THIRTEEN YEARS OF HEAVY QUARK EXPANSION: EXAMPLES FOR ITS PROGRESS AND ITS PROBLEMS*

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The heavy mass expansion has become a standard tool that has significantly evolved since its formulation about thirteen years ago. Some of the major results of the heavy mass expansion, namely the determination of V_{cb} (both exclusive and inclusive) and lifetime calculations, are reviewed and some open problems are pointed out.

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1. Introduction

The heavy mass expansion is applied to decay processes of heavy quarks already for more than thirteen years. After the early work by Shifman, and Voloshin [1] the symmetries that appear in the heavy mass limit have been formulated in a clean way by Isgur and Wise in two famous papers [2, 3], which are among the most often quoted papers on phenomenological particle physics of the last decade. Based on [1–3], one could obtain relations between form factors which are used (in a refined form) in the context of the extraction of *e.g.* V_{cb} from exclusive decays.

This limit of QCD could be formulated as an effective field theory, the so called Heavy Quark Effective Field Theory (HQET) [4, 5]. It allows us to put the expansion in inverse powers of the heavy quark mass $m_{\rm Q}$ on a field-theoretical basis and thus to treat the radiative corrections in the heavy mass limit in a systematic way using the language of Feynman Diagramms. In addition this gives us a way to parameterize the sub-leading terms of the $1/m_{\rm Q}$ expansion in terms of higher dimensional operators.

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Parallel to this development a description of inclusive decays has been worked out [6–9], including also sub-leading terms in the $1/m_{\rm Q}$ expansion as well as radiative corrections. The additional piece of input here is to use the operator product expansion in a similar way as in deep inelastic scattering, making use of the fact that the $m_{\rm Q}$ is a scale much larger than $\Lambda_{\rm QCD}$. Soon it became clear that these two branches have to merge into a unique description which we now know as the heavy quark expansion.

Since then, many detailed calculations have been performed, and many reviews have been written, such that I can quote only a small selection of them [10–14]. In particular, the sub-leading (and in some cases even the sub-sub-leading) terms in $\alpha_s(m_{\rm Q})$ have been computed. Using the data from Cornell, which became more and more accurate, as well as from the new B factories, we have even gained some control over the sub-leading terms of the $1/m_{\rm Q}$ expansion, which involves certain non-perturbative parameters.

The main progress triggered by the discovery of the heavy quark symmetries and the systematic expansion in powers of $1/m_{\rm Q}$ is in the sector of semi-leptonic processes, exclusive as well as inclusive. As far as non-leptonic processes are concerned, the heavy quark expansion can be applied to lifetimes and also to a calculation of the semi-leptonic branching fraction, and some of the results are briefly outlined below. However, up to now no significant progress has been made in the field of exclusive non-leptonic decays, although here a few new ideas are suggested which eventually could lead to some progress even for this class of decays [15, 16].

In this mini-review I shall pick out a few subjects in which over the last years a significant progress has been made. I shall discuss the exclusive heavy-to-heavy decays and the extraction of V_{cb} in the next section, in Sec. 3 I shall consider inclusive transitions, including the determinations of V_{cb} from these processes as well as a short discussion of the status of the lifetime calculations.

2. Exclusive decays

It has become textbook knowledge that in the heavy mass limit the heavy ground state hadrons fall into spin symmetry doublets, corresponding to the two spin directions of the heavy quark. Furthermore, in the heavy mass limit the heavy quark becomes a static source of a color field, which is independent of flavor. These symmetries have simple consequences which can be expressed in terms of a Wigner–Eckard Theorem, very similar to the case of the well known rotational symmetry.

This Wigner–Eckard Theorem implies relations between the form factors for semi-leptonic transitions between heavy mesons, such as the transitions $B \to D$ and $B \to D^*$. It is in the meantime common knowledge that these transitions are described by only a single form factor ξ, the so called Isgur–Wise function, which depends on the scalar product of the four-velocities v and v' of the initial and final state hadrons.

