LEPTONIC DECAYS OF $B^0_{d,s}$ MESONS IN THE MSSM WITH LARGE $\tan\beta$ *

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Using the effective Lagrangian approach we compute the dominant for $\tan\beta \gg 1$ supersymmetric contributions to the leptonic decay rates of $B^0_{d,s}$ mesons. We point out that in this limit the CP violating asymmetries measured in the decays $B^0_d(\bar{B}^0_d) \to \tau^-\tau^+$ can be interesting probes of the complex phases of the MSSM parameters and/or the V_{td} element of the CKM matrix.

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1.

The Minimal Supersymmetric Standard Model (MSSM) with large ratio $v_2/v_1 \equiv \tan\beta$ of vacuum expectation values of the two Higgs doublets attracts recently a renewed attention. Apart from being theoretically interesting (large $\tan\beta$ is needed to unify all third generation Yukawa couplings at the GUT scale) and leading rather naturally to mass values of the Higgs boson h^0 consistent with the lower limit from LEP, the MSSM with large $\tan\beta$ can also easily reproduce the recently measured [1] value of the muon anomalous magnetic moment which is by $2.6\,\sigma$ larger than the value predicted in the Standard Model (SM) [2].

Truly spectacular effects of large $\tan\beta$ can, however, be expected to show up in B-physics where the predictions of the MSSM can differ significantly from those of the SM due to the enhancement of the b-quark Yukawa coupling constant. Particularly interesting in this context are such flavour changing neutral current (FCNC) processes, like $B^0_{d,s} \to l^-l^+$, for which the SM predicts very low rates. In the MSSM, there are new possible contributions to the FCNC processes which for large $\tan\beta$ can dominate over the

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SM ones. Firstly, the effects of the CKM mixing can be amplified through loops involving the charged Higgs boson H^+ and, even more, charginos and squarks. Secondly, the sfermion sector of the MSSM can in general be a new (independent of the CKM mixing matrix) source of FCNC processes: If squark mass matrices are not appropriately aligned with the corresponding quark mass matrices then the gluino couplings become flavour violating[1] leading to rather large rates of various FCNC processes. The requirement that the successful predictions of the SM are not spoiled puts some constraints on the MSSM parameter space: If the CKM mixing matrix is the only source of FCNC processes then existing experimental data impose some correlations on masses and composition of charginos and top squarks with the mass of the charged Higgs boson [3,4]. These correlations, which become weaker with growing sparticle masses, depend in part on the element V_{td} of the CKM matrix which is not directly measured [3, 4]. Simultaneously, the same data severely constrain those flavour violating entries of the squark (and slepton) mass matrices which cause transitions between the first two generations; other entries are, however, considerably less constrained [4, 5].

In this note we report on the recent complete one-loop calculation [6] of the $B^0_{d,s} \to \mu^-\mu^+$ decay branching fraction in the MSSM. This calculation extends previous works [8, 9] by analysing also the case of flavour mixing induced by squark mass matrices. We concentrate here on the most interesting large $\tan\beta$ regime of the MSSM in which the $B^0_{d,s} \to \mu^-\mu^+$ decay amplitude is dominated by the neutral Higgs boson "penguin" diagrams [9] and derive approximate formulae which are most useful for qualitative understanding of the dominant effects in the large $\tan\beta$ regime.

Compared to the original publication [6] we improve here the correlation of the predicted rates for $B^0_{d,s} \to \mu^-\mu^+$ decay with $\mathrm{BR}(B \to X_s\gamma)$ by incorporating the latest refinements (NLO corrections along with the resummation of the terms enhanced for by $\tan\beta$ factors) in computing the latter quantity. Finally, following the recent suggestion [7], we comment on possible effects of CP violation in the $B^0_{d,s} \to \tau^-\tau^+$ decay.

2.

The $B^0_{d,s} \to l^-l^+$ decay amplitude has the following general form

$$-i\mathcal{A}(B^0_{s,d} \to l^-l^+) = \bar{u}(k_1)\left(a\gamma^5 + b\right)v(k_2), \qquad (1)$$

where u, v are the spinors of the outgoing leptons and the factors a and b can be expressed in terms of the Wilson coefficients of the standard effective

[1] The same is true for the neutralino vertices. Also the chargino couplings are no longer proportional to the CKM matrix elements.

Hamiltonian for the $|\Delta B| = 1$ processes evaluated at some hadronic scale[2] $\mu_h \sim m_b$:

$$\begin{aligned} a &= \frac{f_{B_I}}{4}\Big\{2m_l \left[C^{\rm V}_{\rm LL} - C^{\rm V}_{\rm LR} + C^{\rm V}_{\rm RR} - C^{\rm V}_{\rm RL}\right] \\ &\qquad -\frac{M^2_{B_I}}{m_b}\left[C^{\rm S}_{\rm LL} - C^{\rm S}_{\rm LR} + C^{\rm S}_{\rm RR} - C^{\rm S}_{\rm RL}\right]\Big\}, \\ b &= \frac{f_{B_I}}{4}\frac{M^2_{B_I}}{m_b}\left[C^{\rm S}_{\rm LL} + C^{\rm S}_{\rm LR} - C^{\rm S}_{\rm RR} - C^{\rm S}_{\rm RL}\right], \end{aligned} \tag{2}$$

where $C^{\rm V}_{XY}$ and $C^{\rm S}_{XY}$ are the Wilson coefficients of the $(\bar d_J\gamma_\mu P_X d_I)(\bar l_B\gamma^\mu P_Y l_A)$ and $(\bar d_J P_X d_I)(\bar l_B P_Y l_A)$ operators, respectively, and the constants f_{B_I} for $I = d, s$, are the only sources of the theoretical uncertainty.

