THE DETERMINATION OF V_{ub} FROM INCLUSIVE SEMILEPTONIC B DECAYS*

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Inspecting limits on $|V_{ub}|$ estimates, great need is seen for improvement. One method to determine the related $|V_{ub}/V_{cb}|$ ratio is proposed. We take into account first-order perturbative and non-perturbative leading-twist QCD corrections. We analyze in detail the impact of model uncertainties on the accuracy of our prediction.

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1. Introduction

The measurement of the CKM matrix element V_{ub}, which play an important role in the determination of the unitarity triangle, will most cleanly be performed in the study of semileptonic $b \to u$ transitions. As concluded in the BaBar Workshop [1], a variety of methods will be employed for the best result.

One of the theoretically cleanest possibilities is to use inclusive semileptonic decays. Placing a cut on the hadronic invariant mass of the final state can in principle eliminate the charm contribution, which otherwise would be overwhelming. This cut on the hadronic invariant mass can be implemented at the asymmetric B factories, making this method experimentally feasible.

From the theoretical point of view, the cut on hadronic invariant mass implies the necessity to replace the power expansion in terms of $1/m_Q$ with an expansion in twist. This kinematical region is described by the "shape function" of purely non-perturbative origin.

Our method is one of several that have been proposed. One of them [2] avoids the twist expansion and relies only on a standard $1/m_Q$ allowing a clean determination of V_{ub}. Still another approach has been put forth

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in [3, 4] where the factorization into soft jet and hard sub-processes [5] has been employed to relate the radiative b decays to the semileptonic ones in a way which explicitly reduces the impact of the shape function uncertainties. This approach is similar to the one in [6].

Exclusive decays will open a completely different window on V_{ub}; however, in these decays a certain model dependence seems to be unavoidable, unless lattice data become reasonably precise.

In this paper, we will elaborate on a method in which a cut is applied on the hadronic invariant mass in semileptonic B decays to select the $b \to u$ transitions. The advantage is that the hadronic invariant mass may be easier to measure, however, this method involves the shape function and potentially has larger theoretical uncertainties than the inclusive method using the leptonic invariant mass. The method based on the hadronic invariant mass spectrum has already been discussed in [7], where the main focus was on the perturbative contributions of order α_s and $\beta_0\alpha_s^2$.

The purpose of this paper is to consider this approach in detail and to try to estimate the uncertainties, including perturbative as well as non-perturbative contributions and, in addition, a cut on the lepton energy. It turns out that the main uncertainties originate from the heavy quark mass m_b (or, equivalently, from $\bar{\Lambda} = M_B - m_b$ where M_B is the B-meson mass) and the strong coupling α_s, while the uncertainties introduced by the shape function are small.

The next section deals with the kinematics. Then, in Sec. 3, we give the radiative corrections for the partonic process $b \to u\ell\bar{\nu}_\ell$ to order α_s and discuss the leading-twist non-perturbative effects; this requires the introduction of the light-cone distribution function for the heavy quark, for which we use a simple parametrization. We combine the perturbative and non-perturbative corrections and study the uncertainties in the determination of $|V_{ub}|/|V_{cb}|$ in Sec. 4.

2. Kinematics

We shall first define the kinematic variables for the partonic process $b \to u\ell\bar{\nu}_\ell$. We refer the reader to [8] for further discussion of the $b \to c$ decay process. The initial state b quark has a momentum $p_b = m_b v$, where v is the velocity of the B meson. With the momentum transfer to the leptons $q = k + k'$ the variable $p = q - m_b v$ is the partonic momentum of the final state. Writing $E_\ell = vk$ for the energy of the lepton, $E_p = vp$ for the partonic energy of the final state and p^2 for the partonic invariant mass, we define the following re-scaled variables:

$$x = \frac{2E_\ell}{m_b}\,, \qquad x_p = \frac{2E_p}{m_b}\,, \qquad y = \frac{q^2}{m_b^2}\,, \qquad z = \frac{p^2}{m_b^2}\,. \tag{1}$$

Using these definitions, we compute the triple differential partonic rate to order α_s and order $1/m_b$, and split it into the tree-level term Γ^0 and the $\mathcal{O}(\alpha_s)$ correction Γ^1:

$$d\Gamma^{\rm pert} = d\Gamma^{\rm parton,0} + \frac{2\alpha_s}{3\pi}\, d\Gamma^{\rm pert,1}\,. \tag{2}$$

The radiative corrections to $\mathcal{O}(\alpha_s)$ have recently been calculated by [9]; we have checked their result and find full agreement with ours. We have adapted the tedious formulae found therein to the purposes of our evaluation. They can be found in the Appendix to [10].