Even more importantly, the heavy quark symmetries also yield an absolute normalization of the form factors in the heavy quark limit, such that

$$\xi(vv' = 1) = 1.$$

In particular, one obtains an absolute normalization for some of the form factors relevant in weak decays. The most prominent example is the decay $B \to D^* \ell \bar{\nu}$ where one may compute the energy spectrum $d\Gamma/d(vv')$ of the outgoing D^*; an extrapolation to the normalization point yields

$$\lim_{v\to v'} \frac{1}{\sqrt{(vv')^2-1}} \frac{d\Gamma}{d(vv')} = \frac{G_{\rm F}^2}{4\pi^3} \left|V_{cb}\right|^2 (m_B - m_{D^*})^2 m_{D^*}^3 \left|\xi_{\rm A1}(vv'=1)\right|^2, \quad (1)$$

where $\xi_{\rm A1}$ is one of the axial-vector form factors which becomes the Isgur–Wise function in the heavy mass limit, thus $\xi_{\rm A1}(vv' = 1) = \xi(vv' = 1) = 1$.

The machinery of HQET allows us to calculate or at least parameterize the corrections to this relation. The most important property of the corrections is the absence of terms of the order $1/m_c$[1]. This fact is due to a theorem which has been known as the Ademollo–Gatto Theorem [17] in the context of current algebra and has been applied to the case at hand by Luke [18].

The radiative corrections to the normalization statement (1) have been computed in the meantime to sub-leading order, *i.e.* $\mathcal{O}(\alpha_s^2(m_{\rm Q}))$ [19], and a complete BLM re-summation of the terms of order $(\beta_0\alpha_s)^n$ has been performed in [20]. The result is that the radiative corrections are small, they change $\xi_{\rm A1}(1)$ by

$$\Delta_{\rm A1}^{\rm radcorr} = -0.01 \pm 0.01\,, \quad (2)$$

where the conservative estimate of the uncertainty is given by the size of the $\mathcal{O}(\alpha_s^2)$ contributions.

The terms of order $1/m_c^2$ are much harder to discuss, since there is yet no systematic way to compute them. They involve non-perturbative matrix elements which either have to be modelled or have to be taken from future lattice calculations. Various estimates have been discussed over the last ten

[1] The corrections depend in fact on the parameter

$$\Delta = \frac{1}{m_c} - \frac{1}{m_b}\,,$$

so the absence of $1/m_c$ corrections means that the sub-leading corrections are of the order Δ^2.

years, and it has become clear that early estimates (*e.g.* [21, 22]) tended to underestimate their size. The methods which are used today to estimate the size of these corrections are based on a sum rule making use of the zero-recoil limit; from this a typical value is [23, 24]

$$\Delta_{\rm A1}^{1/m^2} = -0.08 \pm 0.04\,. \tag{3}$$

It is important to notice that the uncertainty quoted in (3) is again an estimate which will probably match the taste of most authors who worked in this field. In other words, the uncertainty is based on "common sense" and, in particular, cannot be interpreted as a statistical uncertainty with *e.g.* a Gaussian distribution. The uncertainty quoted in (3) contains also an estimate of the contributions of order $1/m^3$, which, however, are assumed to be small.

A detailed discussion of the central values as well as of the uncertainties of the normalization (1) has been performed in the context of the BaBar Workshop [25], where the number

$$\xi_{\rm A1}(vv'=1) = 0.913 \pm 0.007_{\rm pert} \pm 0.024_{1/m^2} \pm 0.011_{1/m^3} \tag{4}$$

is given. Although this number represents some consensus among the theorists who participated in the BaBar Workshop, concerns have been expressed (see *e.g.* [26]), that the central value as well as the uncertainties should be reconsidered. At some occasions lower central values as well as larger errors have been used, *e.g.* in [26] a value of

$$\xi_{\rm A1}(vv'=1) = 0.89 \pm 0.08 \tag{5}$$

is quoted and the LEP heavy flavor working group used

$$\xi_{\rm A1}(vv'=1) = 0.89 \pm 0.05 \tag{6}$$

for their analysis.

It is important to point out once more that all these values are compatible within the ranges of uncertainties given. Clearly the issue here is the estimation of the uncertainties and their interpretation; even an uncertainty as large as the one quoted in [26] cannot be excluded, although it is considered to be very unlikely. On the other hand, if the uncertainties are overestimated, theorists will be on the safe side, but on the expense of loosing predictive power.

