

SIZE OF PENGUIN POLLUTION OF
THE CKM CP VIOLATING PHASE IN $\bar{B}_s \rightarrow \rho K_S^*$

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We use the perturbative QCD methods of Lepage and Brodsky to calculate the rate for $\bar{B}_s \rightarrow \rho K_S$, with an eye toward the CP violating unitarity triangle angle γ . We show that, although the penguins are large, there are regions of the allowed parameter space of the Cabibbo–Kobayashi–Maskawa (CKM) mixing matrix wherein γ is measurable in the sense that penguins change of the value of $\sin(2\gamma)$ one would extract from the attendant time dependent asymmetry measurement by less than 29%, so that a 3σ measurement of $\sin(2\gamma)$ as being different from 0 is allowed by the corresponding theoretical uncertainty. This would establish CP violation in B_s decays. The rates which we find tend to favour the type of luminosities now envisioned for hadron-based B-factories.

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1. Introduction

Now that there are two asymmetric e^+e^- colliding beam B-factories, the SLAC-LBL-LLNL and KEK asymmetric B-factories, as well as several other B-factory type machines, such as HERA-B, the CESR upgrade, and the Tevatron upgrade, for example, under construction, the systematic exploitation of these machines for CP violation studies is not far away. To realize the true potential of these studies, it is important that the complete set of Standard Model CP violation parameters for the B -system be explored, if it is at all possible. In particular, this means that all CP violating angles α , β and γ of the unitarity triangle should be measured, where we use the notation of [1] for these angles. The angle β is the “gold plated”

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angle of the triangle, as it will be presumably the most readily measurable of the three angles, via the modes $B \rightarrow \Psi/JK_S, \Psi/JK_+^*$. It (β) is in fact used to specify the minimal requirements for the B-factory machine and detector system to be successful. (Here, K_+^* denotes the CP + neutral K^* meson.) Accordingly, the B decay modes needed for measurement of the angles α and γ must also be identified and assessed. In this connection, the mode $\bar{B}_s^0 \rightarrow \rho + K_S$ is worthy of some attention; for, were it not for the possible contamination from penguins, this mode would be a candidate mode for the measurement of γ [1]. Indeed, the potential contamination from penguins is just as substantial as it is for the mode $\bar{B} \rightarrow \pi^0\pi^0$ in connection with the measurement of α , for which the authors [2] have devised isospin methods to combine the measurements of the modes $B \rightarrow \pi^+\pi^-, \pi^0\pi^0$ and $B^+ \rightarrow \pi^+\pi^0$ to extract α independent of the size of the penguin contamination — the main experimental problem of course is the measurement of the $\pi^0\pi^0$ mode. It is desirable to address these penguin CP violation pollution effects from a dynamical approach which aims to quantify them directly, thereby isolating just where a measurement may still be made, in view of the available parameter space in the respective CKM mixing matrix. Indeed, in a recent paper [3], we analysed the theoretical expectations for the size of these penguins in the basic mode $\bar{B} \rightarrow \pi^+\pi^-$ as well as in the companion mode $\bar{B} \rightarrow \pi^0\pi^0$. We have found that, in a large region of the parameter space, the asymmetric B-factory devices at SLAC and KEK will be able to extract the fundamental CP violating angle α without depending on the penguin trapping methods in Ref. [2]. The natural question to ask is whether an analogous region exists in the case of the measurement of the angle γ in the $B_s \rightarrow \rho K_S$ decay? It is this question that we address in the following theoretical development.

Specifically, we will use the approach of Lepage and Brodsky [4], as it is represented in our analysis of $D \rightarrow \pi^+\pi^-, K^+K^-$ in Ref. [5]. In this realization of perturbative QCD for hard exclusive processes, as we shall illustrate explicitly below, the exclusive amplitude is represented as a convolution of a hard scattering kernel (referred to as T_H in Ref. [4]) with distribution amplitudes that sum the respective large QCD collinear logarithms associated with radiation from the external legs of the constituent partons. These distribution amplitudes, therefore, obey a rigorous QCD evolution equation derived from QCD perturbation theory in Ref. [4]. We refer to this representation of hard exclusive hadron processes as the Lepage–Brodsky method. It was already formulated in Ref. [6] in the context of the exclusive two-body B decays to light mesons of the type of interest to us here. See also Refs. [7,8] for further illustrations of the method we shall use. As we explain in Ref. [3], we expect the accuracy of our methods as used here to be at least as accurate as the 25% accuracy determined in the work in Ref. [5]. We present both the

absolute decay rates and the ratio of branching ratios corresponding to such rates, with and without the penguins included in the respective calculations. In this way, we expect to minimise the sensitivity of our results to the uncertainty of the normalisation of the distribution amplitudes which we do use. Indeed, in the respective CP asymmetry parameter $\sin(2\gamma)$ analysis, we compute its apparent shift away from its expected value in the absence of penguins in ratio to that expected value, $\Delta \sin(2\gamma)/\sin(2\gamma)$. We refer to this shift as the penguin shift of $\sin(2\gamma)$. The analog of this shift plus unity was already introduced in Ref. [9] in the study of the time-dependent CP violating asymmetry in the $\pi^+\pi^-$ decay mode. We will exhibit a formula for the penguin shift of $\sin(2\gamma)$ here for definiteness in complete analogy with what we have already published in Ref. [3] for the corresponding shift of the analogous CP violating asymmetry parameter $\sin(2\alpha)$ for the $\pi^+\pi^-$ decay mode. Evidently, the normalisation of our distribution amplitudes also drops out of the penguin shift of $\sin(2\gamma)$.

Concerning the CKM matrix itself, we follow the conventions of Gilman and Kleinknecht in Ref. [10] for the CP violating phase $\delta_{13} \equiv \delta$ and in view of the current limits on it we consider the entire range $0 \leq \delta \leq 2\pi$. For the CKM matrix parameters V_{td} and V_{ub} we also consider their extremal values from [10] (the Particle Data Group (PDG) compilation). To parametrise these extremes, we use the notation defined in Ref. [11] for $|V_{ub}/V_{cb}|$ in terms of the parameter $R_b = 0.385 \pm 0.166$ [10]. All other CKM matrix element parameters are taken at their central values [10].

We should emphasise that the decay under study here is not the only way to study the CP violating angle γ . Indeed, due to the very small rates which we shall find, it will be seen that the most appropriate machine to pursue the mode under discussion here is a hadron collider type B-factory device. As shown in Refs. [1, 11, 12], the e^+e^- colliding beam type B-factory device can approach γ from other decay mode avenues.

We further emphasise that it is possible to use the methods of Lepage and Brodsky [4], as they are represented in the analyses in Refs. [3, 5], to address both the concept of colour suppression for the $\bar{B}_s \rightarrow \rho K_S$ decay as well as the size of the penguin pollution in its CP violating phase structure as described above. We will take advantage of this opportunity to get a quantitative estimate of the colour suppression effect in this decay under study here. In practice, what this will mean is that, in addition to computing our Branching Ratio (BR) for the decay with and without penguins included, we will also compute it with and without gluon exchange between the would-be spectator \bar{s} and the $q\bar{q}$ lines of the outgoing ρ . Again, we will focus on the respective ratios of BR's to avoid sensitivity to the uncertainty in the normalisation of our distribution amplitudes. Such an estimate of colour suppression has not appeared elsewhere.