SM particles contribute only to $C^{\rm V}_{\rm LL}$ (the W^+W^- box and Z^0 penguin) and to $C^{\rm V}_{\rm LR}$ (the Z^0 penguin) yielding [10] (see also [11])

$$a = f_{B_I} m_l \frac{G_{\rm F}\alpha}{\sqrt{2}\pi s^2_W}\lambda_{tI} Y(x_t)\,, \quad b = 0\,, \tag{3}$$

where $\lambda_{tI} \equiv V^*_{tb}V_{tI}$ and $Y(x_t) \approx 0.997$ (for $\bar m_t(\bar m_t) = 166$ GeV). This gives

$${\rm BR}(B^0_s \to \mu^-\mu^+) = 2.7\times 10^{-9}\left(\frac{\tau(B_s)}{1.49\ {\rm ps}}\right)\left(\frac{f_{B_s}}{200{\rm MeV}}\right)^2\left(\frac{|V_{ts}|}{0.040}\right)^2, \tag{4}$$

$${\rm BR}(B^0_d \to \mu^-\mu^+) = 8.7\times 10^{-11}\left(\frac{\tau(B_d)}{1.55\ {\rm ps}}\right)\left(\frac{f_{B_d}}{175{\rm MeV}}\right)^2\left(\frac{|V_{td}|}{0.008}\right)^2. \tag{5}$$

The $\tan\beta$ enhanced contribution of the charged scalar $H^\pm$ present in the Higgs sector of the MSSM enters through the $W^\pm H^\mp$ box diagram and through the neutral Higgs boson penguin diagrams (all these contribution appear in $C^{\rm S}_{\rm LR}$ and $C^{\rm S}_{\rm LL}$ (for $\Delta B = 1$) or $C^{\rm S}_{\rm RL}$ and $C^{\rm S}_{\rm RR}$ (for $\Delta B = -1$) Wilson coefficients). In the limit $\tan\beta \gg 1$ these effects have been correctly computed within the general Two Higgs Doublet Model of type II (2HDM(II)) in [12]. The same result applies to the MSSM charged Higgs boson (the derivation is even simpler owing to the fact that in this model for $M_A > M_Z$ only the heavier scalar H^0 and the pseudoscalar A^0 couple to the charged leptons, and $M_H \approx M_A$). One gets [12]

$$\delta^H a = \delta^H b = f_{B_I} m_l \frac{G_{\rm F}\alpha}{\sqrt{2}\pi s^2_W}\lambda_{tI}\left[Y(x_t) - \frac{M^2_{B_I}}{M^2_W}\tan^2\beta\frac{\log(r)}{r-1}\right], \tag{6}$$

[2] In the case of the vector operators the QCD running of their Wilson coefficients $C^{\rm V}$ is trivial, *i.e.* $C^{\rm V}(\mu_h) = C^{\rm V}(M_Z)$; for the scalar operators the running is $C^{\rm S}(\mu_h) = [m_b(\mu_h)/m_b(M_Z)]C^{\rm S}(M_Z)$ so that the explicit factor $1/m_b(\mu_h)$ in Eq. (2) is cancelled.

where $r = M_{H^+}^2/m_t^2$. In the 2HDM(II) this result is now not particularly interesting because the recent computation of BR($B \to X_s\gamma$) [13] together with the new CLEO experimental result for this process [14] BR($B \to X_s\gamma$) $=(2.85\pm0.35\,\mathrm{stat}\pm0.22\,\mathrm{sys})\times10^{-4}$ set the bound $M_{H^+} \gtrsim 470$ GeV (valid for $\tan\beta \gtrsim 5$) [13,15]. Thus, in order to enhance the $B^0_{d,s} \to \mu^-\mu^+$ amplitude significantly one would need $\tan\beta \gg 50$ which does not look realistic. On the other hand, although in the MSSM the positive contribution of H^+ to the $b \to s\gamma$ decay amplitude can be compensated by the contribution of sparticles — in which case the effects of H^+ could still enhance $B^0_{d,s} \to \mu^-\mu^+$ by approximately one order of magnitude even for $\tan\beta \approx m_t/m_b$ [6,12] — the sfermion sector contribution is then much more important. As we show, in contrast to the H^+ contribution which grows as $\tan^4\beta$, the chargino/stop and — in the case of flavour violation introduced by squark mass matrices — also the gluino/sbottom contributions grow as $\tan^6\beta$.

3.