In the process of extracting V_{cb} from an extrapolation based on (1) another potential source of uncertainty is the formula used for the extrapolation to the point $v = v'$. However, the slope of the form factor at $v = v'$ can be constrained using dispersion relations [27], which renders this uncertainty negligible.

Clearly the relevant quantity which is extracted from experiment is the product $|\xi_{\rm A1}(1)V_{cb}|$. Fortunately the value of $\xi_{\rm A1}(1)$ enters multiplicatively, so one may always trivially scale out the dependence on the form factor value.

The comparison with data and the extraction of V_{cb} will be discussed in another contribution to these proceedings [28]

3. Inclusive decays

Inclusive decays can also be described using a $1/m_{\rm Q}$ expansion. The main observation is that the heavy quark mass sets a large scale, the presence of which can be exploited in a similar way as in deep inelastic scattering, where the large scale is set by the large momentum transfer.

The basic ingredient may be understood very easily. The inclusive rate is written as

$$\begin{aligned} \Gamma(B \to X) &= \sum_X (2\pi)^4 \delta^4(p_B - p_X) \langle B|H_I|X\rangle\langle X|H_I|B\rangle \qquad (7) \\ &= \int d^4x \, \langle B|H_I(x)H_I(0)|B\rangle \\ &= -2{\rm Im} \int d^4x \, \langle B|T[H_I(x)H_I(0)]|B\rangle \, . \qquad (8) \end{aligned}$$

The effective Hamiltonian H for $\Delta B = \pm 1$ contains one (anti)bottom quark, which can be written as

$$b(x) = \exp(-i\, m_{\rm Q}(vx))\, b_v(x) \, ,$$

where v is the velocity of the decaying B meson. This allows us to make the dependence of the matrix element on the large scale $m_{\rm Q}$ explicit; *i.e.* the matrix element involving only the field b_v may be expanded in inverse powers of $m_{\rm Q}$.

Technically this means that one may perform an Operator Product Expansion (OPE) of the time-ordered product in (7) such that

$$\begin{aligned} &\int d^4x \, T[H_I(x)H_I(0)] \qquad (9) \\ &= C_0\mathcal{O}_0 + \frac{1}{m_b}\sum_i C_1^{(i)}\mathcal{O}_1^{(i)} + \left(\frac{1}{m_b}\right)^2 \sum_i C_2^{(i)}\mathcal{O}_2^{(i)} + \cdots \, , \end{aligned}$$

where $\mathcal{O}_n^{(i)}$ are local operators of dimension $(mass)^n$ and $C_n^{(i)}$ are coefficients that can be calculated in perturbation theory.

The rate is obtained by taking the diagonal matrix elements of (9). These matrix elements still have a mass dependence (*e.g.* through the states) which is expanded in $1/m_{\rm Q}$ using the techniques of HQET. After this expansion, the rate is written as

$$\Gamma = \Gamma_0 + \frac{1}{m_{\rm Q}}\Gamma_1 + \frac{1}{m_{\rm Q}^2}\Gamma_2 + \frac{1}{m_{\rm Q}^3}\Gamma_3 + \cdots . \tag{10}$$

First few terms of this expansion have been investigated in detail. The leading term is in general the decay rate of a free quark, since the matrix element entering this leading term is normalized. Thus one recovers the parton model result as the leading term of a systematic expansion. The contribution Γ_1 vanishes due to a similar argument as it is used in the derivation of Luke's Theorem, and thus the first non-trivial corrections to inclusive b hadron decays are of order $1/m_b^2$. In this order, two parameters appear, which are defined by the matrix elements

$$2M_{\rm H}\lambda_1 = \langle H(v)|\bar{Q}_v(iD)^2Q_v|H(v)\rangle\,, \tag{11}$$

$$6M_{\rm H}\lambda_2 = \langle H(v)|\bar{Q}_v(-i)\sigma_{\mu\nu}[iD^\mu, iD^\nu]Q_v|H(v)\rangle\,. \tag{12}$$

The parameter λ_2 can be obtained from the splitting within the spin symmetry doublet for ground state mesons, $\lambda_2 \approx 0.13$ GeV^2, while λ_1 cannot be easily read off from the spectrum of heavy hadrons. However, it can be determined from the moments of the hadronic invariant mass spectrum in inclusive semi-leptonic decays or the moments of the photon-energy spectrum in $B \to X_s\gamma$. Both determinations yield a consistent result which is in the range of [29] $-0.25\,{\rm GeV}^2 \leq \lambda_1 \leq -0.1\,{\rm GeV}^2$.