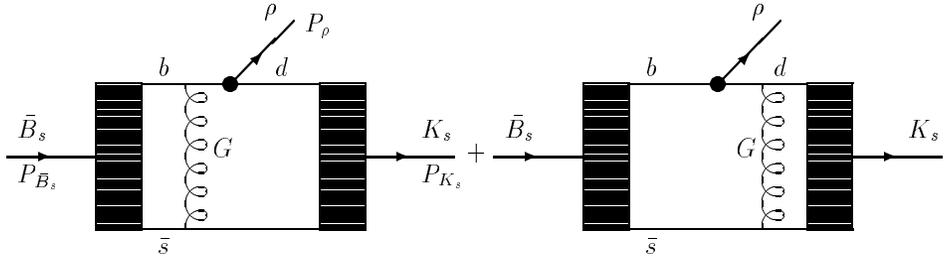


Fig. 1. The process $\bar{B}_s \rightarrow \rho + K_S$. The four-momenta are indicated in the standard manner: P_A is the four-momentum of A for all A . To leading order in the perturbative QCD expansion defined by Lepage and Brodsky in [4], the two graphs shown are the only ones that contribute in the dominant contribution as isolated by the methods of [6] when penguins and colour exchange between the outgoing ρ partons and the outgoing K_S partons are ignored. The remaining graphs in which the gluon G is exchanged between the would-be spectator \bar{s} and the remaining ρ parton lines as well as the penguin type graphs are shown in Figs. 2 and 3, where we see that, for QCD penguins, there is the added possibility that the gluon G interacts with the penguin gluon itself, of course.

Specifically, we note that the QCD corrections to the weak interaction Lagrangian will be represented via the QCD corrected effective weak interaction Hamiltonian \mathcal{H}_{eff} as it is defined in Ref. [11]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[\sum_{j=u,c} V_{jq}^* V_{jb} \left\{ \sum_{k=1}^2 Q_k^{jq} \tilde{C}_k(\mu) + \sum_{k=3}^{10} Q_k^q \tilde{C}_k(\mu) \right\} \right] + \text{h.c.}, \quad (1)$$

where the Wilson coefficients \tilde{C}_i and operators Q_k are as given in Ref. [11], G_F is Fermi's constant, μ is the renormalization scale and is of $\mathcal{O}(m_b)$ and here $q = s$. The application of this effective weak interaction Hamiltonian to our process $\bar{B}_s \rightarrow \rho K_S$ then proceeds according to the realization of the Lepage–Brodsky expansion as described in [6]. This leads to the “dominant” contribution in which the ρ is interpolated into the operator $O_2 = Q_1$ in \mathcal{H}_{eff} via the factorised current matrix element $\langle \rho | \bar{u}(0) \gamma_\mu P_L u(0) | 0 \rangle$, $P_L \equiv (1 - \gamma_5)/2$ so that the respective remaining current in $O_2 = Q_1$ is responsible for the \bar{B}_s to K_S transition shown in Fig. 1. We refer to this contribution as the “Tree” contribution. The complete amplitude for the process under study here, $\bar{B}_s \rightarrow \rho K_S$, is given by the sum of the graphs in Fig. 1 and those in Figs. 2 and 3, to leading order in the Lepage–Brodsky expansion defined in [4] and realized according to the prescription in [6].

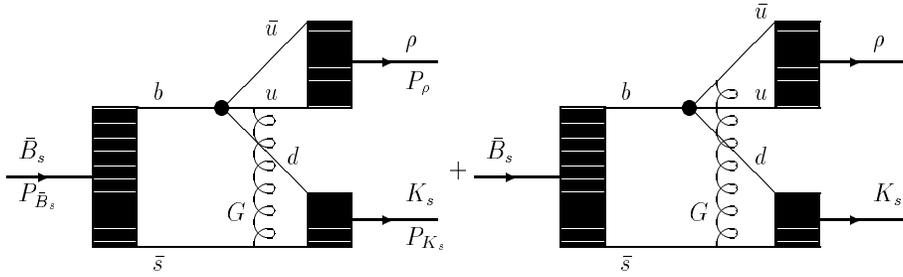


Fig. 2. The colour exchange graphs for the process $\bar{B}_s \rightarrow \rho + K_s$ to leading order in the Lepage–Brodsky expansion in [4, 6], ignoring penguins. The kinematics is as defined in Fig. 1.

In Fig. 2, we show the graphs in which colour is exchanged between the would-be spectator \bar{s} in Fig. 1 and the outgoing ρ parton lines and in Fig. 3 we show the respective penguin graphs: the dominant graphs, according to the prescription in Ref. [6], 3(a), (b), the colour exchange graphs 3(c), (d), and the exchange of the hard gluon G between the would-be spectator \bar{s} and the penguin gluon itself for QCD penguins, 3(e), which we also will classify as colour exchange. To address the issue of factorisation/colour-suppression, we shall present results when graphs in Figs. 2 and 3(c)–(e) are dropped and when they are included. We thus give results for the approximations in

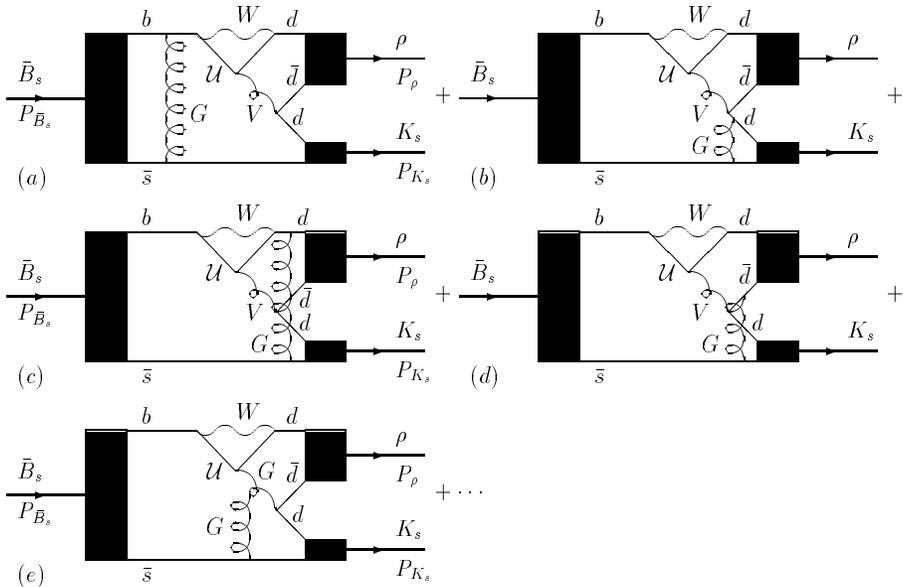


Fig. 3. The penguin graphs for the process $\bar{B}_s \rightarrow \rho + K_s$, to leading order in the Lepage–Brodsky expansion defined in [4, 6]. The kinematics is as defined in Fig. 1.

which only the graphs in Fig. 1 are included (“Tree”), in which the graphs in Figs. 1, 3(a) and (b) are included (“Tree+Penguin”), in which the graphs in Figs. 1, 2, 3(a) and (b) are included (“Tree+Penguin+Tree Colour Exchange (CE_T)”), and in which all graphs in Figs. 1, 2, 3(a)–(e) are included (“Tree+Penguin+Tree and Penguin Colour Exchange (CE_{T+P})”). For the Electro Weak (EW) penguins, there is no penguin gluon with which the would-be spectator \bar{s} could interact. We need to stress that, as shown in Ref. [6], the usual QCD factorisation properties for exclusive amplitudes at large momentum transfer are sufficient to justify the formulation of our amplitude according to the graphs in Figs. 1–3. More phenomenological arguments, such as the current field identity based BSW model in [13], *etc.*, which would lead to the same graphs, are not needed.

