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CP VIOLATION IN HIGGS–BOSON INTERACTIONS*

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We consider a general two-Higgs-doublet model with CP violation in the scalar sector, that leads, at the one-loop level of the perturbation expansion, to CP violation in the process $e^+e^- \to t\bar{t} \to l^\pm \dots$ and $e^+e^- \to t\bar{t} \to \overset{(-)}{b} \dots$. The goal of this study is to include *consistently* CP-violating effects in distributions of top-quark decay products ($l^\pm$ or $\overset{(-)}{b}$) that emerge *both* from $t\bar{t}$ production *and* from t or $\bar{t}$ decay processes.

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1. Introduction

Interactions of the top quark have not been precisely tested yet, in particular, CP violation in the top-quark interactions has not been verified. The classical method for incorporating CP violation into the Standard Model (SM) of electroweak interactions is to make Yukawa couplings of the Higgs boson to quarks explicitly complex, as built into the Kobayashi–Maskawa mixing matrix [1] proposed more than two decades ago. However, CP violation could equally well be partially or wholly due to other mechanisms. The possibility that CP violation derives largely from the Higgs sector itself is particularly appealing in the context of the observed baryon asymmetry, since its explanation requires more CP violation [2] than is provided by the SM. Even the simple two-Higgs-doublet model (2HDM) extension of the one-doublet SM Higgs sector provides a much richer framework for describing CP violation since there spontaneous and/or explicit CP violation is possible in the scalar sector [3]. The model, besides CP violation, offers many other appealing phenomena, for a review see Ref. [4].

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For our analysis, the most relevant part of the interaction Lagrangian takes the following form[1]:

$$\mathcal{L} = -\frac{m_t}{v} h\bar{t}(a + i\gamma_5 b)t + C\frac{h}{v}\left(m_Z^2 Z_\mu Z^\mu + 2m_W^2 W_\mu W^\mu\right) , \qquad (1)$$

where h is the lowest mass scalar, g is the SU(2) coupling constant, v is the Higgs boson vacuum expectation value (with the normalization adopted here such that $v = 2m_W/g = 246$ GeV), a, b and C are real parameters which account for deviations from the SM, $a = 1$, $b = 0$ and $C = 1$ reproduce the SM Lagrangian. Since under CP, $\bar{t}(a + i\gamma_5 b)t \overset{CP}{\to} \bar{t}(a - i\gamma_5 b)t$ and $Z_\mu Z^\mu \overset{CP}{\to} Z^\mu Z_\mu$, one can observe that terms in the cross section proportional to ab or bC would indicate CP violation. The 2HDM is the minimal extension of the SM that provides non-zero ab and/or bC.

In this paper we will focus on CP-violating contributions to the process $e^+e^- \to t\bar{t} \to l^\pm \ldots$ and $e^+e^- \to t\bar{t} \to \overset{(-)}{b} \ldots$ induced within 2HDM. However, the fundamental goal is to seek for the ultimate theory of electroweak interactions. There are several reasons to utilize CP violation in the top physics while looking for physics beyond the SM:

- The top quark decays immediately after being produced as its huge mass $m_t = 174.0 \pm 3.2 \pm 4.0\,\text{GeV}$ [6] leads to a decay width Γ_t much larger than Λ_{QCD}. Therefore, the decay process is not contaminated by any fragmentation effects [7] and decay products may provide useful information on top-quark properties.
- Since the top quark is heavy, its Yukawa coupling is large and therefore, its interactions could be sensitive to a Higgs sector of the electroweak theory.
- At the same time, the TESLA collider design is supposed to offer an integrated luminosity of the order of $L = 500\ \text{fb}^{-1}\text{y}^{-1}$ at $\sqrt{s} = 500\,\text{GeV}$. Therefore, expected number of $t\bar{t}$ events per year could reach 5×10^4 even for $t\bar{t}$ tagging efficiency $\varepsilon_{t\bar{t}} = 15\%$. That should allow to study subtle properties of the top quark, which could *e.g.* lead to CP-sensitive asymmetries of the order of 5×10^{-3}.
- Since the top quark is that heavy and the third family of quarks effectively decouples from the first two, any CP-violating observables within the SM are expected to be tiny, *e.g.*: *(i)* non-zero electric dipole moment of fermions is generated at the three-loop approximation of the perturbation expansion [8], *(ii)* the decay rate asymmetry (being a one-loop effect) is strongly GIM suppressed reaching at most a value 10^{-9} [9]. So, one can expect that for CP-violating asymmetries any SM background could be safely neglected.

[1] One could also consider more general, CP-violating ZZh coupling, see Ref. [5], however here the contribution from such a vertex would be negligible.

Therefore, it seems to be justified to look for CP-violating Higgs effects in the process of $t\bar{t}$ production and its subsequent decay at future linear e^+e^- colliders. Even though 2HDM contributions to various CP-sensitive asymmetries have already been published in the existing literature [10, 11], here we are presenting results (for a detailed discussion see Ref. [12]) of a consistent treatment of CP violation *both* in the production, $e^+e^- \to t\bar{t}$, *and* in the top-quark decay, $t \to bW$. For an extensive review of CP violation in top-quark interactions see Ref. [13].

