INELASTIC RESCATTERRING IN $B \to \pi\pi, K\overline{K}$*

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Inelastic rescatterring effects in B decays into a pair PP of pseudoscalar mesons ($PP = \pi\pi$ or $K\overline{K}$) are discussed. It is assumed that B meson decays through a short-distance tree-diagram process into two jet-like states composed of low-mass resonances $M_1 M_2$ which rescatter into PP. The rescatterring of resonance pair $M_1 M_2$ into the final PP state is assumed to proceed through a Regge flavour exchange. Since such processes constitute a fraction, diminishing with increasing energy, of total inelastic PP scattering, the inelastic rescattering contribution should die out for $m_B \to \infty$. At $m_B = 5.2$ GeV, however, explicit estimates show that rescattering corrections could be substantial, leading to long-distance corrections to $B^0 \to K\overline{K}$, comparable to short-distance penguin contributions.

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Studies of CP violation in B decays must involve Final State Interaction (FSI) effects. Unfortunately, a reliable estimate of such effects is very hard to achieve. In the analyses of $B \to PP$ decays (P — pseudoscalar meson) only some intermediate states, believed to provide nonnegligible contributions, are usually taken into account. Most often studies are restricted to the case of elastic or quasi-elastic rescattering.

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1. Quasi-elastic rescattering

Consider quasi-elastic rescattering in B^0 decays into $\pi^+\pi^-$ and $\pi^0\pi^0$. The Short-Distance (SD) amplitudes are

$$\begin{aligned} w\left(B^0 \to \pi^+\pi^-\right) &= -\frac{1}{\sqrt{2}}(T+P)\,, \\ w\left(B^0 \to \pi^0\pi^0\right) &= \frac{1}{2}P\,, \end{aligned} \tag{1}$$

when only the dominant T (tree) and P (penguin) amplitudes are taken into account (Fig. 1). Since isospin is a good symmetry of strong interactions, rescattering is diagonal in the basis of a definite total isospin. Consequently, one expects that the amplitudes W of definite final isospin I are modified by Watson phases δ_I only:

$$\begin{aligned} W\Big(B \to (\pi\pi)_0\Big) &= -\frac{1}{\sqrt{3}}\Big(T+\frac{3}{2}P\Big)\exp(i\delta_0)\,, \\ W\Big(B \to (\pi\pi)_2\Big) &= -\frac{1}{\sqrt{6}}\,T\exp(i\delta_2)\,. \end{aligned} \tag{2}$$

The physical assumption that goes into Eq. (2) is that the probabilities of SD decays are not changed by such a Long-Distance (LD) effect as rescattering. When decay amplitude into a (*e.g.*) $\pi^+\pi^-$ state is extracted from Eqs. (2) one obtains

$$W\left(B^0 \to \pi^+\pi^-\right) = -\frac{1}{\sqrt{2}}\exp\left(i\delta_2\right)\left(T+\frac{2}{3}T\left(\exp\left(i\varDelta\right)-1\right)+P\exp\left(i\varDelta\right)\right), \tag{3}$$

where $\varDelta = \delta_0 - \delta_2$. The appearance of the second term on the r.h.s. of Eq. (3) indicates that in general final-state interactions cannot be properly

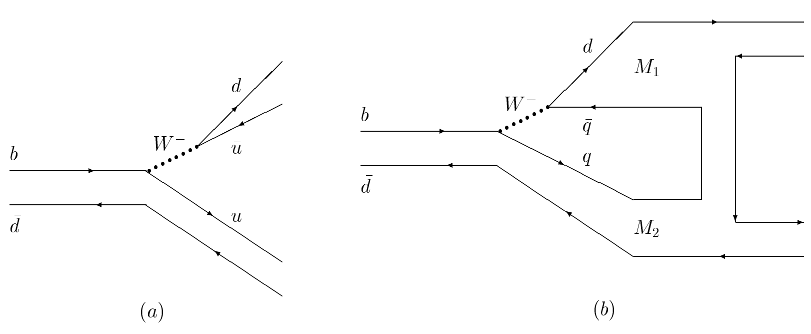

Fig. 1. Dominant diagrams for B decay: (a) tree T and (b) penguin P.

described if one multiplies short-distance tree and penguin amplitudes in Eq. (1) by two different phase parameters [1, 2]. The only exception is when the two phases δ_0 and δ_2 are equal, which occurs if only elastic $\pi\pi$ rescattering is allowed: the Pomeron does not distinguish between $\pi^+\pi^-$ and $\pi^0\pi^0$, or $(\pi\pi)_0$ and $(\pi\pi)_2$.