In the case of the so-called minimal flavour violation (the CKM matrix as the only source of the FCNC processes) the formulae describing the supersymmetric contribution to the $B^0_{d,s} \to \mu^-\mu^+$ decay amplitude for $\tan\beta \gtrsim 30$ can be derived by considering the limit in which all soft SUSY breaking parameters, except for the ones which determine the Higgs potential, are much larger than the electroweak scale. In this limit which allows to work in the symmetric phase of the theory (*i.e.* with $v_i = 0$) and in which sfermions still have definite chirality, we can construct the effective theory by integrating out sparticles (but not the Higgs fields) [9, 16, 17]. Threshold corrections shown in Fig. 1 give then rise to the effective Yukawa interactions of the

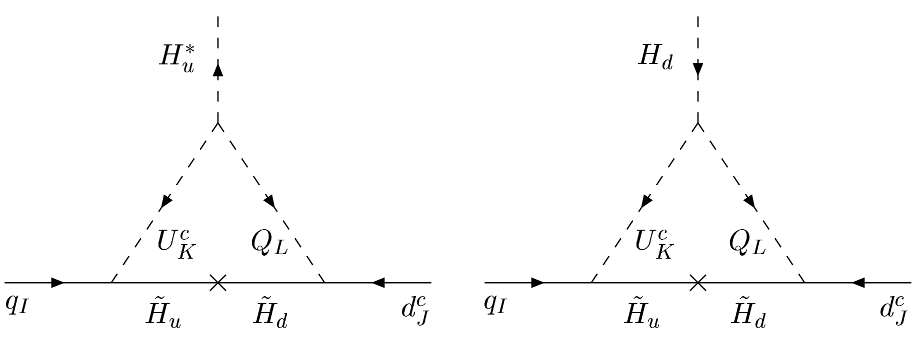

Fig. 1. Diagrams giving rise to $\Delta'_u Y_d$ and $\Delta'_d Y_d$, respectively, in the construction of the effective theory.

down-type quarks summarised by

$$\mathcal{L}_{\rm eff} = -\varepsilon_{ij}\left(Y_d + \Delta_d Y_d\right)^{JI} H^d_i q^I_j d^{c\,J} - \left(\Delta_u Y_d\right)^{JI} H^{*u}_i q^I_i d^{c\,J} + \text{h.c.}\,, \qquad (7)$$

where I, J are the generation indices and we work in the language of two-component Weyl spinors. After diagonalizing the resulting fermion mass matrices one gets from (7) the neutral Higgs boson interactions of the form:

$$\mathcal{L}_{\rm int} \approx \frac{1}{\sqrt{2}} \tan\beta\; d^c_J \left(\Delta'_u Y_d\right)^{JI} d_I\; (H^0 + iA^0) + \text{h.c.} \qquad (8)$$

($\Delta'_u Y_d$ denotes the correction $\Delta_u Y_d$ in the basis in which the original Yukawa couplings are diagonal). Explicit evaluation of the first diagram in Fig. 1 ($\Delta_d Y_d$ does not contribute to the flavour changing neutral Higgs boson interactions) gives

$$\left(\Delta'_u Y_d\right)^{JI} = \frac{1}{16\pi^2}\lambda_{tI}\frac{e^2}{2\sqrt{2}s^3_W}\frac{m^2_t m_{d_J}}{M^3_W}\tan\beta A^*_t \mu^* C_0(m_{C_1}, M_{\tilde{t}_1}, M_{\tilde{t}_2})\,, \qquad (9)$$

where A_t is the top squark mixing parameter and C_0 is the standard three point loop function[3]. In the case of B^0_s decay (*i.e.* for $J = 3$, $I = 2$) Eqs. (8),(9) lead to:

$$\delta^C a = \delta^C b = f_{B_I} m_l \frac{G_{\rm F}\alpha}{\sqrt{2}\pi s^2_W}\lambda_{ts}\left[\frac{M^2_B}{8M^2_W}\frac{m^2_t}{M^2_A}\tan^3\beta A^*_t \eta m_{C_1} C_0\right]. \qquad (10)$$

The origin of the $\tan^3\beta$ factor is as follows: there are $\tan\beta$ factors both in Eq. (8) and in Eq. (9) and the third one comes from couplings of A^0 and H^0 Higgs bosons to the charged lepton pair. It is also important that the effects of charginos are significant only if the scale of the extended Higgs sector (set by M_A) is close to the electroweak scale.

Fig. 2 shows predictions of the MSSM for BR($B^0_s \to \mu^+\mu^-$) for different parameters of the model. Sharp dependence on the stop mixing angle $\theta_t \propto -A_t$ is clearly seen. The results are not very sensitive to the chargino composition (the ratio M_2/μ); it is the mass of the lighter chargino which is important. It is also remarkable that the chargino/stop contribution described by Eqs. (9),(10) does not vanish when all sparticle mass parameters are scaled uniformly: $M_{\tilde{t}_i} \to \lambda M_{\tilde{t}_i}$, $m_{C_i} \to \lambda m_{C_i}$, $\mu \to \lambda\mu$, $A_t \to \lambda A_t$.

[3] In this approach charginos are pure electroweak eigenstates: higgsino with mass $m_{C_1} = |\mu|$ and wino with mass $m_{C_2} = |M_2|$; it is the higgsino that contributes to $\Delta'_u Y_d$. Full diagrammatic calculation shows then that the replacement $|\mu| \to m_{C_1}$ extends the domain of applicability of the approximate formula (10); the factor η keeps then track of the phase of the μ^* parameter.