The paper is organized as follows. In Section 2, we briefly outline the mechanism of CP violation in the 2HDM, introduce the mixing matrix for neutral scalars and derive necessary couplings. In Section 3, we recall current experimental constraints relevant for the CP-violating observables considered in this paper. In Section 4, we collect results for the most attractive energy and angular CP-violating asymmetries. Concluding remarks are given in Section 5.

2. The two-Higgs-doublet model with *CP* violation

The 2HDM of electroweak interactions contains two SU(2) Higgs doublets denoted by $\Phi_1 = (\phi_1^+, \phi_1^0)$ and $\Phi_2 = (\phi_2^+, \phi_2^0)$. It is well known [3] that the model allows for both spontaneous and explicit CP violation[2].

After SU(2)$\times$U(1) gauge symmetry breaking, one combination of neutral Higgs fields, $\sqrt{2}(c_\beta \mathrm{Im}\,\phi_1^0 + s_\beta \mathrm{Im}\,\phi_2^0)$, becomes a would-be Goldstone boson which is absorbed while giving mass to the Z gauge boson. (Here, we use the notation $s_\beta \equiv \sin\beta$, $c_\beta \equiv \cos\beta$, where $\tan\beta = \langle\phi_2^0\rangle/\langle\phi_1^0\rangle$.) The same mixing angle, β, also diagonalizes the mass matrix in the charged Higgs sector. If either explicit or spontaneous CP violation is present, the remaining three neutral degrees of freedom,

$$(\varphi_1, \varphi_2, \varphi_3) \equiv \sqrt{2}(\mathrm{Re}\,\phi_1^0,\ \mathrm{Re}\,\phi_2^0,\ s_\beta \mathrm{Im}\,\phi_1^0 - c_\beta \mathrm{Im}\,\phi_2^0) \tag{2}$$

are not mass eigenstates. The physical neutral Higgs bosons h_i $(i = 1, 2, 3)$ are obtained by an orthogonal transformation, $h = R\varphi$, where the rotation matrix is given in terms of three Euler angles $(\alpha_1, \alpha_2, \alpha_3)$ by

$$R = \begin{pmatrix} c_1 & -s_1c_2 & s_1s_2 \\ s_1c_3 & c_1c_2c_3 - s_2s_3 & -c_1s_2c_3 - c_2s_3 \\ s_1s_3 & c_1c_2s_3 + s_2c_3 & -c_1s_2s_3 + c_2c_3 \end{pmatrix}, \tag{3}$$

where $s_i \equiv \sin\alpha_i$ and $c_i \equiv \cos\alpha_i$.

[2] Here we are considering a model with discrete Z_2 symmetry that prohibits flavor changing neutral currents. In order to allow for CP violation the symmetry has to be broken softly by the term $\mu_{12}^2 \Phi_1^\dagger \Phi_2$ in the potential.

As a result of the mixing between real and imaginary parts of neutral Higgs fields, the Yukawa interactions of the h_i mass-eigenstates are not invariant under CP. They are given by:

$$\mathcal{L} = -\frac{m_f}{v} h_i \bar{f}(a_i^f + i b_i^f \gamma_5) f \,, \tag{4}$$

where the scalar (a_i^f) and pseudoscalar (b_i^f) couplings are functions of the mixing angles. For up-type quarks we have

$$a_i^u = \frac{1}{s_\beta} R_{i2}, \quad b_i^u = \frac{c_\beta}{s_\beta} R_{i3} \tag{5}$$

and for down-type quarks:

$$a_i^d = \frac{1}{c_\beta} R_{i1}, \quad b_i^d = \frac{s_\beta}{c_\beta} R_{i3} \tag{6}$$

and similarly for charged leptons. For large $\tan\beta$, the couplings to down-type fermions are typically enhanced over the couplings to up-type fermions.

In the following analysis we will also need the couplings of neutral Higgs and vector bosons, they are given by

$$g_{VVh_i} \equiv 2\frac{m_V^2}{v} C_i = 2\frac{m_V^2}{v}(s_\beta R_{i2} + c_\beta R_{i1}) \tag{7}$$

for $V = Z, W$. Hereafter we shall denote the lightest Higgs boson by h and its R-matrix index by i.

3. Experimental constraints

Hereafter we will focus on Higgs boson masses in the region, $m_h = 10 \div 100\,\mathrm{GeV}$. As it has been shown in the literature [14, 15] the existing LEP data are perfectly consistent with one light[3] Higgs boson within the 2HDM. It turns out that even precision electroweak tests allow for light Higgs bosons [16].