2. Short-distance penguin and rescattering contributions

The short-distance penguin amplitude P is estimated at 20% of the tree contribution. It is of the order of λ^3, where $\lambda \equiv \sin\theta_{\rm C}$ is one of the four parameters in the Wolfenstein's parametrization of the Cabibbo–Kobayashi–Maskawa (CKM) matrix. Due to the dominance by the intermediate top quark, P has the weak phase β from the V_{td} element of the CKM matrix. Although λ is not small enough to permit real distinction between terms differing by one factor of λ, we observe that if Fig. 1(b) is understood as a diagram representing low-energy rescattering through charmed ($q = c$) or noncharmed ($q = u$) intermediate states, these processes are also of the order of λ^3. For $q = c$, the amplitude of this "charming penguin" has weak phase 0. For $q = u$, the diagram represents quasi-elastic ($\pi\pi \to \pi\pi$) and inelastic ($M_1M_2 \to \pi\pi$) rescattering. The corresponding amplitude has weak phase $-\gamma$ of the V_{ub} element of the CKM matrix. With one of the goals of the present program of CP violation studies being the determination of CP violating phases α, β, γ, it is clearly very important to know which of the penguin-like admixtures to tree diagrams is in fact dominant (if any) and, consequently, which weak phase will be probed by experiments.

3. Simple two-channel model

Since a large part of the (time-reversal invariant) inelastic $\pi\pi$ scattering at energy $\sqrt{s} = m_B = 5.2\,\text{GeV}$ (or larger) goes into multiparticle final states composed of noncharmed mesons, one may conjecture that inelastic rescattering processes shown in Fig. 1(b) (with $q = u$) will be important. A very simplified two-channel model of what should be expected of inelastic rescattering was presented in [3, 4]. In this model there are two states: $|f_1\rangle = |\pi\pi\rangle$, and $|f_2\rangle$ representing "everything else" that $\pi\pi$ might scatter into. The most general 2×2 unitary S matrix is

$$S = \begin{bmatrix} \cos 2\theta & i\sin 2\theta \\ i\sin 2\theta & \cos 2\theta \end{bmatrix}, \tag{4}$$

with the top left element describing $\pi\pi \to \pi\pi$ scattering.

If one accepts that final state interactions cannot modify the probability of the original SD weak decay, it follows that vector $\boldsymbol{W}$ representing the FSI-corrected amplitudes is related to vector $\boldsymbol{w}$ of the original SD amplitudes through [4]

$$\boldsymbol{W} = \boldsymbol{S}^{1/2}\boldsymbol{w} \approx \left(\boldsymbol{1} + \frac{i}{2}\boldsymbol{T}\right) \boldsymbol{w}\,, \tag{5}$$

(where the last equality follows if $\boldsymbol{S}$ is close to $\boldsymbol{1}$, *i.e.* if rescattering is assumed small). The appearance of the square root of the $\mathbf{S}$ matrix is related to Watson's theorem: in the basis of $\mathbf{S}$-matrix eigenstates $|\nu\rangle$, the above equation reduces to $W_\nu = \mathrm{e}^{i\delta_\nu} w_\nu$, *i.e.* the condition of unchanged probability ($|W_\nu| = |w_\nu|$) admits Watson phases only.

