ± 0.25	1.41 ± 0.29
$\frac{\mathrm{Br}(\pi^\mp K^\pm)}{2\mathrm{Br}(\pi^0 K^0)}$	0.92 ± 0.08	1.17 ± 0.08	1.50 ± 0.19	1.83 ± 0.34	0.83 ± 0.22
$\frac{\tau_{B^+}}{\tau_{B^0}}\,\frac{\mathrm{Br}(\pi^\mp K^\pm)}{\mathrm{Br}(\pi^\pm K^0)}$	0.74 ± 0.07	0.91 ± 0.04	1.12 ± 0.07	1.32 ± 0.12	1.06 ± 0.18
$\frac{\mathrm{Br}(\pi^+ \pi^-)}{\mathrm{Br}(\pi^\mp K^\pm)}$	0.96 ± 0.60	0.67 ± 0.38	0.43 ± 0.25	0.27 ± 0.18	0.26 ± 0.06
$\frac{\tau_{B^+}}{\tau_{B^0}}\,\frac{\mathrm{Br}(\pi^+ \pi^-)}{2\mathrm{Br}(\pi^\pm \pi^0)}$	0.96 ± 0.25	0.83 ± 0.20	0.66 ± 0.15	0.50 ± 0.13	0.42 ± 0.14

I gratefully acknowledge an exciting collaboration with M. Beneke, G. Buchalla and M. Neubert, with whom the ideas discussed in this talk have been developed and from whom I have learned so much. I thank the organisers of the Kraków Epiphany Conference for creating such a stimulating atmosphere and the participants for their instructive questions and discussions.

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