In the hadronic picture, the momentum of the initial B meson is

$$M_B\, v = (m_b + \bar{\Lambda})v\,,$$

where we have used the relation between the B meson and the b-quark mass

$$M_B = m_b + \bar{\Lambda} \tag{3}$$

which holds to leading order in the $1/m_b$ expansion. Consequently, the hadronic mass of the final state is

$$M_X^2 = (M_B\, v - q)^2 = (p + \bar{\Lambda}\, v)^2 = p^2 + 2E_p\bar{\Lambda} + \bar{\Lambda}^2$$

and hence involves both the partonic invariant mass and the partonic energy. Thus we have

$$\begin{aligned}\frac{d\Gamma^{\rm parton}}{dM_X^2 dE_\ell}(m_b) \;=\; & \int_0^1 dx \int_{1-x}^{2-x} dx_p \int_{z_{\rm min}}^{z_{\rm max}} dz\, \frac{d^3\Gamma^{\rm pert}}{dx\, dx_p\, dz} \\ & \times \delta(M_X^2 - m_b^2 z - x_p\bar{\Lambda}m_b - \bar{\Lambda}^2)\,\delta\left(E_\ell - \frac{m_b x}{2}\right), \end{aligned} \tag{4}$$

where the kinematic limits of the z integration depend on the cuts as well as on x_p. They are

$$z_{\rm min} = \begin{cases} 0 & \text{for } x_p \le 1\,, \\ x_p - 1 & \text{for } x_p > 1\,, \end{cases} \tag{5}$$

and

$$z_{\rm max} = (1-x)(x_p + x - 1)\,. \tag{6}$$

3. Perturbative and leading twist corrections

The analysis we perform requires the knowledge of the triple differential partonic rate to order α_s, as can already be seen from Eq. (4) as well as from the final formula, Eq. (9). The Born approximation reads

$$\frac{d^3\Gamma^{\mathrm{pert},0}}{dx\,dx_p\,dz} = 12\Gamma_0\,(2-x-x_p)(x+x_p-1)\,\delta(z), \tag{7}$$

where

$$\Gamma_0 = \frac{G_{\mathrm{F}}^2 m_b^5}{192\pi^3}\,. \tag{8}$$

As is well known, the virtual one-loop correction contains an infrared divergence that cancels with the real gluon emission. However, incorporating the leading twist corrections implies a smearing of the partonic mass over a reasonable range. While choosing an excessively small value for this window would still yield a meaningless result, a region of size $\bar{\Lambda} m_b$ is believed to exist where this smearing provides a realistic approximation. Instead of keeping track of the virtual and real parts, one can now use an appropriately integrated distribution [10], which is subjected to the smearing procedure.

It has been shown that the leading-twist non-perturbative corrections can be implemented at tree level by redefining the heavy-quark mass and a subsequent convolution with a so-called shape function [11–15]. This convolution corresponds to an integration over the light-cone variable k_+, namely

$$d\Gamma^{\mathrm{hadron}} = \int\limits_{-m_b}^{\bar{\Lambda}} d\Gamma^{\mathrm{parton}}(m_b+k_+)\,f(k_+)\,dk_+ \tag{9}$$

in $d\Gamma^{\mathrm{HQET}}$. Even though this formula has been found to contain spurious contributions of sub-leading twist [17] and probably fails to hold upon incorporating radiative corrections [18], we stick with this naive formulation as suggested in [16]. It is at least as consistent as using the ACCMM model [19] beyond tree level, which is common practice. In fact, the connection between the ACCMM model and the shape function formalism has been pointed out in [20].

The shape function is a nonperturbative function which has to be determined either from experiment or by some model. A few relations are known for the moments of the shape function, which are met by the ansatz [21]

$$f(x) = N\,\mathrm{e}^{cx}\,(1-x)^a\,, \qquad x = 1-\frac{M_B-m^*}{\bar{\Lambda}}\,. \tag{10}$$

We can relate the parameters in this formula to the HQET parameters

$$N = \frac{c^c}{\bar{\Lambda}\, \mathrm{e}^c \Gamma(c)}\,, \qquad a = c - 1\,, \qquad c = -\frac{3\bar{\Lambda}^2}{\lambda_1}\,. \tag{11}$$

We shall discuss the dependence of our results on this ansatz in the next section.