With

$$\mathbf{S}^{1/2} = \left[\begin{array}{cc} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{array} \right], \tag{6}$$

one immediately obtains

$$W_1 = w_1\cos\theta + iw_2\sin\theta \equiv |W_1|\,\mathrm{e}^{i\delta_1}\,. \tag{7}$$

Since from πN data extrapolated to $\pi\pi$ one gets $\cos 2\theta \approx 0.6$–0.7, one finds

$$\tan\delta_1 = \frac{w_1}{w_2}\tan\theta \approx 0.45\frac{w_1}{w_2}\,. \tag{8}$$

If B meson decays with similar strengths into the $|\pi\pi\rangle$ and $|f_2\rangle$ states, one calculates $\delta_1 \approx 20^\circ$–$25^\circ$. In principle, therefore, the effect of inelastic rescattering may be large.

4. Inelastic rescattering at high energy

In the previous example all states produced in the first stage of the decay could be rescattered into $\pi\pi$. This is an overpessimistic situation. Consider a little more general case with three states $|f_1\rangle \equiv |\pi\pi\rangle$, $|f_2\rangle$, and $|f_3\rangle$, into which $\pi\pi$ scattering may go. It may happen that B meson does not decay into state $|f_3\rangle$. If $\pi\pi$ inelastic scattering is dominated by $|\pi\pi\rangle \to |f_3\rangle$ transitions with $|\pi\pi\rangle \to |f_2\rangle$ small or negligible, then the total inelastic rescattering contribution corresponds to $|B\rangle \to |f_2\rangle \to |\pi\pi\rangle$ and must be fairly weak. In such a case neglecting final state interaction altogether might be a good approximation: elastic FSIs result in the multiplication of SD amplitudes by the same phase for all isospin (or SU(3) symmetry) related final states.

In order to know whether neglecting inelastic rescattering is or is not justified, we have to estimate the contribution from states of type $|f_2\rangle$ (*i.e.* those into which B decays) in the unitarity relation for the $l = 0$ partial wave in $\pi\pi$ scattering

$$|\langle\pi\pi|S|\pi\pi\rangle|^2 + \sum_k |\langle\pi\pi|S|f_k\rangle|^2 = 1\,. \tag{9}$$

Inelastic production of particles in high energy $\pi\pi$ scattering is expected to proceed after the projectiles have exchanged a gluon. Colour separation is prevented by the production of many $q\bar{q}$ pairs whose number increases with energy. Colour singlet quark-antiquark pairs materialize later as resonances whose momenta are approximately parallel to the axis of $\pi\pi$ collision in c.m.s. Thus, a typical inelastic process may be represented by the diagram shown in Fig. 2(a), in which the ordering of quark lines from top to bottom may be roughly correlated with the ordering in rapidity of produced resonances. Any transfer of flavour quantum numbers over large distances in rapidity is suppressed. This corresponds to suppression of flavour-exchange in quasi-two-body production processes $\pi\pi \to M_1 M_2$ at high energies (the time-reversed process of this type is shown on the r.h.s. of the diagram in Fig. 1(b) with $q = u$). It is through the presence of many free quark-antiquark pairs of *unconstrained* flavour that the combinatorial factors beat suppressions of flavour transfer and, through the production of more and more resonances with increasing energy, they maintain approximately constant inelastic cross section.

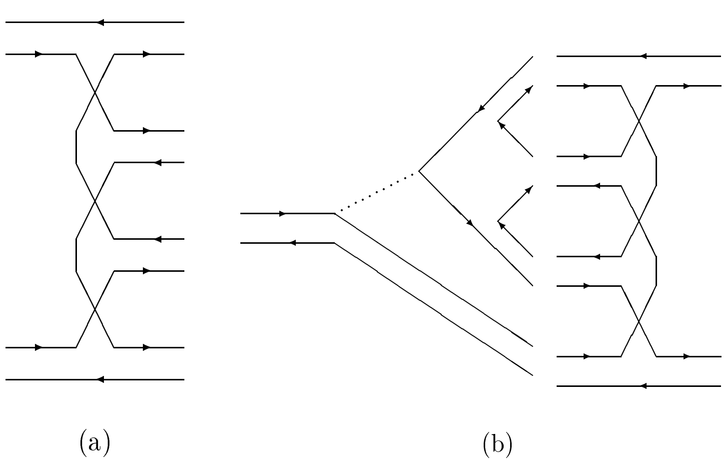

Fig. 2. (a) Multiparticle production at high energy. (b) Suppression mechanism for $B \to |f_4\rangle \to PP$.