In order to amplify the form factors calculated in this paper we have adopted for an illustration $\tan\beta = 0.5$. However, there exist experimental constraints on $\tan\beta$ from $K^0 - \bar{K}^0$ and $B_d - \bar{B}_d$ mixing [17], $b \to s\gamma$ decay [25] and $Z \to b\bar{b}$ decay [18]. Since small $\tan\beta$ enhances $H^\pm tb$ coupling, in order to maintain $\tan\beta = 0.5$ we have to decouple charged Higgs effects and therefore, we assume that $m_{H^\pm} \gtrsim 500 \div 600\,\mathrm{GeV}$.

[3] Sum rules discussed in Ref. [14] prove that even within the CP-violating version of the 2HDM one can satisfy LEP experimental constraints with one light Higgs boson.

The constraints on the mixing angles α_i that should be imposed in our numerical analysis are as follows:

- The ZZh couplings, C_i^2, are restricted by non-observation of Higgs-strahlung events at LEP1 and LEP2, see Ref. [19]
- The contribution to the total Z-width from $Z \to Z^* h_i \to f\bar{f}h_i$ is required to be below 7.1 MeV, see Ref. [20].

It turns out that the restriction on the ZZh coupling from its contribution to the total Z-width is always weaker than the one from Zh production if $m_h \gtrsim 10\,\mathrm{GeV}$.

The LEP constraints on the ZZh coupling restrict the following entries of the mixing matrix R_{ij}:

$$|\sin\beta R_{i2} + \cos_\beta R_{i1}| \le C_i^{\exp}\,, \tag{8}$$

where $C_i^{\exp}$ stands for the upper limit for the relative strength of ZZh coupling determined experimentally in Ref. [19] up to the Higgs mass $m_h = 105\,\mathrm{GeV}$. As we have concluded in the previous section, CP-violating phenomena, we are considering, are enhanced by small $\tan\beta$. In that case one can see from Eq. (8) that the LEP constraints mostly restrict R_{i1}. Through the orthogonality the restriction on R_{i1} is transferred to constrain $|R_{i2}R_{i3}| = |R_{i2}\sqrt{1 - R_{i1}^2 - R_{i2}^2}|$ which multiplies leading contributions to all CP-violating asymmetries considered here[4]. The final result for the upper limit on $|R_{i2}R_{i3}|$ as a function of $\tan\beta$ is shown in Fig. 1. In fact the bound

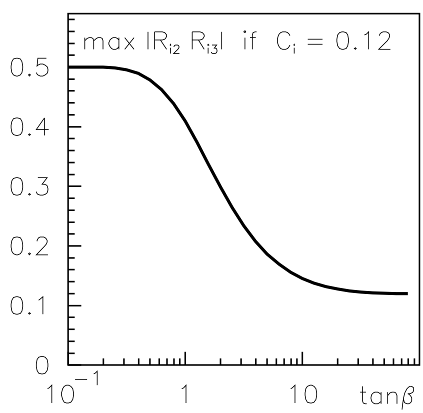

Fig. 1. Maximal value of $|R_{i2}R_{i3}|$ allowed by the LEP constraints on ZZh_i coupling as a function of $\tan\beta$.

[4] As it has been shown in Ref. [12] the other contribution that is proportional to $R_{i1}R_{i3}$ is by 1-2 orders of magnitude smaller.

on $|R_{i2}R_{i3}|$ depends on the Higgs mass, however, in order to be conservative, we have assumed $C_i^{\rm exp}=0.12$ that is the most restrictive experimental limit[5] (obtained for $m_h \simeq 18\,{\rm GeV}$).

As it is seen from Fig. 1 the constraints for $|R_{i2}R_{i3}|$ are weak for small $\tan\beta$. Therefore for $\tan\beta \simeq 0.5$ it should be legitimate to assume $|R_{i2}R_{i3}| \simeq \frac{1}{2}$ which is the maximal value consistent with orthogonality.

4. *CP*-violating asymmetries

Hereafter we assume that there exists only one light Higgs boson h and possible effects of the heavier scalar degrees of freedom decouple.

The effective $t\bar{t}\gamma$ and $t\bar{t}Z$ vertices will be parameterized by the following form factors[6]:

$$\Gamma_v^\mu = \frac{g}{2}\bar{u}(p_t)\left[\gamma^\mu(A_v - B_v\gamma_5) + \frac{(p_t - p_{\bar{t}})^\mu}{2m_t}(C_v - D_v\gamma_5)\right]v(p_{\bar{t}}), \qquad (9)$$

where g denotes the SU(2) gauge coupling constant and $v=\gamma, Z$. The SM contributions to the form factors are the following:

$$A_\gamma^{\rm (SM)} = -\frac{4}{3}\sin\theta_W\,, \quad B_\gamma = 0, \quad A_Z^{\rm (SM)} = -\frac{v_t}{2\cos\theta_W}\,, \quad B_Z^{\rm (SM)} = -\frac{a_t}{2\cos\theta_W}$$

for

$$v_t = \left(1 - \frac{8}{3}\sin^2\theta_W\right), \qquad a_t = 1\,.$$

The form factors A_v, B_v, C_v describe CP-conserving while D_v parameterizes CP-violating contributions.