Some discussion of the effective values of the coefficients $C_1 = \tilde{C}_2$, $C_2 = \tilde{C}_1$ in relation to the coefficients a_1 and a_2 as defined in Ref. [13] is now appropriate. Following Ref. [13] and the recent results in Ref. [14], when we use the standard QCD to calculate the diagrams in Fig. 1 and take them alone as our result (this is our definition of factorisation), we use $a_2 \cong 0.24 \cong |C_2(m_b)|$ and when we assess the colour-suppression effect by including the exchange of G between the \bar{s} and the $q\bar{q}$ of the ρ we set $C_1(m_b) \cong 1.1$; these results are consistent with those found in Ref. [14]. We note that the naive relation $a_2 \cong C_2 + C_1/3 \cong 0.127$ would give a value for a_2 that is about a factor of two smaller than what is found in Refs. [13,14] and the references therein. The parameters a_1, a_2 are, therefore, purely phenomenological properties of the hard effective weak interaction process and can be taken from experiment in our analysis: one may view a_2 , for example, as the effective value of $C_2 + C_1/3$ when the current field identity is used to interpolate the ρ into our effective weak interaction vertex. The Lepage–Brodsky formalism then allows us to calculate the recoil corrections associated with the momentum transfer required for the would-be spectator to be kicked from the B_s to the final outgoing K_S using perturbative QCD to describe the respective hard gluon exchange, as we noted above. This “kick” is the defining aspect of our calculation of $\bar{B}_s \rightarrow \rho K_S$ in comparison to those in Ref. [15] and in fact in comparison to the related two body B decay analyses in Ref. [16]. The point is the following. As one can see from the results in Refs. [3,5,8], contrary to what happens in the tree level part of the calculations in Refs. [15,16], the graph in Fig. 1 in which the hard gluon kick to the spectator comes from the b -quark line (the heavy quark line) develops an imaginary part that is treated rigorously in our work so that there is a non-trivial strong phase for our “tree level” contribution compared to those in Refs. [15,16]. This happens because, as $m_B > m_b + m_s$ where m_q are evaluated at the

scale $\sim m_B$, the heavy quark line can reach its perturbative QCD mass shell in the graph in Fig. 1, and in the similar graphs in Figs. 2 and 3. Evidently, this effect is missing in the results in Refs. [15, 16]. Any serious discussion of the CP asymmetries in the amplitudes for exclusive two-body B decays must take this strong phase into account in general (it is different for “Tree+Penguin” contributions for example) as one can see from our formula for time-dependent asymmetry in $\bar{B}_s \rightarrow \rho K_S$ below. Our paper is the first paper to do this systematically.

Here, we should also comment on the recent results of Ref. [17] on the process $B \rightarrow \pi\pi$. The authors in Ref. [17] use the same Feynman diagrams, analogous to those in Figs. 1–3 here, as we have shown already in Ref. [3] and same Lepage–Brodsky expansion formalism except that they assume the graphs analogous to those in Fig. 1 are to be replaced by a real form factor with the appropriate external wave function/decay constant factors. The usual corrections to the diagrams in Fig. 1 are then represented as a power series in α_s times this assumed real form factor. We do not make such an assumption; we calculate systematically in the Lepage–Brodsky expansion. A major difference is that the authors in Ref. [17] miss the recoil phase of the dominant contribution in the analog of Fig. 1 for the $B \rightarrow \pi\pi$ process, although they do calculate the recoil phase in the respective analogs of Figs. 2 and 3. To see what effect this has, we note that, from our Eq. (5) in Ref. [3], we get the direct CP violation result [18] for the $\bar{B} \rightarrow \pi\pi$ process as

$$-0.0086 < \mathcal{A}_{CP}^{\text{dir}} < 0, \quad \text{for } \gamma \in (0, \pi), \quad (2)$$

$$-0.0086 < \mathcal{A}_{CP}^{\text{dir}} < -0.0050, \quad \text{for } \frac{3\pi}{4} \geq \gamma \geq \frac{\pi}{4}, \quad (3)$$

with $\mathcal{A}_{CP}^{\text{dir}}$ monotone decreasing in the second currently preferred [1] region of γ for $\pi/4 \leq \gamma \leq 1.806$ and monotone increasing for $1.806 \leq \gamma \leq 3\pi/4$, whereas in Ref. [17] this asymmetry is predicted to be $-4\% \sin \gamma$. Evidently, experiment will soon be able to distinguish between the two approaches. See Ref. [19] for further discussion of this and related matters.

In this way, using the methods of Ref. [4] we evaluate the graphs illustrated in Fig. 1–3 and arrive at the results in Table I and in Fig. 4 (the explicit expressions for the respective amplitudes may be inferred from those for the process $\bar{B} \rightarrow \pi\pi$ given in Eq. (1) and in Eqs. (A1)–(A4) in Ref. [3], via the appropriate substitutions of momenta and distribution amplitudes; for example, for the factor F_N in (A1) in Ref. [3] we would now have its form obtained by the substitutions:

$$\begin{aligned}
a_1 &\rightarrow a_2 \\
\sqrt{2}f_\pi P_{\pi-\alpha} &\rightarrow f_\rho m_\rho \varepsilon(P_\rho)_\alpha \\
\frac{\sqrt{3}f_\pi}{2}y_1y_2 &\rightarrow \frac{\sqrt{3}f_K}{2\sqrt{2}}y_1y_2(1+3\beta'_K(y_2-y_1)) \\
P_{\pi^+} &\rightarrow P_{K_S} \\
m_u &\rightarrow m_d \\
m_\pi &\rightarrow m_{K_S} \\
4x_2^2y_2E_{\pi^+}^2+m_B^2 &\rightarrow q^2((P_d+q)^2-m_d^2+i\varepsilon) \\
(-2x_2y_2E_{\pi^+}+m_B)(y_1m_B^2-m_b^2+i\varepsilon) &\rightarrow q^2((P_b-q)^2-m_b^2+i\varepsilon), \quad (4)
\end{aligned}$$

wherein $q = P'_s - P_s$, P_f , $f = b$, \bar{s} is 4-momentum of f in the \bar{B}_s in Fig. 1 and P_d, P'_s are the 4-momentum of d, \bar{s} , respectively, in the K_S in Fig. 1, so that we have $P_s \cong x_2 P_{\bar{B}_s}$ and $P'_s \cong y_2 P_{K_S}$, for example, and $\beta'_K \cong 0.418$ is the asymmetry parameter in the Lepage–Brodsy distribution amplitude for the K_S as determined in Ref. [20] and evolved to the scale m_B . We use $f_K \cong 0.112$. In this regard, we further note that the Lepage–Brodsy distribution amplitude for the ρ in the analog of Eq. (A4) in Ref. [3] for the process under study here would substitute $\sqrt{3}f_\rho m_\rho \not{\varepsilon}(P_\rho)z_1z_2(1.348 - 1.74z_1 + 1.74z_1^2)$ for $\sqrt{3}f_\pi z_1z_2\gamma_5(P'_{\pi^0} + m_\pi)$ for example by the standard methods, where we use the Chernyak–Zhitnitsky type result [21] for the ρ distribution amplitude in analogy with our discussion in the Notes Added in Ref. [3]. Here, $\varepsilon(P_\rho)$ and f_ρ are the respective ρ polarisation 4-vector and decay constant with $f_\rho \cong 0.14$ GeV. The \bar{B}_s distribution amplitude is taken in complete analogy with the \bar{B}_d in Ref. [3], so that it is given by:

$$\frac{a_B \phi_B(w_1, w_2)}{\sqrt{2N_c}} = \frac{a_B \delta(w_2 - x_2)}{\sqrt{2N_c}},$$

where $N_c = 3$ is the number of colours, $a_B = f_{B_s}/\sqrt{4N_c}$ and $x_2 \cong 0.0542$ is determined, as we present in our Appendix, following the treatment of heavy mesons in Ref. [4] using potential model parameters such as those in Ref. [22]. Finally, note that the quark masses m_q are the running current quark masses [23]). For completeness, the complete result for the amplitude corresponding to the graphs shown in Figs. 1–3 is given in the Appendix. Moreover, the precise definition of the penguin shift $\Delta \sin(2\gamma)$ is given by the following generalisation to our process $\bar{B}_s \rightarrow \rho K_S$ of the formula of Gronau in Ref. [9] for the corresponding shift of $\sin(2\alpha)$ due to penguins in the $\bar{B}_d \rightarrow \pi\pi$ process:

$$-\sin(2\gamma) - \Delta(\sin(2\gamma)) \equiv \frac{\text{Im } \lambda}{\frac{1}{2}(1 + |\lambda|^2)}, \quad (5)$$