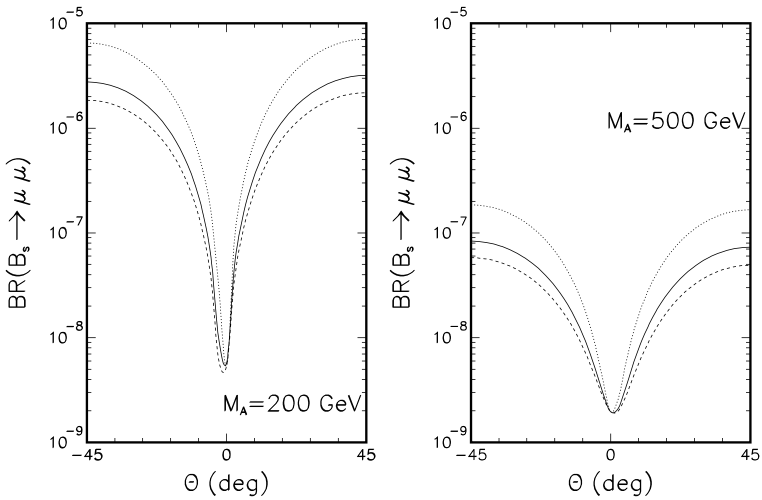

Fig. 2. $\mathrm{BR}(B_s^0 \to \mu^-\mu^+)$ as a function of the stop mixing angle θ_t for $\tan\beta = 50$, the lighter chargino mass 100 GeV, $M_2 = 10\mu$ and two different values of M_A. Solid, dashed and dotted lines correspond to $(M_{\tilde{t}_2}, M_{\tilde{t}_1})$ equal to (240, 500), (400, 700) and (300, 850) GeV, respectively.

To check the correlation of the prediction for $\mathrm{BR}(B_{s,d}^0 \to \mu^-\mu^+)$ with the results for $\mathrm{BR}(B \to X_s\gamma)$ we have performed two different scans over the MSSM parameter space corresponding to two different scenarios for which there exist formulae allowing to compute $\mathrm{BR}(B \to X_s\gamma)$ in the large $\tan\beta$ regime. In Figs. 3(a)–(d) we have taken the following parameter ranges: $500 < m_{C_1} < 1200$ GeV, $0.5 < |M_2/\mu| < 5$, $0.6 < M_{\tilde{t}_2}/m_{C_1} < 3$, $1 < M_{\tilde{t}_1}/M_{\tilde{t}_2} < 3$ and $-40^\circ < \theta_t < 40^\circ$ whereas in Figs. 3(e) and (f) we have used $100 < m_{C_1} < 200$ GeV, $2 < |M_2/\mu| < 6$, $m_{\tilde{g}} = 3.5 M_2$, $100 < M_{\tilde{t}_2} < 220$ GeV, $0.7 < M_{\tilde{t}_1}/m_{\tilde{g}} < 3$ and $-15^\circ < \theta_t < 15^\circ$. We have also rejected all points for which $\Delta\rho_{\mathrm{squarks}} > 6 \times 10^{-4}$. For computing $\mathrm{BR}(B \to X_s\gamma)$ we used the routine based on the recent calculation [13] and incorporated the charged Higgs and supersymmetric contributions following the prescriptions of Ref. [20] appropriate for large $\tan\beta$ and all sparticles heavy and of Refs. [20, 22] appropriate for the case of light one (mostly right-handed) stop and one chargino (mostly higgsino). The results of the scans, shown in Fig. 3 demonstrate that the the CLEO result for $\mathrm{BR}(B \to X_s\gamma)$ does not eliminate the points corresponding to the largest values of $\mathrm{BR}(B_{s,d}^0 \to \mu^-\mu^+)$ although certain correlation of the MSSM predictions for these two processes can be seen in the plots. The $\mathrm{BR}(B_{s,d}^0 \to \mu^-\mu^+)$ in the case of one light stop are generically smaller than in the case of all sparticles heavy because of the presence of one large mass in the C_0 function in Eq. (10).