Further in this paper, the following parameters will be adopted: $m_t = 175\,{\rm GeV}$, $m_Z = 91.187\,{\rm GeV}$, $\Gamma_Z = 2.49\,{\rm GeV}$, $\sin^2\theta_W = 0.23$ and $m_b = 4.2\,{\rm GeV}$.

Direct calculation of appropriate Feynman diagrams leads to the following result [12] in terms of three-point Passarino–Veltman [21] functions defined in the appendix of Ref. [12]:

$$\begin{aligned} D_\gamma &= \frac{\rm i}{2\pi^2}A_\gamma\frac{m_t^2}{v^2}b_i^t a_i^t m_t^2 \boldsymbol{C}_{12}(p_t, p_{\bar{t}}, m_t^2, m_h^2, m_t^2)\,, \\ D_Z &= \frac{\rm i}{2\pi^2}A_Z\frac{m_t^2}{v^2}b_i^t\left[a_i^t m_t^2 \boldsymbol{C}_{12}(p_t, p_{\bar{t}}, m_t^2, m_h^2, m_t^2)\right. \\ &\quad \left. - C_i m_Z^2 \boldsymbol{C}_{12}(p_t, p_{\bar{t}}, m_h^2, m_t^2, m_Z^2)\right]\,. \end{aligned} \qquad (10)$$

[5] For $m_h \simeq 18\,{\rm GeV}$ the limits presented in Fig. 16 of Ref. [19] for the case when no b-tagging and with b-tagging almost coincide. Therefore, our plot in Fig. 1 is not influenced by potential problems concerning the dependence of the Higgs-$b\bar{b}$ and Higgs-$\tau^+\tau^-$ branching ratios on the mixing angles.

[6] Two other possible form factors do not contribute in the limit of zero electron mass.

From Eq. (10) and Eqs.(5),(7) one can find out that all contributions to the form factors D_γ, D_Z are enhanced for small $\tan\beta$.

We will adopt the following parameterization of the Wtb vertex suitable for the t and $\bar{t}$ decays:

$$\begin{aligned} \Gamma^\mu &= -\frac{g}{\sqrt{2}} V_{tb}\left[\gamma^\mu(f_1^{\rm L}P_{\rm L}+f_1^{\rm R}P_{\rm R}) - \frac{i\sigma^{\mu\nu}k_\nu}{M_W}(f_2^{\rm L}P_{\rm L}+f_2^{\rm R}P_{\rm R})\right], \\ \bar{\Gamma}^\mu &= -\frac{g}{\sqrt{2}} V_{tb}^*\left[\gamma^\mu(\bar{f}_1^{\rm L}P_{\rm L}+\bar{f}_1^{\rm R}P_{\rm R}) - \frac{i\sigma^{\mu\nu}k_\nu}{M_W}(\bar{f}_2^{\rm L}P_{\rm L}+\bar{f}_2^{\rm R}P_{\rm R})\right], \end{aligned} \tag{11}$$

where $P_{\rm L/R}=(1\mp\gamma_5)/2$, V_{tb} is the (tb) element of the Kobayashi–Maskawa matrix and k is the momentum of W. In the SM $f_1^{\rm L}=\bar{f}_1^{\rm L}=1$ and all the other form factors vanish. It turns out that in the limit of massless bottom quarks the only form factors that interfere with the SM are $f_2^{\rm R}$ and $\bar{f}_2^{\rm L}$ for the top and anti-top decays, respectively. Currently, there is no relevant experimental bound on those form factors[7].

One can show that the CP-violating and CP-conserving parts of the form factors for t and $\bar{t}$ are not independent:

$$f_1^{\rm L,R} = \pm\bar{f}_1^{\rm L,R} \quad \text{and} \quad f_2^{\rm L,R} = \pm\bar{f}_2^{\rm R,L}, \tag{12}$$

where upper (lower) signs are those for CP-conserving (-violating) contributions [11, 24]. Therefore any CP-violating observable defined for the top-quark decay must be proportional to $f_1^{\rm L,R}-\bar{f}_1^{\rm L,R}$ or $f_2^{\rm L,R}-\bar{f}_2^{\rm R,L}$.

At the one-loop level one gets the following result [12] for CP-violating contribution to ${\rm Re}\,(f_2^{\rm R}|_{CPV})$:

$${\rm Re}\,(f_2^{\rm R}-\bar{f}_2^{\rm L}) = 2{\rm Re}\,(f_2^{\rm R}|_{CPV}) = \frac{g}{16\pi^2}\frac{m_b}{v}m_b m_t b_i^b C_i {\rm Im}\,\boldsymbol{C}_{22}^{bd}\,. \tag{13}$$

Adopting the maximal value of $R_{i2}R_{i3}$ allowed by the orthogonality and the LEP constraints for $\tan\beta=0.5$, we may discuss a possibility for an experimental determination of the calculated form factors at future e^+e^- colliders. A detailed discussion of expected statistical uncertainties for a measurement of the form factors has been performed in Ref. [26]. It has been shown that adjusting an optimal e^+e^- beam polarizations, using the energy and angular double distribution of final leptons and fitting *all nine*