for

$$\lambda = \frac{A_T e^{-i\phi_T + i\delta_T} + \sum_j A_{P_j} e^{-i\phi_{P_j} + i\delta_{P_j}}}{A_T e^{+i\phi_T + i\delta_T} + \sum_j A_{P_j} e^{+i\phi_{P_j} + i\delta_{P_j}}}, \quad (6)$$

where the amplitude $A_T e^{-i\phi_T + i\delta_T}$ corresponds to the tree-level weak processes in Figs. 1 and 2 and the amplitudes $A_{P_j} e^{-i\phi_{P_j} + i\delta_{P_j}}$ correspond to the respective penguin processes in Fig. 3. Here, we identify the weak phases of the respective amplitudes as ϕ_r , $r = T, P_j$ and the attendant strong phases as δ_r , $r = T, P_j$. In general, $j = 1, 2$ distinguishes the electric and magnetic penguins when this is required, as one can see in our Appendix. In this notation, we have $\gamma \equiv \phi_T$. From the results in Table I, and their ratios with one another, we see that the colour suppression idea does not really hold for this decay. We see, as already anticipated by several authors [1], that the penguins are indeed important. There is a regime, $0^\circ \leq \gamma \lesssim 40.5^\circ$, $102.5^\circ \lesssim \gamma \lesssim 157.9^\circ$, for the central value of R_b for example, wherein the shift of $\sin(2\gamma)$ is less than 29% of its magnitude so that it would be measur-

TABLE I

BR for $\bar{B}_s \rightarrow \rho K_S$ as a function of R_b as defined in the text. The factorised approximation without penguin effects is denoted as ‘‘Tree’’; the corresponding results with the penguin effects (both EW and QCD penguins) included are denoted by ‘‘Tree+Penguin’’; the results corresponding to the inclusion of the gluon exchange between the $u\bar{u}$ in the ρ and the \bar{s} would-be spectator are denoted by ‘‘Tree+Penguin+CE_T’’; and, when the gluon exchanges between the \bar{s} would-be spectator and the outgoing $d\bar{d}$ of the ρ and the penguin gluon itself are included, we denote the result by ‘‘Tree+Penguin+CE_{T+P}’’. All results are given with a factor of $(f_{B_s}/0.141 \text{ GeV})^2 \times 10^{-8}$ removed for a total width $\Gamma(B_s \rightarrow \text{all}) = 4.085 \times 10^{-13} \text{ GeV}$ and for the variation $0 \leq \delta_{13} \leq 2\pi$.

BR($\bar{B} \rightarrow \rho K_S$)/($f_{B_s}/0.141 \text{ GeV}$) ²					
	Tree	Tree+Penguin	Tree+Penguin+CE _T	Tree+Penguin+CE _{T+P}	
R_b	$\times 10^{-8}$				
0.220	0.0352	[0.0296, 0.0875]	[0.0111, 0.823]	[0.000205, 0.646]	
0.385	0.108	[0.0158, 0.117]	[0.236, 1.66]	[0.0805, 1.21]	
0.551	0.221	[0.00624, 0.151]	[0.752, 2.79]	[0.338, 1.95]	

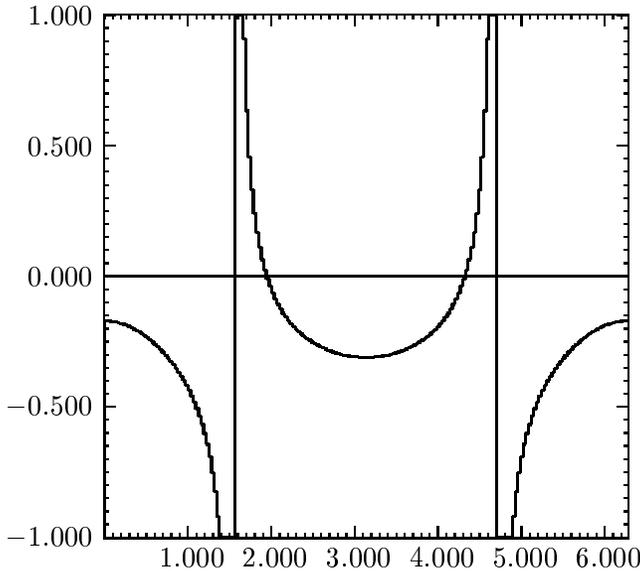


Fig. 4. Penguin shift of the CP asymmetry $\sin(2\gamma)$ in $\bar{B}_s \rightarrow \rho K_S$ for $R_b = 0.385$ for the matrix element approximation corresponding to the last column in Table I. The analogous plots for the $\pm 1\sigma$ values of R_b are discussed in the text.

able in this regime if the luminosity is large enough to provide a sufficient number of events. By measurable, we mean that a 3σ result for its value is not blocked by the uncertainty from penguins. We define this regime in which $|\Delta \sin(\gamma)/\sin(\gamma)|$ is less than 0.29 as the measurability regime. Approximately 34% of this regime of measurability intersects the allowed region given by the limits on γ discussed in Ref. [1], $135^\circ \gtrsim \gamma \gtrsim 45^\circ$. We need to stress the following. When the pollution in the $\sin(2\gamma)$ is $\lesssim |\sin(2\gamma)|$, a 15–20% accuracy calculation of the pollution is sufficient — when we have as well $|\Delta(\sin(2\gamma))| < 0.29 |\sin(2\gamma)|$ $\sin(2\gamma)$ is directly measurable; when $0.29 |\sin(2\gamma)| < |\Delta(\sin(2\gamma))| \lesssim |\sin(2\gamma)|$, we measure a quantity from which we can extract $\sin(2\gamma)$ with $\sim 20\%$ theoretical precision so that $\sin(2\gamma)$ can still be extracted. However, when the pollution is itself dominant and $\sin(2\gamma)$ is $\sim 20\%$ of it, a 20% accuracy knowledge of the pollution will not permit the extraction of $\sin(2\gamma)$. Thus, for a given precision on the pollution, depending on the relative size of the pollution and $\sin(2\gamma)$, one has these three regions and one of these is exactly that addressed as our regime of measurability, one wherein $\sin(2\gamma)$ is measurable. (For the $\pm 1\sigma$ deviations of R_b , the measurability regimes are qualitatively similar in size and location, with the exception that the lower regime is absent for the -1σ deviation case. So, we do not show these $\pm 1\sigma$ deviation measurability regimes separately here — see Ref. [19] for the corresponding plots analogous to that in Fig. 4.)

The question naturally arises as to the sensitivity of our regime of measurability to the parameters in our calculation. We now turn to this. Since the penguin shifts plotted in Fig. 4 are determined from amplitude ratios, they do not depend on the normalisations of the distribution amplitudes, or the hard recoil gluon exchange coupling in Figs. 1–3. What they do depend on are the relative strengths of the leading and non-leading Gegenbauer coefficients [4] in the distribution amplitudes, the relative strengths of the penguin and non-penguin operators in the effective weak Hamiltonian (a_2 and the value of α_s in our one-loop penguins), the quark running masses and the light-cone fraction x_2 of the \bar{s} in the B_s as determined by the Cornell model of B mesons. We have varied all of these parameters systematically as currently allowed by the 1σ limits on them when they are taken from data or theory together with data [19]. We find that the first part of the regime of measurability varies from $[0^\circ, 19^\circ]$ to $[0^\circ, 58^\circ]$. Thus, it may even be true that some of the allowed regime, $45^\circ \lesssim \gamma \lesssim 135^\circ$, overlaps this first part of our regime of measurability. The most important aspect of this variation is that most of it is due to changing the value of the running b -quark mass by just $\pm 3.5\%$ and by varying the value of a_2 between 0.14 and 0.34 (for reference, the variation in α_s is just that generated by the 1σ variation of Λ_{QCD} (see the following), the variations of the non-leading Gegenbauer coefficients are the 1σ variations as determined from their extraction from data in Refs. [20,21], the 1σ variations of the running quark masses are as given in Ref. [23] and the methods therein, and the variation of x_2 , between 0.041 and 0.071, is as given by the parameter variations allowed in Ref. [22] – see Ref. [19] for further details). If we do not vary these two measurable parameters, then the first part of our measurability regime only varies between $[0^\circ, 35^\circ]$ to $[0^\circ, 47^\circ]$, *i.e.*, it is robust to the remaining parameters in our calculation. In the actual precision hadron B-factory environment, we can expect that both m_b and a_2 will be known much better than we know them currently from comparison with data, either experimental or theoretical (lattice) data. The current large sensitivity to m_b and a_2 of the upper boundary on the first part of our regime of measurability is mainly academic because this regime is already outside the preferred region of γ and the variations we see with m_b, a_2 still leave most of this first part outside the preferred region. The second part of our regime of measurability begins at 102.5° and ends at 157.9° . Upon variation of our fundamental parameters as we described above, the beginning point varies between $[98.2^\circ, 105.5^\circ]$ and the ending point varies between $[138^\circ, 180^\circ]$, so that the preferred region of γ which overlaps the second and most important part of our regime of measurability, $[102.5^\circ, 135^\circ]$ is only changed by ${}_{-4.3}^{+3.0}$ degrees by the current uncertainties in our fundamental parameters. Again, if we do not vary a_2 and m_b , this already small effect is reduced significantly. We thus have a robust prediction that γ is measurable in the regime $[102.5^\circ, 135^\circ]$.

