

RECOIL PHASE EFFECT IN EXCLUSIVE  $B$  DECAYS:  
IMPLICATIONS FOR  $CP$  VIOLATION\* \*\*

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(Received April 4, 2001)

In the perturbative QCD approach to exclusive  $B$  decays to two light mesons, the leading twist contribution corresponds to those diagrams in the Lepage–Brodsky expansion in which the would be spectator quark receives its recoil momentum via one gluon exchange. We show that the resulting amplitude, which in the spectator model is real, acquires an imaginary part which may be comparable in size to its real part. Thus, this source of the strong interaction phase in the amplitude must be taken into account in general to discuss, reliably, the expectations for  $CP$  violation in  $B$  decays at any B-factory type scenario.

PACS numbers: 12.38.Bx, 13.25.Hw, 11.30.Er

With the start up of the SLAC and KEK and HERA-B B-factories and with the imminent upgrades the CESR and Tevatron machines to  $CP$  violation in  $B$  decays capability comes the need to clarify the theoretical expectations for this phenomenon. One important aspect of this phenomenon is the possible interplay between the strong and weak phases in the respective decay amplitudes. In particular, in decays such as  $B \rightarrow \pi\pi$ , where amplitudes with both tree level and penguin contributions are involved, it is necessary to know all sources of a possible difference in their strong phases as well as their weak phases. In this communication, we point-out an important source of a difference in the strong phases of penguins and tree contributions that is generally overlooked in the literature [1,2]. In Refs. [3–6], we

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\* Presented at the Cracow Epiphany Conference on  $b$  Physics and  $CP$  Violation, Cracow, Poland, January 5–7, 2001.

\*\* Work partly supported by the US Department of Energy Contracts DE-FG05-91ER40627 and DE-AC03-76ER00515.

have always treated this new strong phase source rigorously. As we illustrate below, unless the particular  $CP$  asymmetry parameter manifests itself already with amplitudes that only involve a single strong phase, this new strong phase must be taken into account to get reliable theoretical control of the respective parameter.

More precisely, the situation can already be seen in the diagrams in Fig. 1 for the process  $\bar{B}_s \rightarrow \rho K_S^0$ , which are to be evaluated in the perturbative QCD formalism of Lepage and Brodsky in Ref. [7] following the development of Ref. [8]. See also Ref. [9] for further applications of the methods in Ref. [8]. The graph in Fig. 1(a) has the important property that, because

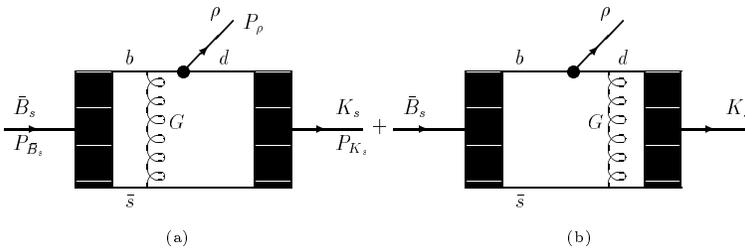


Fig. 1. The process  $\bar{B}_s \rightarrow \rho + K_S^0$ . The four-momenta are indicated in the standard manner:  $P_A$  is the four-momentum of  $A$  for all  $A$ . To leading order in the perturbative QCD expansion defined by Lepage and Brodsky in Ref. [7], the two graphs shown are the only ones that contribute in the factorisation ansatz when penguins and colour exchange between the outgoing  $\rho$  partons and the outgoing  $K_S^0$  partons are ignored. The remaining graphs in which the gluon  $G$  is exchanged between the would-be spectator  $\bar{s}$  and the remaining  $\rho$  parton lines as well as the penguin type graphs are shown in Figs. 2 and 3, where we see that, for QCD penguins, there is the added possibility that the gluon  $G$  interacts with the penguin gluon itself, of course.