These values imply that the non-perturbative corrections in inclusive decays are indeed tiny, namely of the order of $\lambda_{1,2}/m_b^2 = \mathcal{O}(1\%)$. However, for the lifetimes these corrections can be enhanced by large factors of $16\pi^2$. In any case, the non-perturbative corrections are much smaller than the perturbative ones.

As far as the parametric dependence of the corrections on the heavy quark mass is concerned, inclusive B decays have an advantage over exclusive ones, since the corrections are of the order $1/m_b^2$ as compared to $1/m_c^2$ in exclusive decays.

However, inclusive decays also have disadvantages. In the early days it was believed that a precise determination of V_{cb} would not be possible due to the uncertainties induced through the dependence on the b quark mass. Naively, this parameter enters in the fifth power such that even a small uncertainty would make the inclusive decays very uncertain.

Two facts rescue the situation. Firstly, the phase space of the quark decay also depends on the b quark (and on the c quark) mass, in such a way that one may re-express the rate in term of the b quark mass and the difference of the b and the c quark mass, which is known to a better accuracy, since it may be determined from the spin averaged B and D meson masses, up to terms of the order $1/m^2$.

The second fact is the behavior of the radiative corrections. It has been observed that the radiative corrections are large if the mass in the partonic calculation is interpreted as the pole mass [30]. Schematically one obtains

$$\Gamma \propto m_{\rm pole}^5 \left(1 + a_1 \frac{\alpha_s}{\pi} + a_2 \left(\frac{\alpha_s}{\pi} \right)^2 + \cdots \right), \tag{13}$$

with $a_1 = \mathcal{O}(1)$ and $a_2 = \mathcal{O}(-10)$, which means that in this scheme the perturbation series converges very slowly (if at all). Using instead a short distance definition m_{sd}, one can rearrange the perturbative series such that

$$\Gamma \propto m_{\rm sd}^5 \left(1 + b_1 \frac{\alpha_s}{\pi} + b_2 \left(\frac{\alpha_s}{\pi} \right)^2 + \cdots \right), \tag{14}$$

where now $b_1, b_2 = \mathcal{O}(1)$.

It has been argued in [13] that the $\bar{M}S$ mass definition, as it is usually used in perturbative calculations, still does not ensure a good convergence of (14), since it still has an infrared sensitivity. According to [13] one should chose a short-distance mass that involves a hard cut off, *i.e.* involves powers of the cut-off; using such a mass, taken at a low scale, yields a good convergence of (14).

Thus various mass definitions have been proposed to minimize the uncertainties induced by the ignorance of the heavy quark mass. One way that has been proposed and which nicely shows this correlation between the radiative corrections and the quark mass definition is to obtain the quark mass from heavy quarkonia, *i.e.* from the spectrum of Υ states. This calculation involves the application of NRQCD to the energy of the $\Upsilon(1S)$ state, assuming that possible non-perturbative (which means in this case non-coulombic) corrections are small. This approach, suggested by Hoang *et al.* [31, 32], yields a result of the form:

$$\Gamma = \frac{G_{\rm F}^2 |V_{cb}|^2}{192\pi^3} \left(\frac{m_{\Upsilon(1S)}}{2} \right)^5 0.533 (1 - 0.096 - 0.029 + \cdots), \tag{15}$$

where now the quark mass is replaced by $\frac{1}{2} m_{\Upsilon(1S)}$, which is known to a very high precision. The expansion parameter is now not α_s any more, but the resulting series seems to converge reasonably well.

Clearly, there are also other ways to deal with the interplay of radiative corrections and the heavy quark mass, but the message of all this is that the inclusive decays $b \to c \ell \bar{\nu}_\ell$ are indeed at least competitive compared to the exclusive method described in the previous section, if not cleaner.