Recently, several authors [24] have argued that current data actually prefer the regime $36^\circ \leq \gamma \leq 97^\circ$, although more recent theoretical analyses [25, 26] would question this conclusion. Here, we stress that, from our results in Fig. 4, we can see that, in this new so-called preferred region, except for the small region $86.6^\circ \leq \gamma \leq 92.7^\circ$, the penguin shift is bounded in magnitude by a factor of 2 relative to the actual value of $\sin(2\gamma)$ so that, as we have a $\sim 15\%$ accurate knowledge of this shift, we still may use our results in the Appendix to radiatively correct this pollution out of $\sin(2\gamma)$ to the $\sim 30\%$ accurate level, allowing again a 3σ measurement of $\sin(2\gamma)$. The use of this technique to make fundamental tests of the SM is well-known [27].

The BR's in Table I, which remain qualitatively similar to their values shown here under the variations of parameters just considered, however, tend to indicate that the required luminosity would be more appropriate to hadron machines than to an e^+e^- annihilation B-factory. We note that the results in Table I are somewhat lower than the general range of similar results in Refs. [15, 16]. For example, our highest values for the BR just reach the lowest values in latter references. The recent and upcoming measurements of rare B processes can then already discriminate among various models of these processes on the basis of decay rates alone. To illustrate this, we note that in Ref. [3] we used our methods to compute the range

$$1.87 \times 10^{-6} \left(\frac{g_s^2(m_B^2)}{g_s^2(m_B^2)} \Big|_{\Lambda_{\text{QCD}}^{(5)}=0.1 \text{ GeV}} \right)^2 \left(\frac{f_{B_d}}{0.136 \text{ GeV}} \right)^2 \\ \leq \text{BR}(\bar{B}_d \rightarrow \pi^+\pi^-) \leq 2.63 \times 10^{-6} \left(\frac{g_s^2(m_B^2)}{g_s^2(m_B^2)} \Big|_{\Lambda_{\text{QCD}}^{(5)}=0.1 \text{ GeV}} \right)^2 \left(\frac{f_{B_d}}{0.136 \text{ GeV}} \right)^2. \quad (7)$$

We note that, according to Ref. [28], the current two-loop value of $\Lambda_{\text{QCD}}^{(5)}$ is 237_{-24}^{+26} MeV and according to Ref. [29] the best value of $\sqrt{2}f_{B_d}$ is now 210 ± 30 MeV so that we have the estimate

$$\left(\frac{g_s^2(m_B^2)}{g_s^2(m_B^2)} \Big|_{\Lambda_{\text{QCD}}^{(5)}=0.1 \text{ GeV}} \right)^2 \left(\frac{f_B}{0.136 \text{ GeV}} \right)^2 \cong 1.7. \quad (8)$$

This means that our result in Eq. (7) is consistent with the recent CLEO result [30] $\text{BR}(\bar{B}_d \rightarrow \pi^+\pi^-) = 4.7_{-1.5}^{+1.8} \pm 0.6 \times 10^{-6}$. Nonetheless, even if we allow the entire range which we and the authors in Refs. [15, 16] find for $\text{BR}(\bar{B}_s \rightarrow \rho K_S)$, we are led to suggest that the B-factory of the SLAC-LBL/KEK type should focus its attention on other possible roads to γ . Others [1] have reached a similar conclusion.

Finally, we stress that we have found that the assumption of colour suppression (factorisation) does not appear to work very well in our calculations.

This is consistent with the results in Refs. [31,32] on the analysis of the data on the processes $B \rightarrow \Psi/J K^*$. We will take up the corresponding analysis with our methods elsewhere [19].

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Appendix A

In this Appendix, we record for completeness the amplitude which we have evaluated from Figs. 1–3. Specifically, following the usual Feynman rules and the prescription given in Ref. [4], as already illustrated in Ref. [3], we get the amplitude

$$\mathcal{M}(\bar{B}_s \rightarrow \rho K_S) = \frac{(2\pi)^4 \delta(P_{\bar{B}_s} - P_\rho - P_{K_S})}{2m_B 2E_\rho 2E_{K_S} (2\pi)^{9/2}} \times \left(A_T e^{-i\phi_T} e^{i\delta_T} + \sum_j A_{P_j} e^{-i\phi_{P_j}} e^{i\delta_{P_j}} \right), \quad (\text{A.1})$$

where the “would-be tree level” contribution to the amplitude is, from Figs. 1 and 2, given by:

$$\begin{aligned} A_T e^{-i\phi_T} e^{i\delta_T} = & \int d[y] d[w] \\ & \times \text{Tr} \left[\left[\frac{f_K \phi_K \gamma_5 (\mathcal{P}_{K_S} + m_{K_S})}{\sqrt{2}\sqrt{2}} \frac{(-iG_F a_2 V_{ub} V_{ud}^*)}{\sqrt{2}} m_\rho f_\rho \not{\epsilon}^*(P_\rho) (1 - \gamma_5) \right. \right. \\ & \times \frac{i}{\not{P}_b - \not{q} - m_b + i\varepsilon} (-ig_s \lambda^c \gamma_\alpha) \frac{a_B \phi_B \gamma_5 (\mathcal{P}_B - m_B)}{\sqrt{2N_c}} (-ig_s \lambda^c \gamma^\alpha) \\ & + (-ig_s \lambda^c \gamma_\alpha) \frac{f_K \phi_K \gamma_5 (\mathcal{P}_{K_S} + m_{K_S})}{\sqrt{2}\sqrt{2}} (-ig_s \lambda^c \gamma^\alpha) \frac{i}{\not{P}_d + \not{q} - m_d + i\varepsilon} \\ & \left. \left. \times \frac{(-iG_F a_2 V_{ub} V_{ud}^*)}{\sqrt{2}} m_\rho f_\rho \not{\epsilon}^*(P_\rho) (1 - \gamma_5) \frac{a_B \phi_B \gamma_5 (\mathcal{P}_B - m_B)}{\sqrt{2N_c}} \right] \frac{(-i)}{q^2} \right. \\ & + r_{ce} \int d[z] \left[\text{Tr} \left\{ (-ig_s \lambda^c \gamma_\alpha) \frac{f_K \phi_K \gamma_5 (\mathcal{P}_{K_S} + m_{K_S})}{\sqrt{2}\sqrt{2}} \right. \right. \\ & \left. \left. \times \frac{(-iG_F a_2 V_{ub} V_{ud}^*)}{\sqrt{2}} \lambda^e \gamma^\mu (1 - \gamma_5) \frac{a_B \phi_B \gamma_5 (\mathcal{P}_B - m_B)}{\sqrt{2N_c}} \right\} \right. \\ & \left. \times \left(\frac{f_\rho \phi_\rho \not{\epsilon}^*(P_\rho) m_\rho}{\sqrt{2}\sqrt{2}} (-ig_s \lambda^c \gamma_\alpha) \frac{i}{\not{P}_u + \not{q} - m_u + i\varepsilon} \lambda^e \gamma_\mu (1 - \gamma_5) \right. \right. \\ & \left. \left. + \frac{i}{-\not{P}_u - \not{q} - m_u + i\varepsilon} (-ig_s \lambda^c \gamma_\alpha) \frac{f_\rho \phi_\rho \not{\epsilon}^*(P_\rho) m_\rho}{\sqrt{2}\sqrt{2}} \lambda^e \gamma_\mu (1 - \gamma_5) \right) \right] \frac{(-i)}{q^2}, \quad (\text{A.2}) \end{aligned}$$