[7] There exists, however, direct experimental constraint from the Fermilab Tevatron on the form factor $f_1^{\rm R}$ that are obtained through the determination of the W-boson helicity. Pure $V-A$ theory for massless bottom quarks predicts an absence of positive helicity W^+ bosons, therefore the upper limit on the helicity $\mathcal{F}_+$ implies an upper limit on the $V+A$ coupling $f_1^{\rm R}$, however, the resulting limit is rather weak [22]. There exist an indirect, but much stronger bound [23] on the admixture of right-handed currents, $\bar{f}_1^{\rm R}$, coming from data for $b\to s\gamma$, namely $-0.05\lesssim\bar{f}_1^{\rm R}\lesssim 0.01$.

form factors leads to the following statistical errors for the determination of CP-violating form factors: $\Delta[\mathrm{Re}\,(D_\gamma)] = 0.08$ and $\Delta[\mathrm{Re}\,(D_Z)] = 14.4$ for $\varepsilon_{t\bar{t}} \simeq 15\%$. It is seen that only $\mathrm{Re}\,(D_\gamma)$, could be measured with a high precision. We have found (see plots in Ref. [12]) that $\mathrm{Re}\,(D_\gamma)$ may reach at most a value of 0.10, therefore one shall conclude that several years of running with yearly integrated luminosity $L = 500\ \mathrm{fb}^{-1}\mathrm{y}^{-1}$ should allow for an observation of $\mathrm{Re}\,(D_\gamma)$ generated within 2HDM, provided the lightest Higgs boson mass is not too large. On the other hand, the expected [26] precision for the determination of the decay form factors is much more promising: $\Delta[\mathrm{Re}\,(f_2^{\mathrm{R}} - \bar{f}_2^{\mathrm{L}})] = 0.014$. However, it has been found in Ref. [12] that the maximal expected[8] size of $\mathrm{Re}\,(f_2^{\mathrm{R}} - \bar{f}_2^{\mathrm{L}})$ is 5×10^{-5} (for $m_h > 10\,\mathrm{GeV}$), therefore either an unrealistic growth of the luminosity, or other observables (besides the energy and angular double distribution of final leptons) are required in order to observe CP-violating form factors in the top-quark decay process. The results of Ref. [26] assumed simultaneous[9] determination of *all 9 form factors*, therefore another chance to reduce of $\Delta[\mathrm{Re}\,(f_2^{\mathrm{R}} - \bar{f}_2^{\mathrm{L}})]$ is to have some extra independent constraints on the top-quark coupling coming from other colliders, like the Fermilab Tevatron or LHC.

Looking for CP violation one can directly measure in a model independent way [26] all the form factors including those which are odd under CP. However, another possible attitude is to construct certain asymmetries sensitive to CP violation. In this section we will discuss several asymmetries that could probe CP violation in the processes $e^+e^- \to t\bar{t} \to l^\pm \ldots$ and $e^+e^- \to t\bar{t} \to \overset{(-)}{b} \ldots$. We will systematically drop all contributions quadratic in non-standard form factors and calculate various asymmetries keeping only interference between the SM and D_γ, D_Z or $\mathrm{Re}\,(f_2^{\mathrm{R}} - \bar{f}_2^{\mathrm{L}})$.

4.1. Integrated lepton-energy asymmetry

Let us introduce the rescaled lepton energy, x, by

$$x \equiv \frac{2E_l}{m_t}\left(\frac{1-\beta_t}{1+\beta_t}\right)^{1/2}, \tag{14}$$

where E_l is the energy of l in e^+e^- c.m. frame and $\beta_t \equiv \sqrt{1 - 4m_t^2/s}$.

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[8] It turns out (see Ref. [12] for details) that $\mathrm{Re}\,(f_2^{\mathrm{R}} - \bar{f}_2^{\mathrm{L}})$ is by 2–4 orders of magnitude below $\mathrm{Re}\,D_\gamma$ or $\mathrm{Re}\,D_Z$ even for large b-quark Yukawa coupling ($\tan\beta = 50$). The suppression is caused both by the experimental limit on $|C_i|$ (for $m_h < 105$ GeV) and by an extra suppression factor of $(m_b/m_t)^2$ (relative to $\mathrm{Re}\,D_{\gamma,Z}$).

[9] Obviously, that leads to reduced precision for the determination of the form factors.