3.1. Determination of V_{cb} from inclusive decays

After these preliminaries on advantages and disadvantages of the $1/m_{\rm Q}$ expansion in inclusive decays we shall now apply this to the determination of V_{cb}. This issue has been investigated in some detail in the context of the BaBar Workshop [25], where

$$|V_{cb}| = \xi_{\rm th} \left(\frac{{\rm BR}(\bar{B} \to X_c \ell \bar{\nu})}{10.5\%} \right)^{1/2} \left(\frac{1.6{\rm ps}}{\tau_B} \right)^{1/2} \tag{16}$$

with

$$\xi_{\rm th} = 0.0403(1 \pm 0.030 \pm 0.024 \pm 0.025 \pm 0.012) \tag{17}$$

is quoted; the uncertainties are due to higher order radiative corrections, the mass difference $m_b - m_c$, the b quark mass and the corrections of order $1/m_b^3$ or higher.

The latest data on this were given at the ICHEP2000 in Osaka [33], which are

$$V_{cb} = (40.66 \pm 0.36) \times 10^{-3} \times \left(1 \pm 0.015_{\rm pert} \pm 0.010_{m_b} \pm 0.012_{1/m_b^3} \right) . \tag{18}$$

It is noteworthy that for both determinations of V_{cb} the uncertainties are already dominated by the theoretical ones. This relation between the uncertainties will become worse, since data will improve further over the next years, while progress on the theoretical side is not foreseeable.

Finally one small caveat needs to be mentioned, which is the use of duality in inclusive decays. Clearly, the assumption of parton hadron duality is the central point in any inclusive calculation, and most theorists believe it to be a safe assumption. I shall return to this point at the end of the next section. This caveat has to be kept in mind when discussing V_{cb} extractions from inclusive decays, while the exclusive method described in the last section is safe against possible duality violations.

3.2. Lifetimes

Another inclusive quantity which can be calculated in the framework of the $1/m_b$ expansion are lifetimes of heavy hadrons. Since in the $1/m_b$ expansion the leading term is the free quark decay the leading order prediction for b hadrons is that

$$\tau(B^0) = \tau(B^+) = \tau(B_s) = \tau(\Lambda_b) = \tau(b) . \tag{19}$$

Furthermore, the first non-perturbative corrections are of order $\lambda_{1,2}/m_b^2$ and thus very small. In particular, they are the same for the charged and the neutral B mesons and thus a lifetime difference can (and will) emerge only at the level of $1/m_b^3$ corrections and are expected to be very small. Clearly, perturbative corrections to the free quark decay (*i.e.* the parton model) are short distance contributions and consequently will also not contribute to the lifetime difference.

A lifetime difference between the Λ_b and the B meson can in principle arise already at the level $1/m_b^2$, since the kinetic energy operator is different for Λ_b and B; in addition, $\lambda_2(\Lambda_b) = 0$ due to heavy quark spin symmetry. The difference between the kinetic energies can be estimated by comparing charm and bottom hadrons [34]

$$\begin{aligned} &[M(\Lambda_b) - M(\Lambda_c)] - [\bar{M}(B) - \bar{M}(D)] \\ &= (\Lambda_1(B) - \Lambda_1(\Lambda_b)) \left(\frac{1}{m_c} - \frac{1}{m_b} \right) + \mathcal{O}(1/m_c^3)\,, \end{aligned} \tag{20}$$

where $\bar{M}(H) = (M(H_{0^-}) + M(H_{1^-})/4$ is the spin averaged mass of the mesons. From this estimate one finds that the $\mathcal{O}(1/m_b^2)$ contribution to the lifetime difference between Λ_b and B is as small as 2%.

On the other hand, experimentally the lifetime of the Λ_b is significantly smaller than the one of the B meson, by about 20% [35]. In view of the small $1/m_b^2$ contributions, this effect needs to be explained entirely by $1/m_b^3$ or even higher order terms.