where contribution of Fig. 2 is (not) included for $r_{ce} = 1(0)$ and where

$$\begin{aligned}\frac{f_K}{\sqrt{2}\sqrt{2}}\phi_K &= \frac{f_K}{\sqrt{2}\sqrt{2}}\phi_K(y_1, y_2), \\ \frac{f_\rho}{\sqrt{2}\sqrt{2}}\phi_\rho &= \frac{f_\rho}{\sqrt{2}\sqrt{2}}\phi_\rho(z_1, z_2), \\ \frac{a_B}{\sqrt{2}N_c}\phi_B &= \frac{a_B}{\sqrt{2}N_c}\phi_B(w_1, w_2),\end{aligned}$$

are the Lepage–Brodsky distribution amplitudes:

$$\begin{aligned}\phi_K(y_1, y_2) &= y_1 y_2 (1 + 3\beta'_K(y_2 - y_1)), \\ \phi_\rho(z_1, z_2) &= z_1 z_2 (1.348 - 1.74z_1 + 1.74z_1^2), \\ \phi_B(w_1, w_2) &= \delta(w_2 - x_2),\end{aligned}$$

are as indicated in the text above with

$$x_2 = \left(m_s^c - \frac{(m_s^c + m_b^c - m_B)m_b^c}{(m_s^c + m_b^c)} \right),$$

following the treatment of heavy mesons suggested by Ref. [4] based on non-relativistic potential model considerations for example. Here, the constituent quark masses are taken as [22] $m_s^c \cong 0.51$ GeV and $m_b^c \cong 5.1$ GeV, so that $x_2 \cong 0.0542$ when we take $m_B \cong 5.369$ GeV, as we should according to Ref. [10]. From Ref. [3] we have $a_B = f_B/\sqrt{12}$ where f_B is the B decay constant. Here, P_A is the 4-momentum of A for all A and, when a parton-type occurs in two external wave functions a prime is used to distinguish the two 4-momenta in an obvious way. To be precise, let us list these internal parton momenta as follows for Fig. 1:

$$\begin{aligned}P_b^+ &= x_1 m_B, & P_b^- &= \frac{m_b^2 + Q_\perp^2(B)}{x_1 m_B}, & \vec{P}_{b\perp} &= \vec{Q}_\perp(B), \\ P_{\bar{s}}^+ &= x_2 m_B, & P_{\bar{s}}^- &= \frac{m_s^2 + Q_\perp^2(B)}{x_2 m_B}, & \vec{P}_{\bar{s}\perp} &= -\vec{Q}_\perp(B), \\ Q_\perp^2(B) &= x_1 x_2 m_B^2 - x_2 m_b^2 - x_1 m_s^2, \\ P_d^+ &= y_1(E_K + P_{Kz}), & P_d^- &= \frac{m_d^2 + Q_\perp^2(K)}{y_1(E_K + P_{Kz})}, & \vec{P}_{d\perp} &= \vec{Q}_\perp(K), \\ P_{\bar{s}}'^+ &= y_2(E_K + P_{Kz}), & P_{\bar{s}}'^- &= \frac{m_s^2 + Q_\perp^2(K)}{y_2(E_K + P_{Kz})}, & \vec{P}'_{\bar{s}\perp} &= \vec{Q}_\perp(K), \\ Q_\perp^2(K) &= y_1 y_2 m_K^2 - y_2 m_d^2 - y_1 m_s^2,\end{aligned}$$

where we always work to leading order in Q_\perp^2/m_B^2 for all Q_\perp , and where we use the usual light-cone notation with $E_K = P_K^0$ and $P_{Kz} = P_K^3$ so that $P_K^\pm = P_K^0 \pm P_K^3$, etc. The λ^e are the QCD colour matrices generating the vector representation carried by the quarks so that g_s is the QCD coupling constant. Thus, Eq. (A.2) illustrates explicitly how the Feynman diagrams in Figs. 1–3 are evaluated for readers unfamiliar with the methods we used in Ref. [3], for example. The standard trace and integration manipulations, taking into account the definition [4] $[dx] \equiv dx_1 dx_2 \delta(1 - x_1 - x_2)$, then lead from Eq. (A.2) to the result:

$$\begin{aligned}
 & A_T e^{-i\phi_T} e^{i\delta_T} \\
 &= \frac{-iG_F}{\sqrt{2}} \mathcal{F} \frac{V_{ud}^* V_{ub}}{C_F} \frac{a_2}{\sqrt{2}} \frac{P_{\text{CMS}} m_B^2}{Q^2} \\
 &\times \left[I_{21} \left(m_B - 2(m_K + m_b) + \frac{m_K m_b}{m_B} \right) + I_{22} \left(2(m_K - E_K) + \frac{m_K^2}{m_B} \right) \right. \\
 &+ \left(x_2 m_B - 2(x_2 E_K + x_1 m_K) + m_d + \frac{m_K(m_K - m_d)}{m_B} \right) \frac{0.291}{D_2} \\
 &+ \frac{-4C_1 \sqrt{2} r_{ce}}{m_B a_2} \left(\frac{0.291}{2m_B^2} (-m_B^2 + m_K^2 + m_\rho^2) (1.348\hat{a}_1 - 3.088\hat{a}_2 + 3.48\hat{a}_3 - 1.74\hat{a}_4) \right. \\
 &\left. \left. - \frac{0.166666m_\rho^2}{m_B^2} (-0.253\hat{b}_0 + 2.758\hat{b}_1 - 2.505\hat{b}_2) \right) \right], \tag{A.3}
 \end{aligned}$$

where the various mathematical symbols are defined below. Continuing in this way, using the entirely similar methods, we find that the penguin graphs in Fig. 3 correspond to the contributions to the amplitude in Eq. (A.1) given by:

$$\begin{aligned}
 & A_{P_1} e^{-i\phi_{P_1}} e^{i\delta_{P_1}} \\
 &= \frac{-iG_F}{\sqrt{2}} \mathcal{F} \left\{ \frac{m_B P_{\text{CMS}}}{Q^2} \alpha_P^{(a)} I_{31} m_B^2 \left[\left(x_2 - \frac{2(x_2 E_K + x_1 m_K)}{m_B} + \frac{m_K^2}{m_B^2} \right) \frac{1}{D_2} \right. \right. \\
 &+ \left. \left(1 - \frac{2(m_b + m_K)}{m_B} + \frac{m_K m_b}{m_B^2} \right) \frac{I_{21}}{0.291} + \left(-1 + \frac{2m_K}{m_B} + \frac{m_\rho^2}{m_B^2} \right) \frac{I_{22}}{0.291} \right] \\
 &\left. + \left(\mathcal{P}_{cea} \alpha_P^{(a)} \Big|_g + \mathcal{P}_{ceb} \alpha_P^{(a)} \right) r_{cep} \right\},
 \end{aligned}$$