CP symmetry could be tested using the following leptonic double energy distribution [27]:

$$\frac{1}{\sigma}\frac{d^2\sigma}{dx\ d\bar{x}} = \sum_{i=1}^{3} c_i f_i(x,\bar{x})\,, \tag{15}$$

where x and $\bar{x}$ are for l^+ and l^-, respectively, and

$$c_1 = 1\,, \quad c_2 = \xi\,, \quad c_3 = \tfrac{1}{2}\mathrm{Re}\,(f_2^{\mathrm{R}} - \bar{f}_2^{\mathrm{L}})$$

for

$$\begin{aligned}\xi \equiv{}& \frac{1}{(3-\beta^2)D_V + 2\beta^2 D_A}\\ &\times\frac{-1}{\sin\theta_W}\mathrm{Re}\left[\frac{2}{3}\,D_\gamma + \frac{s^2}{(s-m_Z^2)^2}\frac{(v_e^2+a_e^2)v_t}{64\sin^3\theta_W\cos^3\theta_W}D_Z\right.\\ &\left.-\frac{s}{s-m_Z^2}\left(\frac{v_e v_t}{16\sin^2\theta_W\cos^2\theta_W}D_\gamma + \frac{v_e}{6\sin\theta_W\cos\theta_W}D_Z\right)\right],\end{aligned}$$

for

$$\begin{aligned}D_V &= \left(v_e v_t d - \tfrac{2}{3}\right)^2 + (a_e v_t d)^2,\\ D_A &= (v_e a_t d)^2 + (a_e a_t d)^2\,,\end{aligned}$$

with the SM neutral-current parameters of e: $v_e = -1 + 4\sin^2\theta_W$, $a_e = -1$ and a Z-propagator factor

$$d \equiv \frac{s}{s-m_Z^2}\frac{1}{16\sin^2\theta_W\cos^2\theta_W}\,.$$

The definitions of the functions $f_i(x,\bar{x})$ are to be found in Ref. [28].

The coefficients c_2 and c_3 measure the degree of CP violation in the $t\bar{t}$ production and their subsequent decays, respectively. The following asymmetry could be defined [12] to extract $\mathrm{Re}\,D_\gamma$, $\mathrm{Re}\,D_Z$ and $\mathrm{Re}\,\left(f_2^{\mathrm{R}} - \bar{f}_2^{\mathrm{L}}\right)$ form the double energy distribution:

$$A_{CP}^{ll} \equiv \frac{\displaystyle\int\!\!\int_{x<\bar{x}} dx d\bar{x}\frac{d^2\sigma}{dx d\bar{x}} - \int\!\!\int_{x>\bar{x}} dx d\bar{x}\frac{d^2\sigma}{dx d\bar{x}}}{\displaystyle\int\!\!\int_{x<\bar{x}} dx d\bar{x}\frac{d^2\sigma}{dx d\bar{x}} + \int\!\!\int_{x>\bar{x}} dx d\bar{x}\frac{d^2\sigma}{dx d\bar{x}}}\,. \tag{16}$$

In order to estimate a relative strength of various sources[10] of CP violation it is worth to decompose the asymmetry as follows:

$$A^{ll}_{CP} = g^{ll}_{\gamma t\bar{t}}\,\mathrm{Re}\,D_\gamma + g^{ll}_{Zt\bar{t}}\,\mathrm{Re}\,D_Z + g^{ll}_{Wtb}\,\mathrm{Re}\,\left(f_2^{\mathrm{R}} - \bar{f}_2^{\mathrm{L}}\right). \tag{17}$$

In Table I we show the coefficients g for various c.m. energies. Firstly, it is clear that for any given $\sqrt{s}$ the coefficient $g^{ll}_{Zt\bar{t}}$ is the smallest one. Secondly, it is seen that just above the threshold for $t\bar{t}$ production there is an enhancement of relative contributions from the decay, however, still not sufficient to overcome the suppression of $\mathrm{Re}\,(f_2^{\mathrm{R}} - \bar{f}_2^{\mathrm{L}})$. Therefore, we can conclude that the leading contribution is provided by CP violation in the $\gamma t\bar{t}$ vertex.

TABLE I

The energy dependence of the coefficients g defined in Eq. (17).

$\sqrt{s}$[GeV]	$g^{ll}_{\gamma t\bar{t}}$	$g^{ll}_{Zt\bar{t}}$	g^{ll}_{Wtb}
360	0.0509	0.00954	0.410
500	0.386	0.0684	0.291
1000	0.602	0.102	0.235

Fig. 2 illustrates the Higgs-boson-mass dependence of the leading (proportional to $R_{i2}R_{i3}$) contribution to the integrated lepton-energy asymmetry. It turns out that $\sqrt{s} = 500\,\mathrm{GeV}$ provides the largest asymmetry. Using results of Ref. [27] one can find out an expected statistical error for the determination of A^{ll}_{CP} at any given e^+e^- collider. Assuming $\sqrt{s} = 500\,\mathrm{GeV}$, $L = 500\ \mathrm{fb}^{-1}\mathrm{y}^{-1}$ and lepton tagging efficiency, $\varepsilon_l = 60\%$ we get $\Delta A^{ll}_{CP} = 0.014$. As it is seen from Fig. 2 an observation of the asymmetry would require several years of running at the assumed luminosity.