$m_B > m_b + m_s$ , it is possible for the (heavy)  $b$  quark propagator to reach its perturbative QCD mass shell. This generates an imaginary part for this graph in comparison to the graph in Fig. 1(b). Similar conclusions hold for the graphs in Figs. 2 and 3 as well — the graphs in which the would-be spectator receives its recoil 4-momentum from the heavy quark line acquire an imaginary part. We refer to this effect as the recoil phase effect [3–6, 10]. This effect was always treated properly in our analyses in Refs. [3–6]. In Ref. [10], it was also treated properly. In Refs. [1, 2], it is not taken into account. In a recent analysis of the process  $B \rightarrow \pi\pi$  in Ref. [11], the dominant ‘Tree’ recoil phases in the analogue of Fig. 1 is neglected whereas the recoil phase in the diagrams in Fig. 2 and 3 are treated in some approximation. Thus, the issue is quantitative. Does it really matter whether one treats this recoil phase effect or not?

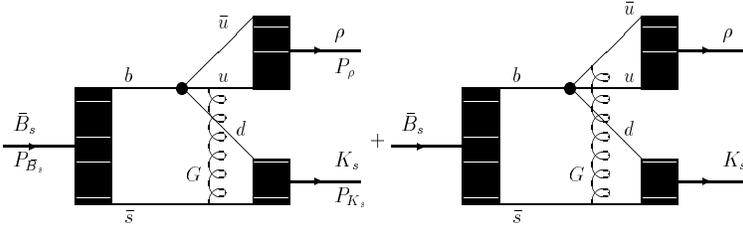


Fig. 2. The colour exchange graphs for the process  $\bar{B}_s \rightarrow \rho + K_S^0$  to leading order in the Lepage–Brodsky expansion in Ref. [7], ignoring penguins. The kinematics is as defined in Fig. 1.

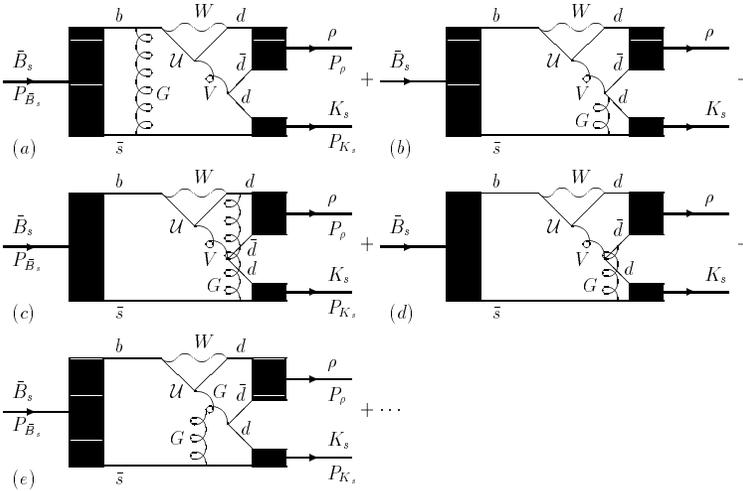


Fig. 3. The penguin graphs for the process  $\bar{B}_s \rightarrow \rho + K_S^0$ , to leading order in the Lepage–Brodsky expansion defined in Ref. [7]. The kinematics is as defined in Fig. 1.

To answer this question, we use the results [6] we have obtained for the process in Figs. 1–3. Specifically, we compute the decay width  $\Gamma(\bar{B}_s \rightarrow \rho K_S^0)$  and the penguin shift of the  $CP$  violating angle  $\gamma$ 's sine,  $\sin \gamma$ , where  $\gamma$  is defined as in Ref. [12]. Here, following Ref. [13], we define the respective shift as  $\Delta \sin \gamma$  which is given by

$$-\sin(2\gamma) - \Delta(\sin(2\gamma)) \equiv \frac{\Im \Lambda}{\frac{1}{2}(1 + |\Lambda|^2)} \quad (1)$$

for

$$\Lambda = \frac{A_T e^{-i\phi_T + i\delta_T} + \sum_j A_{P_j} e^{-i\phi_{P_j} + i\delta_{P_j}}}{A_T e^{+i\phi_T + i\delta_T} + \sum_j A_{P_j} e^{+i\phi_{P_j} + i\delta_{P_j}}}, \quad (2)$$