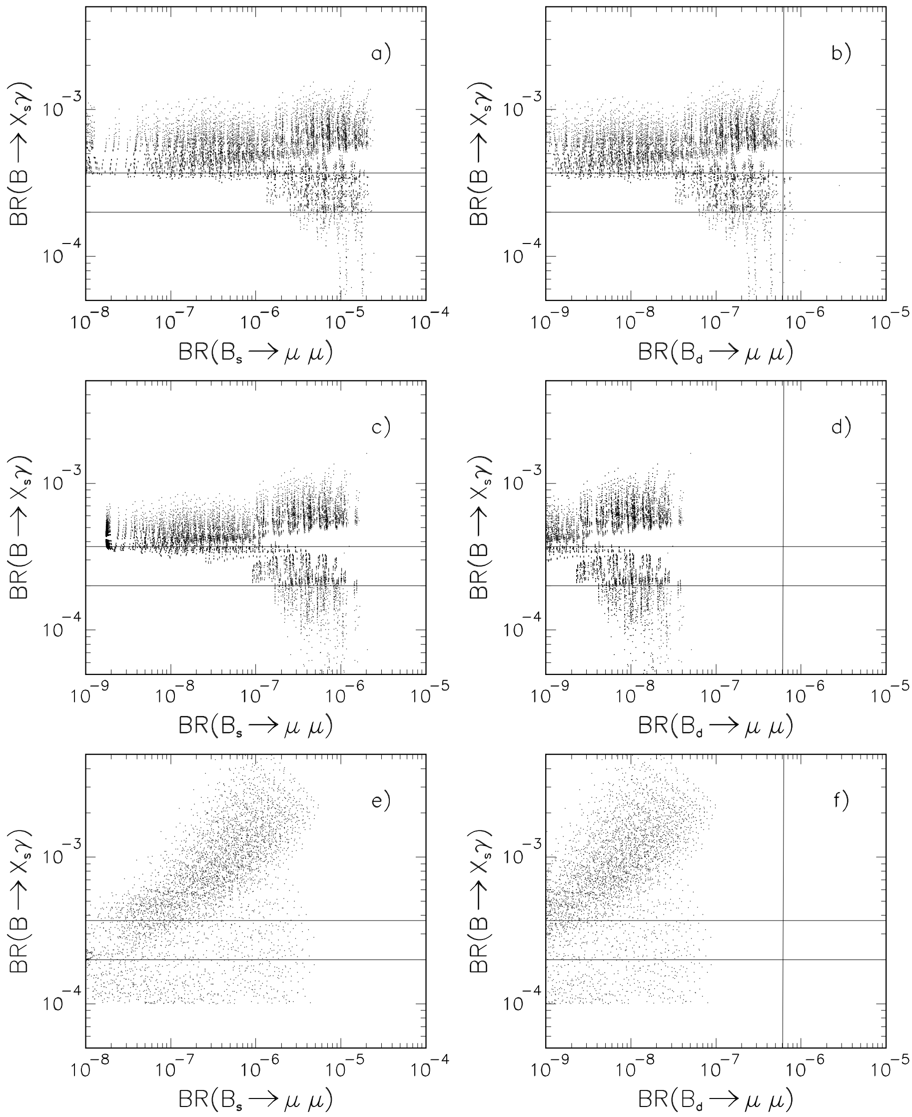

Fig. 3. BR($B \to X_s\gamma$) versus BR($B^0_{s,d} \to \mu^-\mu^+$) for $\tan\beta = 50$, $M_A = 200$ GeV in panels (a), (b) and (e), (f), and $\tan\beta = 50$, $M_A = 400$ GeV in panel (c) and (d). Panels (a)–(d) correspond to the case of heavy sparticles $\gtrsim \mathcal{O}(500\ \mathrm{GeV})$. Panels (e)–(f) correspond to the case of light stop and chargino. Limits from CLEO on BR($B \to X_s\gamma$) and on BR($B^0_d \to \mu^-\mu^+$) are also shown by solid lines. In panel (a) points corresponding to BR($B^0_d \to \mu^-\mu^+$) exceeding this limit have been eliminated.

The predictions for $\mathrm{BR}(B_d^0 \to \mu^+\mu^-)$ are obtained from Eq. (10) with λ_{ts} replaced by λ_{td}. In principle, as mentioned in Section 1, the CKM element V_{td} is not directly measured, and for each set of the MSSM parameters it should be consistently determined by fitting the MSSM predictions to the experimental results for the $B_d^0 - \bar{B}_d^0$ mass difference and the ε_K parameter [3,4]. We have done this (in Fig. 3(e) and (f)) by applying the approximate prescription of Refs. [21] (which takes into account only the contribution of stops and chargino through the standard $(V-A)\times(V-A)$ effective operator). It should be clear however, that for heavy sparticles the effects of chargino/stop loops in $B_d^0 - \bar{B}_d^0$ mixing and in ε_K are negligible and $|V_{td}|$ assumes its SM value.

4.

Effects of the flavour violation in the squark mass matrices can be computed in the same effective Lagrangian approach as described in the previous section. It is convenient to work in the so-called mass insertion [5] approximation in which the off-diagonal entries of the squark mass matrices are treated as additional (bilinear) interactions of squarks. One then finds additional contributions to the quantity $\Delta'_u Y_d$ arising from (dominant because of the strong coupling constant) diagrams shown in Fig. 4:

$$\begin{aligned}\left(\Delta'_u Y_d\right)^{JI} &= \frac{1}{16\pi^2}\frac{8}{3}g_s^2\frac{e}{\sqrt{2}s_W}\tan\beta\mu^* m_{\tilde{g}} D_0(M_\mathrm{D}, M_\mathrm{D}, M_\mathrm{D}, m_{\tilde{g}}) \\ &\quad\times\left[\frac{m_{d_J}}{M_W}\left(\mathcal{M}_\mathrm{D}^2\right)_\mathrm{LL}^{JI} + \left(\mathcal{M}_\mathrm{D}^2\right)_\mathrm{RR}^{JI}\frac{m_{d_I}}{M_W}\right], \end{aligned} \tag{11}$$

where g_s is the strong coupling constant, $m_{\tilde{g}}$ is the gluino mass, $\left(\mathcal{M}_\mathrm{D}^2\right)_\mathrm{LL}^{JI}$ and $\left(\mathcal{M}_\mathrm{D}^2\right)_\mathrm{RR}^{JI}$ are the off-diagonal entries of the left–left and right–right blocks of the down-type squark mass squared matrix. In the four point function D_0 we have taken for simplicity a common value M_D for all down-type squarks. In the case of B^0 decays (*i.e.* $J=3$, $I=s,d$) Eq. (11) leads to:

$$\begin{aligned}\delta^G a &= f_{B_I} m_l \frac{G_\mathrm{F}\alpha}{\sqrt{2}\pi s_W^2}\frac{M_B^2}{M_A^2}\frac{2}{3}g_s^2\frac{s_W^2}{e^2}\tan\beta^3 m_{\tilde{g}}^* D_0 \\ &\quad\times\left[\mu^*\left(\mathcal{M}_\mathrm{D}^2\right)_\mathrm{LL}^{3I} + \mu\left(\mathcal{M}_\mathrm{D}^2\right)_\mathrm{RR}^{3I}\right]\end{aligned} \tag{12}$$

and $\delta^G b$ is obtained by changing $+$ to $-$ in the square bracket.

It is important that the contribution of gluinos depends only on the LL and RR mass insertions $(\delta^\mathrm{D}_{\mathrm{LL(RR)}})^{I3} \equiv M_\mathrm{D}^{-2}\left(\mathcal{M}_\mathrm{D}^2\right)^{I3}_{\mathrm{LL(RR)}}$ which unlike the insertions $(\delta^\mathrm{D}_\mathrm{LR})$ are not so severely constrained [4,5]:

$$|(\delta^\mathrm{D}_{\mathrm{LL(RR)}})^{31}| < 0.2\times(M_\mathrm{D}/1\ \mathrm{TeV}), \quad |(\delta^\mathrm{D}_{\mathrm{LL(RR)}})^{32}| < 30\times(M_\mathrm{D}/1\ \mathrm{TeV})^2.$$

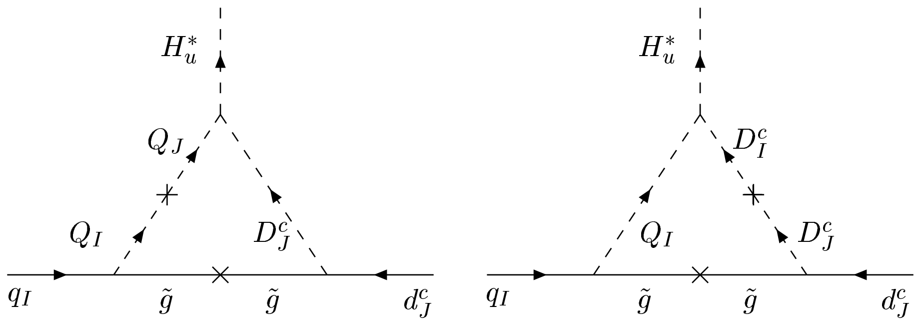

Fig. 4. Additional contributions to $\Delta_u' Y_d$. Crosses on squark lines denote bilinear flavour changing mass insertions.

As is clear from Eq. (12), the effects of nonzero mass insertions, similarly to those of charginos, are proportional to $\tan^3\beta/M_A^2$ and do not vanish if all supersymmetric mass parameters are scaled uniformly (provided the value of the insertion is kept fixed). It is also important that because the 31 and 32 insertions are *a priori* unrelated, the rates of leptonic decays of B_d^0 and B_s^0 induced by flavour violation in squark mass matrices are not correlated and can even be comparable. We have checked [6] that respecting all existing bounds $\mathrm{BR}(B_s^0 \to \mu^-\mu^+)$ induced by gluino exchange can be as large as $10^{-5} - 10^{-4}$, whereas $\mathrm{BR}(B_d^0 \to \mu^-\mu^+)$ can easily exceed the CLEO bound [18] (in which case CLEO eliminates these points of the MSSM parameter space).

5.

Finally, we comment on possible effects of CP violation in the $B^0 \to l^- l^+$ decay. We follow the formalism developed in Ref. [7]. Since the decay amplitude does not involve any strong phases, there can be no direct CP violation. It can however be seen in the interference of mixing and decays provided one is able to measure separately the decay rates into two possible helicity final states $|l^-_{\mathrm{L}} l^+_{\mathrm{L}}\rangle$ and $|l^-_{\mathrm{R}} l^+_{\mathrm{R}}\rangle$ which are mutually CP conjugate. Using the standard formalism [23] and defining the "physical" $|B^0_{\mathrm{ph}}(t)\rangle$ and $|\bar{B}^0_{\mathrm{ph}}(t)\rangle$ states as

$$
\begin{aligned}
|B^0_{\mathrm{ph}}(t)\rangle &= g_+(t)|B^0\rangle + \left(\frac{q}{p}\right) g_-(t)|\bar{B}^0\rangle\,, \\
|\bar{B}^0_{\mathrm{ph}}(t)\rangle &= g_+(t)|\bar{B}^0\rangle + \left(\frac{p}{q}\right) g_-(t)|B^0\rangle\,,
\end{aligned}
\qquad (13)
$$

where q and p have the standard meaning and[4]

$$
\begin{aligned}
g_{+}(t) &= \mathrm{e}^{-iM_Bt}\mathrm{e}^{-\frac{\Gamma}{2}t}\cos\left[\Delta M_B\frac{t}{2}\right],\\
g_{-}(t) &= \mathrm{e}^{-iM_Bt}\mathrm{e}^{-\frac{\Gamma}{2}t}\; i\;\sin\left[\Delta M_B\frac{t}{2}\right]
\end{aligned}
\tag{14}
$$