[10] It should be noticed that the general formulae (see Refs. [26–29]) for the asymmetries considered here have been obtained assuming $m_b = 0$. As it is seen from Eq. (13), the contribution to CP violation in the decay process, $\mathrm{Re}\,\left(f_2^{\mathrm{R}} - \bar{f}_2^{\mathrm{L}}\right)$, turns out to be proportional to m_b^2. Therefore, strictly speaking, CP violation in the decay process should either be disregarded or all the CP-violating contributions of the order of m_b^2 should be calculated. The latter effects are definitely negligible in the 2HDM comparing to contributions from the production process. However, we have found it useful for future applications within other possible models [30] to preserve hereafter contributions from $\mathrm{Re}\,\left(f_2^{\mathrm{R}} - \bar{f}_2^{\mathrm{L}}\right)$ in formulae and corresponding figures for all the asymmetries considered in this study.

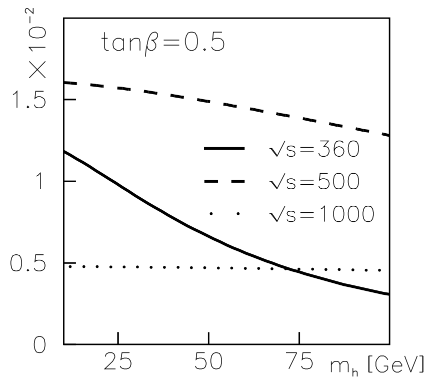

Fig. 2. Higgs-boson-mass dependence of the coefficient of $R_{i2}R_{i3}$ for the asymmetry given by Eq. (16) for $\sqrt{s}$=360 (solid), 500 (dashed), 1000 GeV (dotted) for $\tan\beta = 0.5$.

4.2. Integrated angular asymmetry

Another CP-violating asymmetry could be constructed using the angular distributions of the bottom quarks or leptons originating from the top-quark decay:

$$\frac{d\sigma}{d\cos\theta_f} = \frac{3\pi\beta\alpha_{\rm EM}^2}{2s} B_f \left(\Omega_0^f + \Omega_1^f \cos\theta_f + \Omega_2^f \cos^2\theta_f \right) , \tag{18}$$

where $f = b, l$, B_f is an appropriate top-quark branching ratio, θ_f is the angle between the e^- beam direction and the direction of f momentum in the e^+e^- c.m. frame and Ω_i^f are coefficients calculable in terms of the form factors, see Ref. [29]. The following asymmetry provides a signal of CP violation:

$$\mathcal{A}_{CP}^f(P_{e^-}, P_{e^+}) = \frac{\displaystyle\int_{-c_m}^{0} d\cos\theta_f \frac{d\sigma^{+(*)}(\theta_f)}{d\cos\theta_f} - \int_0^{+c_m} d\cos\theta_f \frac{d\sigma^{-(*)}(\theta_f)}{d\cos\theta_f}}{\displaystyle\int_{-c_m}^{0} d\cos\theta_f \frac{d\sigma^{+(*)}(\theta_f)}{d\cos\theta_f} + \int_0^{+c_m} d\cos\theta_f \frac{d\sigma^{-(*)}(\theta_f)}{d\cos\theta_f}} , \tag{19}$$

where P_{e^-} and P_{e^+} are the polarizations of e and $\bar{e}$ beams, $d\sigma^{+/-(*)}$ is referring to f and $\bar{f}$ distributions, respectively, and c_m expresses the experimental polar-angle cut. In order to discuss possible advantages of polarized initial beams we are considering here the dependence of the asymmetry on the polarization. Hereafter we will discuss the same polarization for e and $\bar{e}$: $P \equiv P_{e^-} = P_{e^+}$.

Again we decompose the asymmetry as follows:

$$\mathcal{A}_{CP}^f(P) = g_{\gamma t\bar{t}}^f(P) \,\mathrm{Re}\, D_\gamma + g_{Zt\bar{t}}^f(P) \,\mathrm{Re}\, D_Z + g_{Wtb}^f(P) \,\mathrm{Re}\, \left(f_2^{\rm R} - \bar{f}_2^{\rm L} \right) . \tag{20}$$

In Table II we show the coefficient functions g calculated for various energy and polarization choices assuming the polar angle cut $|\cos\theta_f| < 0.9$, *i.e.* $c_m = 0.9$ in Eq. (19), for both leptons and bottom quarks[11]. It could be seen that a positive polarization leads to higher coefficients $g^f_{\gamma t\bar{t}}$ and $g^f_{Zt\bar{t}}$. Since $\mathrm{Re}\,(D_\gamma) > \mathrm{Re}\,(D_Z) \gg \mathrm{Re}\,(f_2^{\mathrm{R}} - \bar{f}_2^{\mathrm{L}})$ that implies that maximal asymmetry could be reached for $P = +0.8$ and the dominant contribution is originating from $\mathrm{Re}\,(D_\gamma)$. Since the number of events does not drop drastically when going from unpolarized beams to $P = +0.8$, it turns out that the positive polarization is the most suitable for testing the integrated angular asymmetry. It is clear from the table that the asymmetry for final leptons should be larger by a factor $3 \div 4$ than the one for bottom quarks and their signs should be reversed.