$$\begin{aligned}
 & A_{P_2} e^{-i\phi_{P_2}} e^{i\delta_{P_2}} \\
 &= \frac{-iG_F}{\sqrt{2}} \mathcal{F} \frac{m_B P_{\text{CMS}}}{Q^2} \alpha_P^{(b)} \left[\left(m_b m_\rho^2 \left(\frac{x_2 m_B}{2} - m_K \right) \frac{1}{D_2} \right) I_{33} \right. \\
 &+ \left(m_b m_K x_1 (m_\rho^2 - m_K^2) + m_b m_B \left(\left(1 + \frac{3x_1}{2} - x_1^2 \right) m_K^2 \right. \right. \\
 &\quad \left. \left. - x_2 m_B^2 + \left(1 - \frac{x_1}{2} \right) x_2 m_\rho^2 \right) \right) \frac{I_{32}}{D_2} \\
 &+ \left(m_b m_B^3 x_1 - \frac{1}{2} x_1 m_b^2 m_B^2 - 4x_1 m_b m_B^2 E_K \left(1 - \frac{m_b}{2m_B} \right) \right. \\
 &+ \left. 3x_1 m_b m_B^2 m_K \left(\frac{1}{2} - \frac{m_b}{m_B} \right) \right) \frac{I_{32} I_{21}}{0.291} \\
 &+ \left(3x_1 m_b m_B m_K^2 - x_1 m_b m_K \left(m_B^2 + \frac{m_K^2}{2} - \frac{m_\rho^2}{2} \right) \right) \frac{I_{32} I_{22}}{0.291} \\
 &\left. - m_b m_\rho^2 m_B \left(1 - \frac{m_b}{2m_B} \right) \frac{I_{33} I_{21}}{0.291} + \frac{m_b m_K m_\rho^2}{0.582} I_{33} I_{22} \right]. \tag{A.4}
 \end{aligned}$$

In Eqs.(A.2)–(A.5), the following definitions have been used:

$$\begin{aligned}
 \mathcal{F} &= f_K f_\rho a_B C_F^2 g_s^2 \sqrt{3}, \\
 Q^2 &= \frac{E_K + P_{\text{CMS}}}{2m_B} \left(x_1 m_B^2 - m_b^2 + m_s^2 \right), \\
 I_{21} &= \frac{-0.253}{m_B^2} \ell_{21} + \frac{2.505}{m_B^2} \ell_{22}, \\
 I_{22} &= \frac{-0.253}{m_B^2} \ell_{22} + \frac{2.505}{m_B^2} \ell_{23}, \\
 \text{for } \ell_{21} &= 0.403041 - 2.202003i, \\
 \ell_{22} &= -0.3794583 - 0.6585764i, \\
 \ell_{23} &= -0.5097241 - 0.1969674i, \\
 I_{31} &= 0.0485, \\
 I_{32} &= \frac{0.291}{0.3} \left(\frac{0.195517}{m_B^2} - \frac{0.064303i}{m_B^2} \right), \\
 I_{33} &= \frac{0.291}{0.3} \left(\frac{0.132055}{m_B^2} - \frac{0.0608173i}{m_B^2} \right),
 \end{aligned}$$

$$D_2 = m_b^2 + m_\rho^2 - (E_\rho + P_{\text{CMS}})x_1 m_B - (E_\rho - P_{\text{CMS}})\left(x_2 m_B + \frac{m_b^2}{m_B}\right) - m_d^2,$$

$$\hat{a}_1 = -1.0 - x_2 \ln \frac{x_1}{x_2} - x_2 \pi i,$$

$$\hat{a}_2 = -0.5 - x_2 - x_2^2 \ln \frac{x_1}{x_2} - x_2^2 \pi i,$$

$$\hat{a}_3 = -\frac{1}{3} - \frac{x_2}{2} - x_2^2 + x_2^3 \ln \frac{x_1}{x_2} - x_2^3 \pi i,$$

$$\hat{a}_4 = -0.25 (x_1^4 - x_2^4) - \frac{4}{3} (x_1^3 + x_2^3) x_2 - 3x_2^2 (x_1^2 - x_2^2) - 4x_2^3 - x_2^4 \ln \frac{x_1}{x_2} - x_2^4 \pi i,$$

$$\hat{b}_0 = -\ln \frac{x_1}{x_2} - \pi i, \quad \hat{b}_1 = \hat{a}_1, \quad \hat{b}_2 = \hat{a}_2,$$

$$\alpha_P^{(a)}|_g = \frac{\alpha_s}{2\pi} V_{jd}^* V_{jb} \left[\left[\frac{1}{12} \left(\frac{1}{x_j - 1} \right) + \frac{12}{13} \left(\frac{1}{x_j - 1} \right)^2 - \frac{1}{2} \left(\frac{1}{x_j - 1} \right)^3 \right] x_j + \left[\frac{2}{3} \left(\frac{1}{x_j - 1} \right) + \left(\frac{2}{3} \left(\frac{1}{x_j - 1} \right)^2 - \frac{5}{6} \left(\frac{1}{x_j - 1} \right)^3 + \frac{1}{2} \left(\frac{1}{x_j - 1} \right)^4 \right) x_j \right] \ln x_j \right],$$

$$\begin{aligned} \alpha_P^{(a)} = & \alpha_P^{(a)}|_g + \frac{\alpha_{\text{em}}}{8\pi s_W^2 C_F} \left(-s_W^2 \frac{16}{27} V_{cd}^* V_{cb} \ln \frac{x_u}{x_c} \right. \\ & + V_{td}^* V_{tb} \left\{ -4 \left(3x_t^2 \frac{\ln x_t}{4(x_t - 1)^2} + \frac{x_t}{4} - \frac{3x_t}{4(x_t - 1)} \right) \right. \\ & - \frac{5x_t}{(x_t - 1)} \left(1 - \frac{\ln x_t}{x_t - 1} \right) \\ & - 2 |V_{td}|^2 \left(-3x_t^3 \frac{\ln x_t}{2(x_t - 1)^3} - x_t \left(0.25 - \frac{9}{4(x_t - 1)} - \frac{3}{2(x_t - 1)^2} \right) \right) \\ & + \frac{4s_W^2}{3} \left(0.641 - x_t \left(\frac{7}{3(x_t - 1)} + \frac{13}{12(x_t - 1)^2} - \frac{1}{2(x_t - 1)^3} \right) \right. \\ & - x_t \ln x_t \left(\frac{1}{6(x_t - 1)} - \frac{35}{12(x_t - 1)^2} - \frac{5}{6(x_t - 1)^3} + \frac{1}{2(x_t - 1)^4} \right) \\ & \left. \left. + \frac{4}{9} \ln x_u \right) - \left(\frac{x_t}{2} - \frac{3}{4(x_t - 1)} + 3 (2x_t^2 - x_t) \frac{\ln x_t}{4(x_t - 1)^2} - \frac{3}{4} \right) \right\} \right), \end{aligned}$$

$$x_j = m_j^2/M_W^2, \quad j = u, c, t,$$

$$\alpha_P^{(b)} = \frac{-\alpha_s}{2\pi} V_{td}^* V_{tb} (-0.195) + \frac{\alpha_{em}}{6\pi C_F} V_{td}^* V_{tb} \\ \times \left[0.641 + x_t \left(\frac{1}{2(x_t - 1)} + \frac{9}{4(x_t - 1)^2} + \frac{3}{2(x_t - 1)^3} \right) - \frac{3x_t^3 \ln x_t}{2(x_t - 1)^4} \right],$$