As a particular realisation of these ideas we may consider the multi-Regge model of resonance production discussed over twenty years ago [5]. In this model inelastic $\pi\pi$ collisions produce a variable number of resonances ordered along the rapidity axis with Regge exchanges in between the resonances. As energy (m_B in our case) increases, the average number of Regge exchanges in question and the average number of resonances produced must increase, so that no flavour exchange over a large distance in rapidity space occurs. Thus, at high energy one expects the production of two-resonance states $|f_2\rangle$ to die out. The same will happen to the production of three-resonance states, and so on, while the majority of the inelastic cross section will shift to the production of k resonance states with average k increasing with energy. This situation might be described by replacing $|f_3\rangle$ with a set of k resonance states $|f_k\rangle$. (In fact, each of states $|f_k\rangle$ should be labelled with an additional index to distinguish between states consisting of different k resonances.) A first thought is that states $|f_k\rangle$ for $k \geq 3$ cannot contribute to rescattering in B decays. However, since resonances may decay to lighter ones, interference terms may occur. We will argue, however, that such terms should be small and vanishing at high energy.

As an example, let us consider the case when m_B is large and the heavy resonance M_1 from state $|f_2\rangle$ decays into three lighter resonances, which later rescatter through $|f_4\rangle \to |\pi\pi\rangle$ (the argument is better when the number of resonances into which resonance M_1 decays is larger, *i.e.* when m_{M_1} and m_B are larger). The rescattering part of the process (the diagram on the r.h.s. of Fig. 2(b)) cannot contain quark lines connecting outgoing pions: such processes are suppressed at high energies. Thus, there must be quark–antiquark pairs produced out of the vacuum (*e.g.* the line connecting the first and the third topmost resonances). When one takes the square of the amplitude corresponding to the diagram on the r.h.s. of Fig. 2(b) to evaluate its contribution to inelastic $\pi\pi$ scattering, one obtains amplitude represented by a diagram with free quark loops of unconstrained flavour. These loops lead to a large combinatorial factor mentioned earlier. However, when the left- and right-hand sides of the diagram of Fig. 2(b) are combined to represent the decay followed by rescattering, the combinatorial factor is dramatically reduced: there are no quark lines of unconstrained flavour. Within the language of ref. [5], the resulting combined diagrams are even more suppressed: namely those parts of a diagram in which quark lines of an intermediate resonance "cross" (the second resonance from the top in Fig. 2(b)) represent cancellations between resonances of opposite C-parities. Thus, only low-mass resonances M_1 and M_2 will contribute to the rescattering. Since with increasing energy inelastic $\pi\pi$ scattering is dominated by channels with an ever-increasing number of produced resonances, it follows that inelastic rescattering in B decays should die out at $m_B \to \infty$. It is interesting to note that the dissappearing of rescattering effects at $m_B \to \infty$ was found also in a different framework [6].

The mismatch between the left- and right-hand sides of the diagram in Fig. 2(b) should be attributed to the difference in the original separation of colour: receding quarks of the decaying M_1 resonance stretch a 3–$\bar{3}$ colour string, while in the inelastic rescattering the dominant part comes from octet separation. The latter requires an exchange (diminishing with increasing energy) of a quark to reduce it to 3–$\bar{3}$. Since vanishing of inelastic rescattering at $m_B \to \infty$ is related to colour mismatch, this vanishing should be present in all models.