TABLE II

The energy and polarization dependence of the coefficients $g^f_{\gamma t\bar{t}}(P)$, $g^f_{Zt\bar{t}}(P)$ and $g^f_{Wtb}(P)$ defined in Eq. (20) for leptons ($f = l$) and bottom quarks ($f = b$).

$\sqrt{s}$[GeV]	P	quark b			lepton	
		$g^b_{\gamma t\bar{t}}(P)$	$g^b_{Zt\bar{t}}(P)$	$g^b_{Wtb}(P)$	$g^l_{\gamma t\bar{t}}(P)$	$g^l_{Zt\bar{t}}(P)$
360	0.0	0.00844	0.00106	0.142	–0.0162	–0.00203
	0.8	0.00983	–0.00555	–0.259	–0.0493	0.0278
	–0.8	0.00758	0.00510	0.388	–0.0106	–0.00713
500	0.0	0.113	0.0136	0.121	–0.224	–0.0270
	0.8	0.131	–0.0718	–0.247	–0.627	0.343
	–0.8	0.101	0.0661	0.347	–0.149	–0.0968
1000	0.0	0.332	0.0389	0.0678	–0.722	–0.0845
	0.8	0.422	–0.225	–0.167	–1.55	0.824
	–0.8	0.284	0.181	0.194	–0.507	–0.322

Using the general formula for the asymmetry from Ref. [26] and adopting results for the CP-violating form factors we plot $\mathcal{A}^f_{CP}(P_{e^-}, P_{e^+})$ in Fig. 3 as a function of the Higgs-boson-mass both for bottom quarks and leptons. It is clear that the largest asymmetry could be expected for $P_{e^-} = P_{e^+} = +0.8$ for final leptons at $\sqrt{s} = 500\,\mathrm{GeV}$. With the maximal mixing, $R_{i2}R_{i3} = 1/2$ the 1% asymmetry could be expected for the Higgs boson with mass $m_h = 10 \div 20\,\mathrm{GeV}$. Since the statistical error expected [26] for the asymmetry is of

[11] Note that in Table II there is no column corresponding to the coefficient of $\mathrm{Re}\,(f_2^{\mathrm{R}} - \bar{f}_2^{\mathrm{L}})$. That happens since the angular distribution for leptons is not influenced by corrections to the top-quark decay vertex, see Refs. [29, 31] and [26].

the order of 5×10^{-3}, we can conclude that the asymmetry $\mathcal{A}^f_{CP}(P_{e^-}, P_{e^+})$ is the most promising one, leading to 2σ effect for light Higgs boson and $\tan\beta = 0.5$. As it is seen from Fig. 3 it is relevant to have polarized e^+e^- beams.

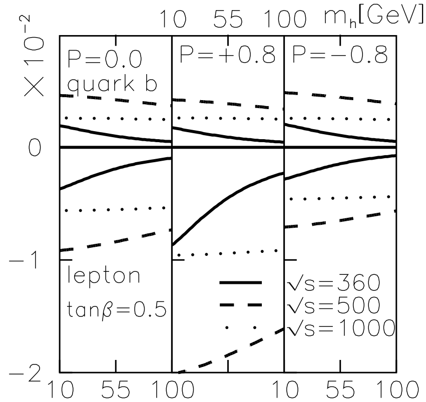

Fig. 3. The Higgs-boson-mass dependence of the coefficient of $R_{i2}R_{i3}$ for the angular asymmetry defined by Eq. (19) for bottom quarks (upper) and leptons (lower) at $\sqrt{s}$=360 (solid), 500 (dashed), 1000 GeV (dotted) with unpolarized beams (left), $P = +0.8$ (middle) and $P = -0.8$ (right) for $\tan\beta = 0.5$.

5. Summary

We have considered a general two-Higgs-doublet model with CP violation in the scalar sector. Mixing of the three neutral Higgs fields of the model leads to CP-violating Yukawa couplings of the physical Higgs bosons. CP-asymmetric form factors generated at the one-loop level of perturbation theory has been calculated within the model. Although in general the existing experimental data from LEP1 and LEP2 constraint the mixing angles of the three neutral Higgs fields, their combination relevant for CP violation is not bounded for small $\tan\beta$ which is the region of our interest. We have shown that the decay form factors are typically smaller then the production ones by 2-3 orders of magnitude. The dominant contribution to CP violation in the production is coming from $\gamma t\bar{t}$ coupling. Several energy and angular CP-violating asymmetries for the process $e^+e^- \to t\bar{t} \to l^\pm \ldots$ and $e^+e^- \to t\bar{t} \to \overset{(-)}{b} \ldots$ has been considered using the form factors calculated within the two-Higgs-doublet model. It turned out that the best test of CP invariance would be provided by the integrated angular asymmetry $\mathcal{A}^f_{CP}(P_{e^-}, P_{e^+})$ for positive polarizations of e^+e^- beams. For one year of running at TESLA collider with the integrated luminosity $L = 500\ \mathrm{fb}^{-1}\mathrm{y}^{-1}$ one could expect 2σ effect for the asymmetry for light Higgs boson and $\tan\beta = 0.5$.

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