one easily computes various CP asymmetries. For simplicity we consider here only the time integrated asymmetries A_1

$$
A_1=\frac{\int_0^\infty dt\Gamma(B^0_{\rm ph}(t)\to l^-_{\rm L}l^+_{\rm L})-\int_0^\infty dt\Gamma(\bar{B}^0_{\rm ph}(t)\to l^-_{\rm R}l^+_{\rm R})}{\int_0^\infty dt\Gamma(B^0_{\rm ph}(t)\to l^-_{\rm L}l^+_{\rm L})+\int_0^\infty dt\Gamma(\bar{B}^0_{\rm ph}(t)\to l^-_{\rm R}l^+_{\rm R})},
\tag{15}
$$

and A_2 obtained by interchanging LL and RR in Eq. (15). One finds:

$$
A_1=\frac{\frac{1}{2}x^2\left(|\lambda_{\rm L}|^2-|\lambda_{\rm R}|^{-2}\right)-x{\rm Im}\left(\lambda_{\rm L}-\lambda_{\rm R}^{-1}\right)}{2+x^2+\frac{1}{2}x^2\left(|\lambda_{\rm L}|^2+|\lambda_{\rm R}|^{-2}\right)-x{\rm Im}\left(\lambda_{\rm L}+\lambda_{\rm R}^{-1}\right)}
\tag{16}
$$

(A_2 is obtained by the interchange $\lambda_{\rm L}\leftrightarrow\lambda_{\rm R}$) where $x\equiv\Delta M_B/\Gamma$ and

$$
\lambda_{\rm L}\equiv\frac{q}{p}\frac{\bar{\mathcal{A}}_{\rm L}}{\mathcal{A}_{\rm L}},\quad \lambda_{\rm R}\equiv\frac{q}{p}\frac{\bar{\mathcal{A}}_{\rm R}}{\mathcal{A}_{\rm R}}.
\tag{17}
$$

The amplitudes $\mathcal{A}_{\rm L(R)}\equiv\langle l^-_{\rm L(R)}l^+_{\rm L(R)}|B^0\rangle$, $\bar{\mathcal{A}}_{\rm L(R)}\equiv\langle l^-_{\rm L(R)}l^+_{\rm L(R)}|\bar{B}^0\rangle$ are given in terms of the coefficients a and b defined in Eq. (1) as

$$
\begin{aligned}
\mathcal{A}_{\rm L}&=a+b\sqrt{1-\frac{4m_l^2}{M_B^2}},\quad \mathcal{A}_{\rm R}=a-b\sqrt{1-\frac{4m_l^2}{M_B^2}},\\
\bar{\mathcal{A}}_{\rm L}&=\bar{a}+\bar{b}\sqrt{1-\frac{4m_l^2}{M_B^2}},\quad \bar{\mathcal{A}}_{\rm R}=\bar{a}-\bar{b}\sqrt{1-\frac{4m_l^2}{M_B^2}}.
\end{aligned}
\tag{18}
$$

Because there is no direct CP violation the following relations hold:

$$
\bar{a}=a^*,\quad \bar{b}=-b^*.
\tag{19}
$$

These relations imply $|\mathcal{A}_{\rm L(R)}|=|\mathcal{A}_{\rm R(L)}|$ which we have used in arriving at Eq. (16). Also, it follows that

$$
\frac{\bar{\mathcal{A}}_{\rm R}}{\mathcal{A}_{\rm R}}=\left(\frac{\mathcal{A}_{\rm L}}{\bar{\mathcal{A}}_{\rm L}}\right)^*
\tag{20}
$$

[4] For simplicity we set here $\Delta\Gamma=0$.

so if $|q/p| = 1$ the formulae for the asymmetries A_1 and A_2 simplify to

$$A_1 = \frac{-2x\mathrm{Im}\lambda_\mathrm{L}}{2 + x^2 + x^2|\lambda_\mathrm{L}|^2}\,, \quad A_2 = \frac{-2x\mathrm{Im}\lambda_\mathrm{R}}{2 + x^2 + x^2|\lambda_\mathrm{R}|^2}\,. \tag{21}$$

The maximal possible value of the asymmetry is $A^{\max}_{1,2} = (2+x^2)^{-1/2}$. Thus, for B^0_s mesons $A_{1,2} < 5\%$ and we will, therefore, concentrate on B^0_d mesons for which the maximum of $A_{1,2}$ is $\sim 63\%$.

In the SM the asymmetries $A_{1,2}$ are very small because to a very good accuracy $q/p \approx \lambda_{tI}/\lambda^*_{tI}$ and $\bar{\mathcal{A}}_{\mathrm{L(R)}}/\mathcal{A}_{\mathrm{L(R)}} \propto \lambda^*_{tI}/\lambda_{tI}$ so that $\mathrm{Im}\lambda_{\mathrm{L(R)}} \approx 0$. Any observed asymmetry would be, therefore, a clean signal for physics beyond the SM. If in the MSSM the CKM matrix is the only source of *both*, FCNC and CP violation, then the same argument as for the SM applies and the asymmetry is very small. There are, however, other possibilities. Since for a measurement of the asymmetry a decent statics is needed, we consider here only the situations described in the two preceding sections in which the MSSM predicts $\mathrm{BR}(B^0_d \to l^-l^+)$ much larger than does the SM.