5. Inelastic rescattering at $m_B = 5.2$ GeV

Although inelastic rescattering is expected to die out at high energy, one may ask if the value of $m_B = 5.2\,\mathrm{GeV}$ is sufficiently high to neglect it. In order to make some estimates, we turn to multi-Regge models of multiparticle production processes [5]. In these models the distribution of the number of produced resonances was evaluated as a function of energy. It turns out [7] that in the $\pi\pi$ scattering at $\sqrt{s} = 5.2\,\mathrm{GeV}$, final states composed of just two resonances occur in 50% of cases, while three-resonance states show up in 35% cases, and so on. Only at much higher $\sqrt{s}$ of the order of 20–30 GeV and more, the production of three and more resonances becomes dominant. Thus, inelastic rescattering is not negligible at $m_B = 5.2\,\mathrm{GeV}$, and one has to estimate it explicitly.

In order to do that, we assume that the average amplitude for the $PP \to M_1M_2$ process is approximately equal to the flavour-exchange Regge amplitude in $PP \to PP$. From πN data extrapolated to $\pi\pi$ scattering one can estimate that at $\sqrt{s} = 5.2\,\mathrm{GeV}$, the $l = 0$ partial wave projection of the latter leads to

$$|\langle PP|S_{\mathrm{Reggeon}}|PP\rangle|^2 = 0.025\,, \tag{10}$$

for a typical state $|PP\rangle$ averaged over different SU(3)-flavour representations of the PP system [8, 9]. On the other hand, Pomeron exchange yields

$$\langle PP|S_{\mathrm{Pomeron}}|PP\rangle = \langle\pi\pi|S_{\mathrm{Pomeron}}|\pi\pi\rangle = \cos 2\theta \approx 0.65\,. \tag{11}$$

If one assumes further that all $\langle PP|S|f_k\rangle$ amplitudes are approximately equal to $\langle PP|S_{\mathrm{Reggeon}}|PP\rangle$ (for average state of $|f_2\rangle$ type this assumption is in agreement with phenomenological analyses [8, 10]), one obtains from Eq. (9):

$$|\langle PP|S_{\mathrm{Pomeron}}|PP\rangle|^2 + n_{\mathrm{tot}}|\langle PP|S_{\mathrm{Reggeon}}|PP\rangle|^2 = 1\,, \tag{12}$$

where $n_{\rm tot}$ denotes the number of average channels. Using Eqs. (11), (10) one estimates that

$$n_{\rm tot} \approx 25\,, \tag{13}$$

or more [8]. With 50% of the total cross section at $\sqrt{s} = 5.2\,{\rm GeV}$ being estimated as due to the production of two-resonance states, one finds that the number of two-resonance states is of the order of

$$n_2 \approx 12\,. \tag{14}$$

One may also try to directly estimate the number of M_1M_2 states produced by the short-distance decay mechanism. The SD decay amplitude is dominated by the tree-diagram contribution. The M_1M_2 states produced through this process include both states composed of light pseudoscalar mesons ($\pi\pi$ in the simplest case) as well as various resonances. From the known b-quark SD decay probability

$$v\left(q^2\right) = 2\left(1-\frac{q^2}{m_b^2}\right)^2\left(1+\frac{2q^2}{m_b^2}\right), \tag{15}$$

one can see that for $m_B \approx 5.2\,{\rm GeV}$ even quite massive resonances M_1 (3–4 GeV) could be formed. The M_2 spectrum extends to $m_2 \approx 2.0$ or 2.5 GeV. The average m_1 mass is of the order of 2.5 GeV, while m_2 is a little smaller, around 1.4 GeV. The number of M_1M_2 states produced in the above range of masses may be estimated on the basis of the ISGW2 approach [11]. One obtains [8] around 10–20 different M_1M_2 states, in agreement with Eq. (14).