where the amplitude  $A_T e^{-i\phi_T + i\delta_T}$  corresponds to the tree-level weak processes in Figs. 1 and 2 and the amplitudes  $A_{P_j} e^{-i\phi_{P_j} + i\delta_{P_j}}$  correspond to the respective penguin processes in Fig. 3. Here, we identify the weak phases of the respective amplitudes as  $\phi_r$ ,  $r = T, P_j$  and the attendant strong phases as  $\delta_r$ ,  $r = T, P_j$ . In general,  $j = 1, 2$  distinguishes the electric and magnetic penguins when this is required, as one can see in the Appendix in Ref. [6]. In this notation, we have  $\gamma \equiv \phi_T$ .

The details of our calculation are given in Ref. [6]. Here, for completeness, we summarise the basic theoretical framework. Concerning the Cabibbo–Kobayashi–Maskawa (CKM) matrix itself, we follow the conventions of Gilman and Kleinknecht in Ref. [14] for the  $CP$ -violating phase  $\delta_{13} \equiv \delta$  and in view of the current limits on it we consider the entire range  $0 \leq \delta \leq 2\pi$ . For the CKM matrix parameters  $V_{td}$  and  $V_{ub}$  we also consider their extremal values from Ref. [14] (the Particle Data Group (PDG) compilation). To parametrise these extremes, we use the notation defined in Ref. [15] for  $|V_{ub}/V_{cb}|$  in terms of the parameter  $R_b = 0.385 \pm 0.166$  [14]. All other CKM matrix element parameters are taken at their central values [14]. We note that the QCD corrections to the weak interaction Lagrangian will be represented via the QCD corrected effective weak interaction Hamiltonian  $\mathcal{H}_{\text{eff}}$  as it is defined in Ref. [15]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[ \sum_{j=u,c} V_{jq}^* V_{jb} \left\{ \sum_{k=1}^2 Q_k^{jq} \tilde{C}_k(\mu) + \sum_{k=3}^{10} Q_k^q \tilde{C}_k(\mu) \right\} \right] + \text{h.c.}, \quad (3)$$

where the Wilson coefficients  $\tilde{C}_i$  and operators  $Q_k$  are as given in Ref. [15],  $G_F$  is Fermi's constant,  $\mu$  is the renormalization scale and is of  $\mathcal{O}(m_b)$  and here  $q = s$ . The application of this effective weak interaction Hamiltonian to our process  $\bar{B}_s \rightarrow \rho K_S^0$  then proceeds according to the realization of the Lepage–Brodsky expansion as described in Ref. [8]. This leads to the “dominant” contribution in which the  $\rho$  is interpolated into the operator  $O_2 = Q_1$  in  $\mathcal{H}_{\text{eff}}$  via the factorised current matrix element  $\langle \rho | \bar{u}(0) \gamma_\mu P_L u(0) | 0 \rangle$ ,  $P_L \equiv \frac{1}{2}(1 - \gamma_5)$  so that the respective remaining current in  $O_2 = Q_1$  is responsible for the  $\bar{B}_s$  to  $K_S^0$  transition shown in Fig. 1, to which we refer as the no colour exchange ‘Tree’ contribution (NC<sub>T</sub>). In Fig. 2, we show the graphs in which colour is exchanged between the would-be spectator  $\bar{s}$  in Fig. 1 and the outgoing  $\rho$  parton lines and in Fig. 3 we show the respective penguin graphs: the dominant graphs according to the prescription in Ref. [8] (3(a), 3(b)), the colour exchange graphs (3(c), 3(d)), and the exchange of the hard gluon  $G$  between the would-be spectator  $\bar{s}$  and the penguin gluon itself for QCD penguins, 3(e), which we also will classify as colour exchange. The complete amplitude for the process under study

here is given by the sum of the contribution of the graphs in Fig. 1 and those of the graphs in Figs. 2 and 3, to leading order in the Lepage–Brodsky expansion defined in Ref. [7] and realized according to the prescription in Ref. [8] as we have just described for Fig. 1, for example. The complete result for the amplitude for  $\bar{B}_s \rightarrow \rho + K_S^0$  is given in Ref. [6], where its implications for the measurement of the unitarity triangle angle  $\gamma$  are presented. Here, we investigate the recoil phase effect in this amplitude in its various aspects from Figs. 1, 2, and 3, separately.