Suppose first that there is no flavour violation in the squark mass matrices but the product $A_t\mu$ has a CP violating phase φ. We further assume for simplicity that sparticles are heavy enough so that their contribution to the $B^0_d - \bar{B}^0_d$ mixing is very small so that $q/p \approx \lambda^*_{tI}/\lambda_{tI}$ just as in the SM. (For such heavy sparticles the constraints on CP violating phases from the electron and neutron electric dipole moments are not very restrictive [24].) Using Eqs. (10),(19),(18),(17) one finds

$$\lambda_\mathrm{L} = \mathrm{e}^{2i\varphi}\frac{1 - \sqrt{1 - \frac{4m_l^2}{M_B^2}}}{1 + \sqrt{1 - \frac{4m_l^2}{M_B^2}}}\,, \quad \lambda_\mathrm{R} = \mathrm{e}^{2i\varphi}\frac{1 + \sqrt{1 - \frac{4m_l^2}{M_B^2}}}{1 - \sqrt{1 - \frac{4m_l^2}{M_B^2}}}\,, \tag{22}$$

i.e. in this limiting situation the asymmetry is independent of the MSSM parameters except for the phase φ of the product $A_t\mu$. It is clear that for $\mu^-\mu^+$ final state both asymmetries are small because $\lambda_\mathrm{L}(\lambda_\mathrm{R})$ is too small (too large). For the $\tau^-\tau^+$ final state, however, we find[5]

$$A_1 \approx -0.09 \times \sin(2\varphi)\,, \quad A_2 \approx -0.37 \times \sin(2\varphi)\,. \tag{23}$$

Another interesting situation arises in the case in which the decay amplitude is entirely dominated by the flavour off diagonal mass insertions (as in Sec. 4). Consider *e.g.* the effects of nonzero $\left(\mathcal{M}^2_\mathrm{D}\right)^{31}_\mathrm{LL}$ insertion. The

[5] $\mathrm{BR}(B^0_d \to \tau^-\tau^+)$ is obtained from $\mathrm{BR}(B^0_d \to \mu^-\mu^+)$ by multiplying the latter by $m^2_\tau/m^2_\mu = 286$.

asymmetry is then given by Eq. (23) with φ replaced by the $\varphi_{td}+\varphi'$, where $\varphi_{td}=\arg(\lambda^*_{td})$ and φ' is the phase of $\mu^*\left(\mathcal{M}^2_{\rm D}\right)^{31*}_{\rm LL}$. It is interesting to note that in general the asymmetries can be nonzero even if all supersymmetric mass parameters are real (including the off diagonal mass insertions). The asymmetries are then directly determined by the phase of the CKM matrix element V_{td}.

6.

Large enhancement of the SM prediction for rates of the leptonic decays of B^0 mesons can occur for $\tan\beta \gg 1$ provided the MSSM Higgs bosons are not too heavy (the new contributions behave as $1/M^2_A$, where M_A is the mass of the CP-odd neutral Higgs boson which sets the scale of the Higgs sector). The contribution of the extended Higgs sector ($\propto \tan^4\beta$) is less important compared to the effects of the chargino sector which grow as $\tan^6\beta$. For $\tan\beta \sim m_t/m_b$ and substantial mixing of the top squarks these can give $\mathrm{BR}(B^0_{s(d)} \to \mu^-\mu^+)$ up to $5\times 10^{-5}(10^{-6})$ respecting other phenomenological constraints including the measurement of $\mathrm{BR}(B\to X_s\gamma)$. Large effects, growing as $\tan^6\beta$ and exhibiting strong dependence on the μ parameter, can be also induced by the off diagonal elements of the down-type squark mass matrix. $\mathrm{BR}(B^0_{s(d)} \to \mu^-\mu^+)$ is sensitive to the 23 (13) off-diagonal entries of the LL and RR blocks of these matrices, which are not so strongly constrained. For $\tan\beta\sim m_t/m_b$ and $M_A \lesssim 200$ GeV these effects can easily give $\mathrm{BR}(B^0_s \to \mu^-\mu^+)$ larger than 10^{-4} and $\mathrm{BR}(B^0_d \to \mu^-\mu^+)$ exceeding the present CLEO limit [18] which, therefore, already now puts constraints on the MSSM parameter space.

In the case in which the chargino and/or gluino contribution dominate over the SM (and Higgs sector) one, the CP violating asymmetries measured in the decays $B^0_d(\bar{B}^0) \to \tau^-\tau^+$ can be interesting probes of the complex phases of the MSSM parameters and/or the V_{td} element of the CKM matrix.

Finally it is important to stress that, all these effects do not necessarily decrease as sparticles become heavy. They are, however, sensitive to the mass scale of the extended Higgs sector. Thus, large deviation from the SM prediction observed in these decays, or nonzero CP asymmetry seen in $B^0_d \to \tau^-\tau^+$ decays apart from being a signal of supersymmetry, would have important implications on the Higgs search at the LHC.

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