At order $1/m_b^3$ one can indeed identify contributions which can be large, depending on the values of the unknown hadronic parameters. These large terms originate from the fact, that the leading free quark decay is a three particle decay on the partonic level, while the $\mathcal{O}(1/m_b^3)$ contributions that are sensitive to the spectator have only a two particle phase space at the partonic level. Thus, these particular contributions receive an enhancement by the phase space ratio, which introduces a factor of $16\pi^2$. Schematically this is

$$\frac{\tau(B)}{\tau(\Lambda_b)} = 1 + \frac{1}{m_b^3}\left[a_0 + \frac{1}{m_b}a_1 + \cdots\right] + \frac{16\pi^2}{m_b^3}\left[b_0 + \frac{1}{m_b}b_1 + \cdots\right]\,, \tag{21}$$

where a_i and b_i are in general of order one, depending on unknown hadronic parameters. In [34] a scan over the parameter space has been performed with the result that one can marginally fit the data.

However, the lifetime ratios is still considered to constitute a problem. If we assume that the charm quark is also heavy, we would need to explain lifetime ratios such as [35]:

$$\frac{\tau(D^+)}{\tau(D^0)} \sim 2.5\,, \qquad \frac{\tau(\Lambda_c)}{\tau(D^+)} \sim 0.2\,, \tag{22}$$

by effects of order $1/m_b^3$. Although these data are consistent with the naive scaling one infers from the $1/m_b$ expansion, one might feel uneasy explaining these large effects by third order corrections.

This has motivated the search for an alternative explanation. First, one may ask the question if the charm quark is indeed heavy enough to be treated in a $1/m_c$ expansion. However, in applications of HQET to exclusive decays like the determination of V_{cb} this approximation seems to work quite well.

Another issue that has to be discussed is again the assumption of duality in these processes; this has been a very active field in the last few years [36–41]. Lacking a tool to study duality in four dimensions, most of these studies were performed in the 't Hooft model, *i.e.* two dimensional QCD. From these studies it turns out that duality violations are small, at least in these model studies. Whether this is also the case in four dimensions is still open; in particular there is no estimate of the uncertainties which have to be assigned to the fact that duality is used in inclusive heavy quark processes.

4. Conclusions

The heavy quark expansion has become the standard way to describe heavy quark decays. It has put the theoretical description of these processes on a firm theoretical basis and makes in many cases models obsolete. Being a systematic expansion the $1/m_{\mathrm{Q}}$ expansion allows to at least estimate the size of the higher order corrections. Although the estimation of higher order terms in most cases depend on model assumptions (a well known case are the $1/m_{\mathrm{Q}}^2$ corrections in any of the V_{cb} determinations) the $1/m_{\mathrm{Q}}$ expansion is still preferable over using a model from the very beginning. So the $1/m_{\mathrm{Q}}$ expansion is a substantial progress, in particular in the light of the forthcoming experimental data, which now can be used to perform a stringent test of the CKM Sector of the Standard Model.

Clearly, over the time the heavy mass expansion has been explored in detail and problems showed up, some of which have been solved. One problem is the obvious question what the meaning of the expansion parameter is, since many different mass definitions are possible. This question has been clarified in principle, what remains is to settle, which definition minimizes the uncertainties.

Another question which is not clarified yet is whether the charm quark is heavy and can be treated in the $1/m_{\mathrm{Q}}$ expansion. As far as semi-leptonic processes are concerned the approximation of heavy charm quark seems reasonable, *e.g.* the inclusive semi-leptonic width of D mesons comes out satisfactorily. Clearly, the assumption of a heavy charm also enters the determination of V_{cb} *via* exclusive decays, where it also seems to work reasonably well. On the other side are the non-leptonic processes of charm, where the

$1/m_Q$ expansion seems to converge badly. From this point of view one might wonder, why this expansion works so well in the semi-leptonic sector. Probably related to this question of the convergence of the heavy mass expansion is the Λ_b lifetime problem.

One of the major challenges for the $1/m_Q$ expansion are exclusive non-leptonic decays. This class of decays is very important for the determination of the CKM angles α, β and γ, which are related to the CP asymmetries in these processes. Some recent progress in this direction will be described in another contribution to these proceedings, however as of now no clear picture (*e.g.* a description in terms of an operator product expansion) has emerged.

Clearly, the heavy mass expansion has become an indispensable tool which is essential for the test of the CKM sector and the Standard Model CP violation. In particular, if new physics effects indeed show up in B decays, it is essential to have a good control over the hadronic uncertainties, which makes further development of theoretical methods an important issue.

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