We take for definiteness the central CKM values. As the individual phases which we present are purely due to strong interactions, we may proceed in this way without loss of physical information. We also set the value of the effective weak interaction parameter (here, note  $\tilde{C}_1 = C_2$ ,  $\tilde{C}_2 = C_1$ )  $a_2$ , which is  $C_2(m_B) + C_1(m_B)/N_c$  in perturbative QCD, to be the recent phenomenological value  $a_2 \cong 0.24$  as found in Ref. [16], but, as it scales the weak interaction, it will not affect the individual strong phases which we study. When we combine the various contributions from Figs. 1–3 to form the entire amplitude, then the weak parameters are important in determining the total phase variation of the amplitude and its attendant  $CP$  violating properties, as we shall see. More precisely, we first isolate the recoil phase of the contribution to the amplitude from the graphs in Fig. 1. From our formulas in Ref. [6] we get the strong recoil phase (all phases are in radians unless explicitly indicated otherwise)

$$\delta_{\text{NC}_T} = 0.528. \quad (4)$$

Already, this is an important result, as it and its analoga have been missed by all previous analyses of exclusive  $B$  and  $D$  decays to two light mesons except the authors' analyses [3–6] and the analysis in Ref. [10]. Evidently, analyses such as that in Ref. [17] which sometimes assume that  $\delta_{\text{NC}_T}$  is zero are misguided and incorrect. As we have checked following the procedures in Ref. [6], variation of the fundamental parameters in our calculation does not change the strong phases of our amplitude by more than  $\sim 15\%$ , so that the result in (4) and its analoga in similar  $B$  decays must be taken into account in  $CP$  violation studies.

Continuing in this way, we compute the strong recoil phase effect for the graphs in Fig. 2 as

$$\delta_{\text{CE}_T} = 0.295, \quad (5)$$

where we use the notation introduced in Ref. [6] to denote contribution from Fig. 2 as the colour exchange tree contribution  $\text{CE}_T$ . Similarly, the graphs in Fig. 3(a), (b), (c), (d) have the strong recoil phases

$$\delta_{P_1} = 0.471, \quad \delta_{P_2} = 0.360, \quad (6)$$

where  $P_j$  denotes the electric( $j = 1$ ) or magnetic( $j = 2$ ) penguin contribution, respectively. The graphs in Figs. 3(c), (d) have the strong recoil phase

$$\delta_{\text{CEP}} = -0.318, \quad (7)$$

where  $\text{CEP}$  denotes penguin graphs with colour exchange between the quarks, so that the graph in Fig. 3(e), which involves the colour exchange between the quarks in the  $\bar{B}_s$  and  $K_S^0$  mesons and the penguin gluon, has the strong recoil phase  $\delta_{\text{CEGP}}$  which we calculate to be

$$\delta_{\text{CEGP}} = 2.33, \quad (8)$$

where we neglect the magnetic form factor in these last two results. One comment is immediate: the different values of the strong recoil phases we find mean that they can not be ignored as some irrelevant over-all factor in either calculating the rates for the exclusive  $B$  decays or calculating the  $CP$  asymmetries in these decays.