4. The measurement of $|V_{ub}|$

For the measurement of $|V_{ub}|$ we propose to study semileptonic B decays with certain cuts. The first cut is on the lepton energy E_ℓ, which is dictated by the experimental limitations on detecting electrons with small momenta. The second cut is on the hadronic invariant mass M_X of the final state, which serves to suppress charm. We define the semileptonic rates including cuts as

$$\Gamma(M^2_{\rm cut}, E_{\rm cut}) = \int\limits_0^{M^2_{\rm cut}} dM_X^2 \int\limits_{E_{\rm cut}}^{(M_B^2 - M_X^2)/2\,M_B} dE_\ell\, \frac{d\Gamma(B \to X_u e \bar{\nu}_e)}{dM_X^2 dE_\ell}\,. \tag{12}$$

Clearly the region $0 < M_X < M_D$ is dominated by $b \to u$ transitions, and we thus use in this region the expressions for $b \to u$ decays only.

We shall normalize everything to the rate with no cut on the hadronic invariant mass, and thus obtain, for the ratio r:

$$\frac{|V_{ub}|^2}{|V_{cb}|^2} r = \frac{\Gamma(M^2_{\rm cut}, E_{\rm cut})}{\Gamma(M_B, E_{\rm cut})}, \tag{13}$$

where in the denominator we can safely take into account the $b \to c$ channel only, the $b \to u$ contribution being only about 1%.

For the charmless decay rate entering the numerator, we include one-loop and leading-twist corrections. The $b \to c$ decay rate in the denominator is evaluated including $\mathcal{O}(\alpha_s)$ corrections.

It will turn out that the precise form of the shape function is irrelevant for the ratio (13), the main sources of uncertainties being the value of the strong coupling and that of the quark masses. The latter may be replaced by $\bar{\Lambda}$. We thus use $m_b = \overline{M}_B - \bar{\Lambda}$ and $m_c = \overline{M}_D - \bar{\Lambda}$, where $\overline{M}$ denotes the spin-averaged meson mass.

We have examined the ratio defined in Eq. (13), allowing for the variable ranges: 0.4 GeV $\leq \bar{\Lambda} \leq 0.75$ GeV, $0.2 \leq \alpha_s \leq 0.3$ and -0.6 GeV$^2 \leq \lambda_1 \leq -0.1$ GeV^2.

The main uncertainty is induced by α_s and $\bar{\Lambda}$, where $\bar{\Lambda}$ is equivalent to the heavy quark mass. It has been argued that these two quantities are correlated; the size of the radiative corrections depends on the particular

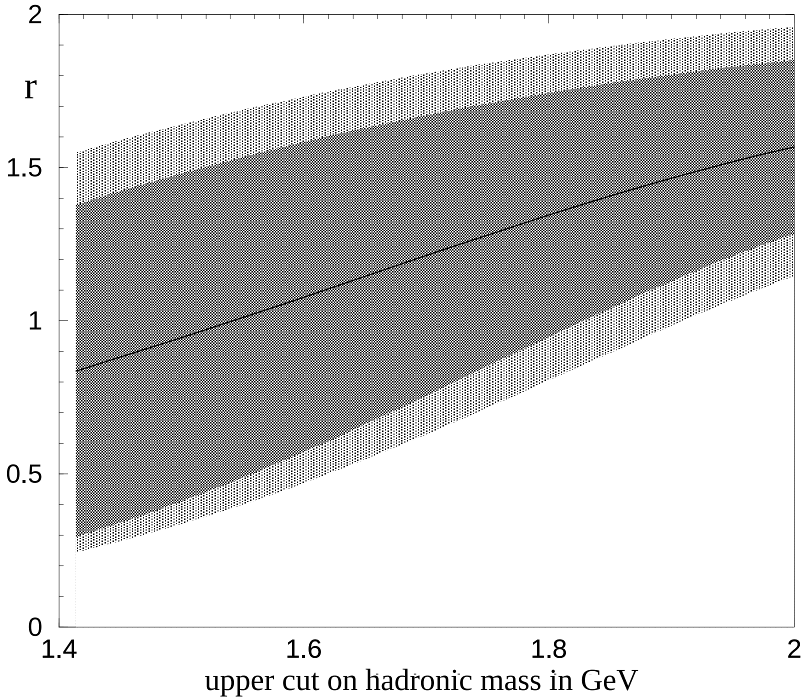

Fig. 1. Ratio r as a function of the hadronic mass cut, with (dark) or without (light) correlation between the strong coupling constant and the pole mass.

choice of the mass. Ignoring this correlation one is led to add the errors on both parameters and then the certainly overestimated uncertainty is (see the lighter band in Fig. 1)

$$1.15 \leq r \leq 1.96 \quad \text{at} \quad M_{\text{cut}}^2 = 4\ \text{GeV}^2\,, \tag{14}$$

where we have used $m_b^{\text{pole}} = (4.75 \pm 0.15)$ GeV and α_s between 0.2 and 0.3. This translates to a theoretical uncertainty of 25% in the $|V_{ub}/V_{cb}|$ ratio.