In order to perform more realistic estimates of rescattering effects and the way they might affect CP violation phenomena, it is important to know the phases of $M_1M_2 \to PP$ amplitudes. These may be estimated by analysing Regge behaviour of such amplitudes in dual models [5]. One finds that for the FSI amplitudes of the r.h.s. of Fig. 1(b) the phase factor arises from

$$\left(-\frac{s}{m_1^2m_2^2}\right)^{\alpha(t)}, \tag{16}$$

where $\alpha(t)$ is the Regge trajectory. If the exchanged Reggeon just interchanges two quarks in the s-channel, the phase is real. After projecting Regge amplitudes onto the $l = 0$ partial wave, the dependence on resonance masses seen in Eq. (16) gives rise to a phase factor depending on the product $(m_1m_2)^2$. For low M_1, M_2 masses, wherefrom the dominant part of rescattering is expected to come, this phase turns out to range from about

-40° at $(m_1m_2)^2 = 1\,\mathrm{GeV}^4$ to $+20^\circ$ at $(m_1m_2) \approx 7\mathrm{GeV}^4$, where Regge approximation starts to become questionable (approximation of Eq. (16) requires values of $s/(m_1m_2)^2$ larger than $4\,\mathrm{GeV}^{-2}$ or so).

Contributions from rescattering through different M_1M_2 states add up according to the distribution of $(m_1m_2)^2$ produced in SD decay. Using Eq. (5) and an appropriately chosen $(m_1m_2)^2$ distribution, the combined effect of rescattering through all intermediate inelastic states can be estimated from:

$$\langle PP|W|B\rangle = \langle PP|1 - \tfrac{1}{2}\mathrm{Im}\,T|PP\rangle\langle PP|w|B\rangle + \tfrac{i}{2}\sum_{j=1}^{n_2}\langle PP|T|f_{2,j}\rangle\langle f_{2,j}|w|B\rangle, \tag{17}$$

where the sum over j from 1 to n_2 in the second term corresponds to the summation over different values of $(m_1m_2)^2$ starting from its smallest value and according to its distribution. The first term in Eq. (17) describes SD decay amplitude with corrections due to elastic rescattering. Since the phases of amplitudes $\langle PP|T|f_{2,j}\rangle$ are not far from zero one expects that the phase of the total contribution from inelastic rescattering in B decays is around 90°.

Numerical estimates of rescattering in $B \to \pi\pi, K\overline{K}$ decays, based on Eq. (17), lead to significant corrections to SD estimates (see [8]). As an example, in Fig. 3 the dependence on the number n of inelastic two-resonance channels included (*i.e.* when n_2 in Eq. (17) is replaced by variable n) is shown for the absolute size of the (zero isospin) $W((K\overline{K})_0)$ amplitude. The amplitude is given in the units of the initial input tree amplitude T. In pure short-distance approach, this amplitude is equal to $W((K\overline{K})_0) = P_{\mathrm{SD}}/2 \approx 0.1\ T$ since the relevant penguin amplitude is usually estimated at $P_{\mathrm{SD}} \approx 0.2\ T$. As shown in Fig. 3, the rescattering-induced contribution to $W((K\overline{K})_0)$ becomes equal to the SD contribution already at $n \approx 5$.

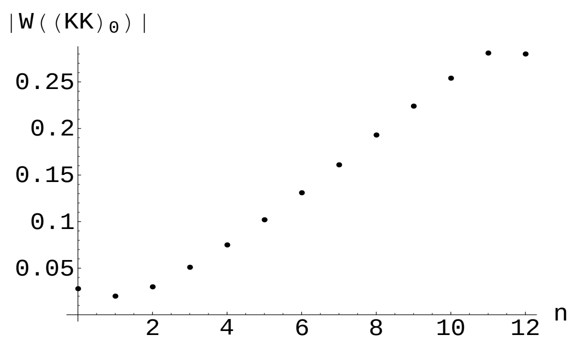

Fig. 3. Dependence of FSI-induced effects on the number of intermediate channels included for $B \to (K\overline{K})_{I=0}$ decays.

One has to conclude that although for $m_B \to \infty$ one may expect that the effects of inelastic rescattering in B decays vanish, for the physical value of $m_B = 5.2\,\text{GeV}$ these effects should be still important. Consequently, unless some unknown reasons justifying the neglect of rescattering exist, the extraction of CP violating parameters from nonleptonic B decays will be most probably very difficult. One may still hope, however, that when enough data on many different B decay channels are accumulated, one will be able to remove the effects of FSIs, thus reaching the parameters of the quark level.

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