Indeed, if we set the phases in Eqs. (4)–(8) to zero, we get a different set of results for the rate for the decay and its penguin pollution of the time dependent asymmetry: we find the total decay rate  $\Gamma(\bar{B}_s \rightarrow \rho K_S^0)$  that satisfies

$$\begin{aligned} 0.221 \times 10^{-20} \text{ GeV} \left( \frac{f_{B_s}}{0.141 \text{ GeV}} \right)^2 &\geq \Gamma(\bar{B}_s \rightarrow \rho K_S^0) \geq \\ 0.160 \times 10^{-20} \text{ GeV} \left( \frac{f_{B_s}}{0.141 \text{ GeV}} \right)^2 & \end{aligned} \quad (9)$$

and we find for example the penguin shift of  $\sin 2\gamma$  plotted in Fig. 4. Thus, the shift is less than 29% (allowing a  $3\sigma$  measurement of  $\sin 2\gamma$ ) for  $0 \leq \gamma \leq 75.1^\circ$  and  $103.4^\circ \leq \gamma \leq 180^\circ$ . These results should be compared with the analogous presented in Ref. [6], where we found, when the recoil strong phases are not set to zero, that

$$\begin{aligned} 0.495 \times 10^{-20} \text{ GeV} \left( \frac{f_{B_s}}{0.141 \text{ GeV}} \right)^2 &\geq \Gamma(\bar{B}_s \rightarrow \rho K_S^0) \geq \\ 0.329 \times 10^{-21} \text{ GeV} \left( \frac{f_{B_s}}{0.141 \text{ GeV}} \right)^2 & \end{aligned} \quad (10)$$

and that the shift is less than 29% for  $0 \leq \gamma \leq 40.5^\circ$  and  $102.5^\circ \leq \gamma \leq 157.9^\circ$ . The differences in these two sets of results show that the recoil phase effect cannot be ignored in exclusive  $B$  decays of the type discussed in this paper.

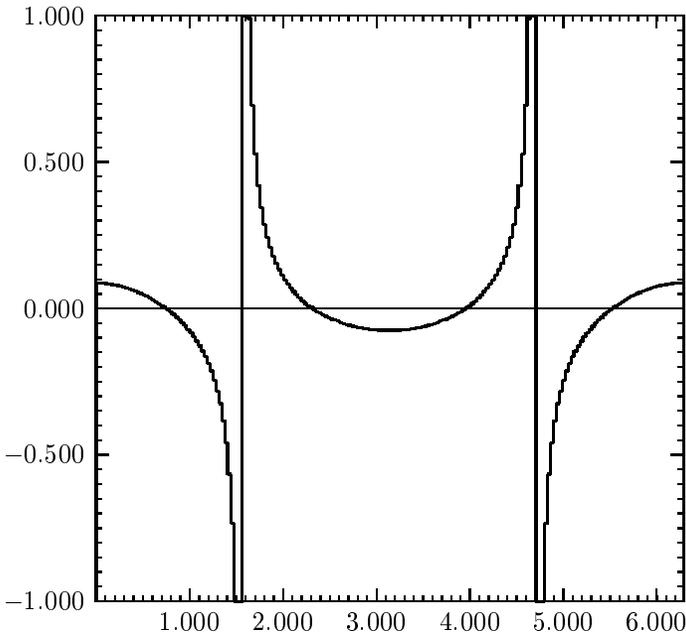


Fig. 4. Penguin shift of the  $CP$  asymmetry  $\sin(2\gamma)$  in  $\bar{B}_s \rightarrow \rho K_S^0$  for  $R_b = 0.385$  for the matrix element with the recoil phase set to zero by using the principle value prescription in the diagrams in Figs. 1–3. The analogous plots obtain for the  $\pm 1\sigma$  values of  $R_b$  as discussed in the text.

This brings us to a comparison of our analysis with those presented in Refs. [1, 2, 11]. To illustrate the size of the recoil phase effect, we use the  $\bar{B} \rightarrow \pi\pi$  process which we have already analysed in Ref. [5] and which Beneke *et al.* have treated in Ref. [11]. From our Eq. (5) in Ref. [5] we see that, if the recoil phases are set to zero in defining the integrals over the light-cone fractions in the analogue of the diagrams in Figs. 1–3 here for the  $\pi^+\pi^-$  case, the decay rates given in Eq. (8) of Ref. [5] are changed by as much as  $\sim 90\%$ . Moreover, if, as Beneke *et al.* do, we set to zero the recoil phase of the ‘dominant’ Tree contribution in the analogue of Fig. 1 here, these decay rates are still changed by as much as  $\sim 90\%$ . Thus, none of the treatments of the recoil phase in Refs. [1, 2, 11] is sufficient.