Another option is to switch to a short-distance mass definition, such as $m_b^{\overline{MS}}$ by replacing

$$m_b^{\text{pole}} = m_b^{\overline{MS}} \left(1 + \frac{4}{3\pi}\alpha_s\right), \tag{15}$$

which reduces the size of the coefficients of the perturbation series, and the perturbative uncertainties thus become smaller. Using a recent value for $M_b^{\overline{MS}}$ [22]

$$m_b^{\overline{MS}} = (4.25 \pm 0.08)\,\text{GeV}, \tag{16}$$

we arrive at an estimate for r with a smaller uncertainty (the darker band in Fig. 1)

$$1.28 \leq r \leq 1.85 \quad \text{at} \quad M^2_{\rm cut} = 4\,{\rm GeV}^2. \tag{17}$$

In order to display the dependence on the input parameters, we choose the "average" values of the three parameters:

$$\bar{\Lambda}^{\rm aver} = 0.55 \text{ GeV}, \qquad \alpha_s^{\rm aver} = 0.25, \qquad \lambda_1^{\rm aver} = -0.35 \text{ GeV}^2, \tag{18}$$

and obtain up to linear terms in the variations

$$r = 1.58 - 0.86\,\frac{\Delta\bar{\Lambda}}{\bar{\Lambda}^{\rm aver}} - 0.26\,\frac{\Delta\alpha_s}{\alpha_s^{\rm aver}} + 0.06\,\frac{\Delta\lambda_1}{\lambda_1^{\rm aver}}\,. \tag{19}$$

This explicitly shows that the dependence on λ_1, which is the second moment of the shape function, is weak. The dependence on even higher moments is expected to be further suppressed by inverse powers of the heavy quark mass.

One may find an approximate formula expressing the ratio $r(M^2_{\rm cut} = 4\,{\rm GeV}^2)$ in terms of the phenomenological parameters above. Using the following expansion up to quadratic terms, one obtains a value which does not depart from our numerical estimate by more than 1%:

$$\begin{aligned} r \;=\; & 1.5815 - 0.85513\,\delta\bar{\Lambda} + 0.06254\,\delta\lambda_1 - 0.2638\,\delta\alpha_s \\ & - 0.2394\,(\delta\bar{\Lambda})^2 + 0.0037\,(\delta\lambda_1)^2 - 0.0385\,(\delta\alpha_s)^2 \\ & + 0.1123\,\delta\bar{\Lambda}\,\delta\lambda_1 - 0.04459\,\delta\bar{\Lambda}\,\delta\alpha_s + 0.03116\,\delta\lambda_1\,\delta\alpha_s\,. \end{aligned} \tag{20}$$

The quantities δx are the relative variations of the respective parameters from their mean values as specified in Eq. (18):

$$\delta x = \frac{x - x^{\rm aver}}{x^{\rm aver}}\,. \tag{21}$$

In conclusion, our best estimate for r is

$$r = 1.57 \pm 0.3\,, \tag{22}$$

which corresponds to a theoretical uncertainty in the determination of $|V_{ub}/V_{cb}|$ of about ten percent

$$\left(\frac{\Delta\,|V_{ub}/V_{cb}|}{|V_{ub}/V_{cb}|}\right)_{\rm theor} \approx 10\%\,. \tag{23}$$

5. Conclusions

We have performed a detailed analysis of one of the possibilities to obtain V_{ub} from inclusive semileptonic B decays by placing a cut on the hadronic invariant mass to get rid of the charm background. This method has been criticized since it depends on the shape function, which describes the endpoint of the hadronic invariant mass spectrum. This function is not very well known and hence it has to be modeled, which will introduce some systematic uncertainty. However, integrating over the window in hadronic invariant masses relevant to $b \to u$ transitions, we have shown that the dependence on the shape function is much smaller than the uncertainties induced by the quark mass and by the truncation of the perturbative series.

The ratio between the semileptonic rates including a cut on the hadronic invariant mass and the semileptonic rate without a cut yields $|V_{ub}/V_{cb}|$ up to a quantity r, which we have computed in leading twist approximation and to order α_s. Based on our calculations the uncertainty in this quantity is 20% leaving us with a 10% theoretical uncertainty in the determination of V_{ub}. This method is thus one of the cleanest possible to obtain V_{ub} at the ongoing B-factory experiments.

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