$$\mathcal{P}_{cea} = \frac{C_G}{C_F} \frac{m_B P_{CMS}}{4x_{c2} d_{bk} d_{bp}} \left[i_{cp00} \left\{ \left(-\frac{1}{2} + 4x_1 \right) m_K^2 + (6 - 4x_1) m_\rho^2 - 3x_1 m_B^2 \right. \right. \\ \left. \left. - 11x_1 m_K m_B + \frac{1}{2} (m_s^2 - m_d^2) \right\} + i_{cp10} \left\{ 5m_B^2 - 5m_\rho^2 - 6m_B m_K \right\} \right. \\ \left. + i_{cp01} \left\{ \frac{5}{2} m_B^2 - \frac{3}{2} m_\rho^2 + \frac{3}{2} m_K^2 \right\} + \frac{2m_B^2}{d_{br}} \left[x_1 i_{dp00} \left\{ (x_1 m_B + m_K) E_K \right. \right. \right. \\ \left. \left. - (2 - x_2) m_B^2 - P_\rho \cdot P_K - x_2 m_B m_K + 4m_K E_\rho \right. \right. \\ \left. \left. + \frac{m_\rho^2}{x_1} \left(1 - \frac{2m_K}{m_B} \right) - m_d^2 \left(\frac{1}{2} + \frac{2}{x_1} \left(1 - \frac{2m_K}{m_B} \right) \right) + \frac{1}{2} (m_K^2 + m_s^2) \right\} \right. \\ \left. + i_{dp01} \left\{ 2E_\rho E_K + 2P_\rho \cdot P_K \left(x_1 - \frac{m_K}{2m_B} \right) + m_K E_\rho \left(1 - 5x_1 - \frac{3m_K}{2m_B} \right) \right. \right. \\ \left. \left. + 2x_1 m_B E_\rho - \frac{E_\rho}{2m_B} m_s^2 - 2m_\rho^2 \left(x_2 + \frac{m_\rho^2}{2x_1 m_B^2} - \frac{E_\rho}{m_B} \right) \right. \right. \\ \left. \left. + \left(\frac{2m_\rho^2}{x_1 m_B^2} + \frac{E_\rho}{2m_B} \right) m_d^2 \right\} - x_1 i_{dp10} \left\{ 2m_B E_K \left(1 - \frac{3m_K}{2m_B} \right) + m_K^2 \right\} \right. \\ \left. + 2i_{dp11} \left\{ P_\rho \cdot P_K \left(1 - \frac{E_K}{m_B} - \frac{3m_K}{2m_B} \right) + E_K E_\rho + \frac{m_K^2}{2m_B} E_\rho \right\} \right. \\ \left. + \frac{E_\rho}{m_B} i_{dp02} \left\{ m_K^2 - m_B^2 + \left(\frac{x_2}{x_1} - 1 \right) m_\rho^2 \right\} - 2 \frac{m_\rho^2}{x_1 m_B^2} i_{dp12} P_\rho \cdot P_K \right] \right],$$

$$\begin{aligned}
 d_{bk} &= m_B^2 + m_K^2 - m_\rho^2 \\
 d_{bp} &= m_B^2 - m_K^2 - m_\rho^2, \\
 d_{br} &= m_B^2 + m_\rho^2 - m_K^2, \\
 x_{c2} &= \frac{2Q^2}{d_{bk}}, \\
 i_{cp00} &= 0.529, \\
 i_{cp10} &= 0.529 \frac{1 - r_\beta}{2}, \\
 i_{cp01} &= \frac{\delta_a}{3} + \frac{\delta_b}{4} + \frac{\delta_c}{5},
 \end{aligned}$$

where $r_\beta = 0.418$, $\delta_a = 1.348$, $\delta_b = -1.74$, $\delta_c = 1.74$, and

$$\begin{aligned}
 i_{dp00} &= \delta_a i_{rdp1}(z_0) + \delta_b i_{rdp2}(z_0) + \delta_c i_{rdp3}(z_0), \\
 i_{dp01} &= \delta_a i_{rdp2}(z_0) + \delta_b i_{rdp3}(z_0) + \delta_c i_{rdp4}(z_0), \\
 i_{dp10} &= (\delta_a i_{rdp1}(z_0) + \delta_b i_{rdp2}(z_0) + \delta_c i_{rdp3}(z_0)) \frac{1 - r_\beta}{2} = i_{dp00} \frac{1 - r_\beta}{2}, \\
 i_{dp02} &= \delta_a i_{rdp3}(z_0) + \delta_b i_{rdp4}(z_0) + \delta_c i_{rdp5}(z_0), \\
 i_{dp11} &= (\delta_a i_{rdp2}(z_0) + \delta_b i_{rdp3}(z_0) + \delta_c i_{rdp4}(z_0)) \frac{1 - r_\beta}{2} = i_{dp01} \frac{1 - r_\beta}{2}, \\
 i_{dp12} &= (\delta_a i_{rdp3}(z_0) + \delta_b i_{rdp4}(z_0) + \delta_c i_{rdp5}(z_0)) \frac{1 - r_\beta}{2} = i_{dp02} \frac{1 - r_\beta}{2}
 \end{aligned}$$

where

$$\begin{aligned}
 i_{rdp1}(z) &= -1 - z \ln \frac{1 - z}{z} - \pi z i, \\
 i_{rdp2}(z) &= -0.5 - z - z^2 \ln \frac{1 - z}{z} - \pi z^2 i, \\
 i_{rdp3}(z) &= -\frac{(1 - z)^3 + z^3}{3} - 3z \frac{(1 - z)^2 - z^2}{2} - 3z^2 - z^3 \ln \frac{1 - z}{z} - \pi z^3 i, \\
 i_{rdp4}(z) &= -\frac{(1 - z)^4 - z^4}{4} - 4z \frac{(1 - z)^3 + z^3}{3} - 3z^2 ((1 - z)^2 - z^2) \\
 &\quad - 4z^3 - z^4 \ln \frac{1 - z}{z} - \pi z^4 i, \\
 i_{rdp5}(z) &= -\frac{(1 - z)^5 + z^5}{5} - 5z \frac{(1 - z)^4 - z^4}{4} - 10z^2 \frac{(1 - z)^3 + z^3}{3} \\
 &\quad - 5z^3 ((1 - z)^2 - z^2) - 5z^4 - z^5 \ln \frac{1 - z}{z} - \pi z^5 i,
 \end{aligned}$$

and for $z_0 = m_b^2/(x_1 d_{br})$

$$\mathcal{P}_{ceb} = \left(1 - \frac{0.5C_G}{C_F}\right) \frac{m_B P_{\text{CMS}}}{2Q^2} (-1 + r_\beta) \left\{ \delta_a (i_{rdp1}(x_1) - i_{rdp2}(x_1)) + \delta_b (i_{rdp2}(x_1) - i_{rdp3}(x_1)) + \delta_c (i_{rdp3}(x_1) - i_{rdp4}(x_1)) \right\}, \quad (\text{A.5})$$

where the kinematics is the usual two-body decay one:

$$m_B = E_K + E_\rho,$$

$$E_K = \frac{d_{bk}}{2m_B},$$

$$P_{\text{CMS}} = \frac{\sqrt{\Delta(m_B^2, m_K^2, m_\rho^2)}}{2m_B},$$

for $\Delta(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$, so that the decay width itself is given by:

$$\Gamma(\bar{B} \rightarrow \rho K_S) = |\mathcal{M}|^2 \frac{P_{\text{CMS}}}{8\pi m_B^2}. \quad (\text{A.6})$$

Here, $C_G = 3$, $C_F = 4/3$ and we have used the values [23]:

$$m_u(1\text{GeV}) \cong 5.0\text{ MeV},$$

$$m_d(1\text{GeV}) \cong 8.9\text{ MeV},$$

$$m_s(1\text{GeV}) \cong 0.175\text{ GeV},$$

$$m_c(m_c) \cong 1.3\text{ GeV},$$

$$m_b(m_b) \cong 4.5\text{ GeV},$$

$$\text{and } m_t(m_t) \cong 176\text{ GeV}.$$

We take $s_W^2 = \sin^2 \theta_W \cong 0.2315$, where θ_W is the usual weak mixing angle and α_{em} is the QED fine structure constant. We note further that we use an average value of the square of the momentum transfer to the would-be spectator in Figs. 1–3 to get $g_s^2 \cong 3.72$ in \mathcal{F} above; the analogous average for the square of the momentum transfer through the penguin yields $\alpha_s \cong 0.25$

in the evaluation of coefficients $\alpha_P^{(i)}$ above. Thus, in both cases, we see that the momentum transfers are large enough that they are well into the perturbative regime where the methods of Ref. [4] apply. This completes our appendix.

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