The situation is entirely similar to the  $\rho K_S^0$  case discussed above insofar as the time dependent  $CP$  violating asymmetry is concerned — neither the complete neglect of the recoil phase in Refs. [1, 2] nor the neglect of the

recoil phase of the dominant ‘Tree’ contribution from the analogue of Fig. 1 here as in Ref. [11] gives the proper result shown in Fig. 4 of Ref. [5] for the dependence of the penguin pollution on  $\delta_{13}$ . To see how big the respective distortion can be on the  $CP$  violating asymmetry itself, we plot in Fig. 5 the value of the direct  $CP$  violating asymmetry [17],  $\mathcal{A}_{CP}^{\text{dir}}(\pi\pi)$ , for the  $\bar{B} \rightarrow \pi^+\pi^-$  case as derived from Eq. (5) in Ref. [5]. This should be com-

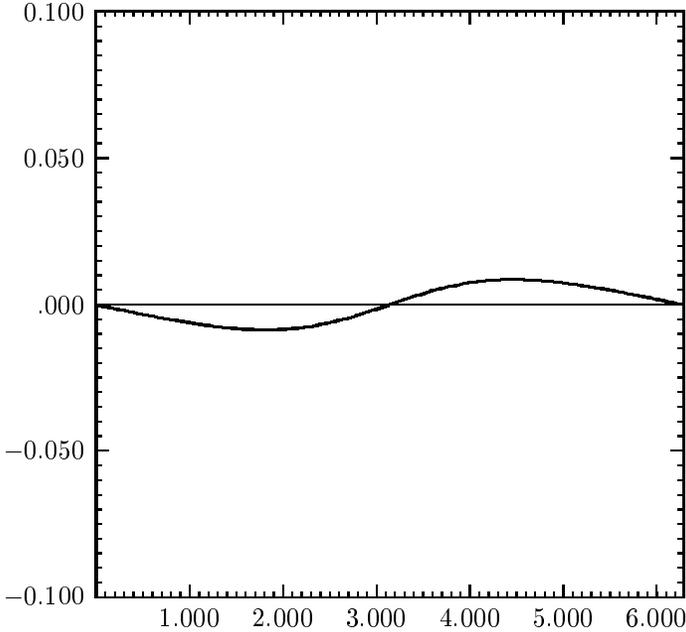


Fig. 5. Direct  $CP$  asymmetry for  $\bar{B} \rightarrow \pi^+\pi^-$ ,  $\mathcal{A}_{CP}^{\text{dir}}(\pi\pi)_d$ , for  $R_b = 0.385$  as calculated from the amplitude in Eq. (5) of Ref. [5], which is derived from the analoga of the diagrams in Figs. 1–3.

pared to the result of Beneke *et al.* [11],  $-0.04 \times \sin \gamma$ . Evidently, experiment will soon distinguish these two results. For reference, we also record the direct  $CP$  violating asymmetry for the  $\bar{B}_s \rightarrow \rho K_S^0$  case,  $\mathcal{A}_{CP}^{\text{dir}}(\rho K_S^0)_s$ , as a function of  $\gamma$  in Fig. 6. We see that it is substantial in a large part of the preferred regime  $45^\circ \leq \gamma \leq 135^\circ$ , just as it has a large part of its most nonzero value in this region in the case of  $\mathcal{A}_{CP}^{\text{dir}}(\pi^+\pi^-)_d$ . The recoil phase effect is an essential part of the results in Figs. 5 and 6. For proving  $CP$  violation in the  $B$  system, these modes suggest that a measurement of  $\mathcal{A}_{CP}^{\text{dir}}$  may be a reasonable way to proceed.

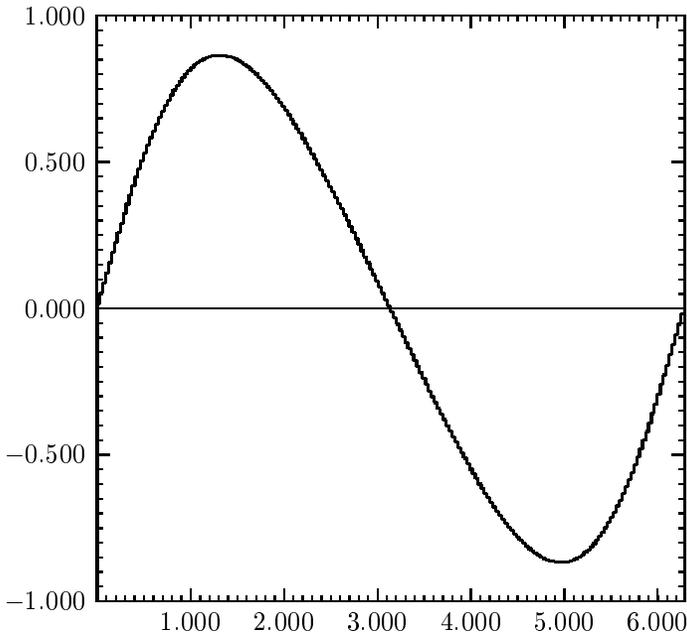


Fig. 6. Direct  $CP$  asymmetry for  $\bar{B}_s \rightarrow \rho K_S^0$ ,  $\mathcal{A}_{CP}^{\text{dir}}(\rho K_S^0)_s$ , for  $R_b = 0.385$  as calculated from the diagrams in Figs. 1–3.

Next, we turn to the case of the modes  $D^* \pi$ , where we follow the notation of Ref. [17] and refer to  $f = D^{*+} \pi^-$ ,  $\bar{f} = D^{*-} \pi^+$ . A strategy advocated in Ref. [17] is to measure the combination  $2\beta + \gamma$  in the time-dependent asymmetries for  $\bar{B} \rightarrow f$  and  $\bar{B} \rightarrow \bar{f}$  using the fact that the product  $\xi_f^{(d)} \times \xi_{\bar{f}}^{(d)}$  yields  $e^{-2i(2\beta+\gamma)}$  if we define (here,  $\phi_d$  is the  $B_d$  mixing phase  $2\beta$ ,  $\lambda \equiv |V_{us}|$ )

$$\begin{aligned} \xi_f^{(d)} &= -e^{-i\phi_d} \frac{A(\bar{B}_d^0 \rightarrow f)}{A(B_d^0 \rightarrow f)} = -e^{-i(\phi_d+\gamma)} \left( \frac{1-\lambda^2}{\lambda^2 R_b} \right) \frac{\bar{M}_f}{M_{\bar{f}}}, \\ \xi_{\bar{f}}^{(d)} &= -e^{-i\phi_d} \frac{A(\bar{B}_d^0 \rightarrow \bar{f})}{A(B_d^0 \rightarrow \bar{f})} = -e^{-i(\phi_d+\gamma)} \left( \frac{\lambda^2 R_b}{1-\lambda^2} \right) \frac{M_{\bar{f}}}{\bar{M}_f}, \end{aligned} \quad (11)$$

for the amplitudes  $A(\bar{B}_d^0 \rightarrow f, \bar{f})$  and their  $CP$  conjugates, respectively. Thus,  $\bar{M}_f$ ,  $M_{\bar{f}}$  are the respective strong interaction matrix elements defined in Eq. (3.26) of Ref. [17]. The point is that, in the actual extraction of the time dependent asymmetry, the strong recoil phase effect gives a non-trivial

value to the strong phase  $\Delta_S$ , as defined in Ref. [17], in the ratio  $\bar{M}_f/M_{\bar{f}}$ . In Ref. [17], this phase has been set to 0 to estimate how accurately the weak phase could be measured in the LHCb environment. Upon calculating the analogue of Fig. 1 for these processes, we find that the value of  $\Delta_S$  is  $-253.6^\circ$ . Thus, the analysis in Ref. [17] should address non-trivial values of  $\Delta_S$  also.

The analysis in Ref. [17] also attempts to use  $u$ -spin and SU(3) symmetry to isolate  $\gamma$  in several modes,  $B \rightarrow \pi K$ ,  $\bar{B}_{s,d} \rightarrow \Psi/JK_S^0$ , and  $\bar{B} \rightarrow \pi\pi$ ,  $KK$  modes. Here, we discuss the perturbative QCD expectations for these assumptions. Since the tree and penguin contributions enter with different CKM coefficients,  $V_{UD}^*V_{Ub}$ , to show the inadequacy of  $u$ -spin symmetry, it is enough to focus on the analogue of Fig. 1 for these decays. The complete predictions from the analogue of all the graphs in Figs. 1–3 will appear elsewhere [18]. For the processes  $\bar{B}_{s,d} \rightarrow \Psi/JK_S^0$  we find for the analoga of Fig. 1 the recoil phases

$$\delta_T(B_s) = 0.982, \quad \delta_T(B_d) = 2.24, \quad (12)$$

and the ratio of strong transition amplitude moduli squared

$$\frac{|\mathcal{A}'|^2}{|\mathcal{A}|^2} = 1.81, \quad (13)$$

where  $\delta_T(B_s)$ ,  $\delta_T B_d$  are the respective strong recoil phases for the graphs in Fig. 1 for the  $\bar{B}_d$  and  $\bar{B}_s$  cases respectively and  $\mathcal{A}'$ ,  $\mathcal{A}$  are the respective strong transition amplitudes. Evidently, the assumption of SU(3) and  $u$ -spin symmetry in exclusive  $B$  decays to light mesons is completely unfounded and the recoil phase effect makes the situation even more acute; for, if the recoil phase is ignored, the 1.81 in (13) becomes 2.24.

In summary, we have shown that the physical phenomenon of the recoil phase effect is important for  $CP$  violation studies in  $B$  decays to two light mesons. We have shown how to take it into account in Refs. [3–6]. We look forward to its further application to the exciting field of  $CP$  violation studies in exclusive  $B$  decays.

The author acknowledges the kind hospitality of Prof. C. Prescott and SLAC Group A and helpful discussions with Drs. P. Dauncey and Robert Fleischer and Prof. L. Lanceri at various stages of this work. The author thanks Prof. M. Jezabek and the Organizing Committee for inviting him to lecture in the 2001 Cracow Epiphany Conference.

### Notes added

1. The imaginary parts which we find in the recoil exchanges in Figs. 1–3 are all leading twist effects. They arise from the (anomalous) solutions of the respective Cutkowsky–Landau–Bjorken equations associated with these graphs, as described in the book by J.D. Bjorken and S.D. Drell, *Relativistic Quantum Fields*, McGraw-Hill, Menlo Park, 1965. Any consistent dispersive treatment of these graphs has to take all of these solutions into account, both anomalous and non-anomalous solutions.
2. As the semi-leptonic decay distribution has the form  $d\Gamma(B \rightarrow X_{\mathcal{U}} + \ell + \nu_{\ell}) = |V_{\mathcal{U}b}|^2 |F_{\text{QCD}}^{\mathcal{U}}|^2 d\text{LIPS}$ ,  $\mathcal{U} = u, c$ , where  $d\text{LIPS}$  is the respective Lorentz invariant phase space factor and both the moduli  $|V_{\mathcal{U}b}|$  and the strong interaction transition amplitude factor  $F_{\text{QCD}}^{\mathcal{U}}$  are  $CP$  invariant, it follows that the analogue of the recoil phase in Fig. 1 for the semi-leptonic decays does not generate  $CP$  violation in these decays.
3. We finally stress that the Lepage–Brodsky expansion in Ref. [7] is an exact re-arrangement of the exact Bethe–Salpeter bound state transition amplitude. Only when authors make arbitrary truncations of the expansion, for example, treating the endpoint contributions at higher twist without including the respective Sudakov resummation that makes them finite, do unknown parameters appear in the application of the expansion to hard interaction processes such as exclusive  $B$  decays to two light mesons.

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