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TOWARD NEUTRINO TEXTURE DOMINATED BY MAJORANA LEFTHANDED MASS MATRIX*

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A form of mixing matrix for three active and three sterile, conventional Majorana neutrinos is proposed. Its Majorana lefthanded part arises from the popular bimaximal mixing matrix for three active neutrinos that works satisfactorily in solar and atmospheric experiments if the LSND effect is ignored. One of three sterile neutrinos, effective in the Majorana righthanded and Dirac parts of the proposed mixing matrix, is responsible perturbatively for the possible LSND effect by inducing one of three extra neutrino mass states to exist actively. The corresponding form of neutrino mass matrix is derived. If all three extra neutrino mass states get vanishing masses, the neutrino mass matrix is dominated by its specific Majorana lefthanded part. Then, the observed qualitative difference between mixings of neutrinos and down quarks may be connected with this Majorana lefthanded dominance realized for neutrinos. If $m_1^2 \simeq m_2^2$ for two of three basic neutrino mass states, the sum rule $\sin^2 2\theta_{\rm sol} + \sin^2 2\theta_{\rm Chooz}/2 + \sin^2 2\theta_{\rm LSND} = 1$ holds in the two-flavor approximation (for each of three cases). Thus, the solar neutrino oscillation amplitude, not fully maximal, leaves some room for the LSND effect, depending on the magnitude of Chooz effect (not observed so far).

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1. Introduction

Although the recent experimental results for atmospheric ν_μ's as well as solar ν_e's are in favor of excluding the hypothetical sterile neutrinos from neutrino oscillations [1], the problem of the third neutrino mass difference manifested in the possible LSND effect for accelerator ν_μ's still exists [2], implying a further discussion on mixing of sterile neutrinos with three active

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flavors $\nu_e\,,\,\nu_\mu\,,\,\nu_\tau$. In the present note we contribute to this discussion by constructing a particular 6×6 texture involving three active and three sterile, conventional Majorana neutrinos. The construction extends (or rather adapts) the familiar bimaximal 3×3 texture [3] working in a satisfactory way for three active neutrinos in solar and atmospheric experiments if the LSND effect is ignored. Then, one of three sterile neutrinos is responsible perturbatively [4] for the possible LSND effect by inducing one of three extra neutrino mass states to exist actively.

As is well known, three sterile Majorana neutrinos

$$\nu^{(s)}_\alpha = \nu_{\alpha\mathrm{R}} + (\nu_{\alpha\mathrm{R}})^c \quad (\alpha = e\,,\,\mu\,,\,\tau) \tag{1}$$

can be always constructed in addition to three active Majorana neutrinos

$$\nu^{(a)}_\alpha = \nu_{\alpha\mathrm{L}} + (\nu_{\alpha\mathrm{L}})^c \quad (\alpha = e\,,\,\mu\,,\,\tau) \tag{2}$$

if there are righthanded neutrino states $\nu_{\alpha\mathrm{R}}$ beside their familiar lefthanded partners $\nu_{\alpha\mathrm{L}}$ participating in Standard Model gauge interactions [of course, $\nu^{(a)}_{\alpha\mathrm{L}} = \nu_{\alpha\mathrm{L}}$ and $\nu^{(s)}_{\alpha\mathrm{L}} = (\nu_{\alpha\mathrm{R}})^c$]. Whether such sterile neutrino states are physically realized depends on the actual shape of the neutrino mass term whose generic form is

$$\begin{aligned} -\mathcal{L}_{\mathrm{mass}} &= \frac{1}{2}\sum_{\alpha\beta}(\overline{\nu^{(a)}_\alpha}\,,\,\overline{\nu^{(s)}_\alpha})\left(\begin{array}{cc} M^{(\mathrm{L})}_{\alpha\beta} & M^{(\mathrm{D})}_{\alpha\beta} \\ M^{(\mathrm{D})*}_{\beta\alpha} & M^{(\mathrm{R})}_{\alpha\beta}\end{array}\right)\left(\begin{array}{c}\nu^{(a)}_\beta \\ \nu^{(s)}_\beta\end{array}\right) \\ &= \frac{1}{2}\sum_{\alpha\beta}\left(M^{(\mathrm{D})}_{\alpha\beta} + M^{(\mathrm{D})*}_{\alpha\beta}\right)\left(\overline{\nu_{\alpha\mathrm{L}}}\,\nu_{\beta\mathrm{R}} + \overline{\nu_{\beta\mathrm{R}}}\,\nu_{\alpha\mathrm{L}}\right) \\ &\quad + \frac{1}{2}\sum_{\alpha\beta} M^{(\mathrm{L})}_{\alpha\beta}\left[\overline{\nu_{\alpha\mathrm{L}}}\,(\nu_{\beta\mathrm{L}})^c + \overline{(\nu_{\alpha\mathrm{L}})^c}\,\nu_{\beta\mathrm{L}}\right] \\ &\quad + \frac{1}{2}\sum_{\alpha\beta} M^{(\mathrm{R})}_{\alpha\beta}\left[\overline{\nu_{\alpha\mathrm{R}}}\,(\nu_{\beta\mathrm{R}})^c + \overline{(\nu_{\alpha\mathrm{R}})^c}\,\nu_{\beta\mathrm{R}}\right], \end{aligned} \tag{3}$$

where$M^{(\mathrm{L,R})*}_{\beta\alpha} = M^{(\mathrm{L,R})}_{\alpha\beta}$, but $M^{(\mathrm{D})*}_{\beta\alpha} \neq M^{(\mathrm{D})}_{\alpha\beta}$ in general. The 6×6 neutrino mass matrix

$$M = \left(\begin{array}{cc} M^{(\mathrm{L})} & M^{(\mathrm{D})} \\ M^{(\mathrm{D})\dagger} & M^{(\mathrm{R})}\end{array}\right) \tag{4}$$

appearing in Eq. (3) is hermitian, $M^\dagger = M$. Here, $M^{(\mathrm{D,L,R})} = \left(M^{(\mathrm{D,L,R})}_{\alpha\beta}\right)$ are 3×3 neutrino mass matrices: Dirac, Majorana lefthanded and Majorana righthanded, respectively. Further on, for six neutrino flavor states we will use the notation $\nu_\alpha \equiv \nu_\alpha^{(a)}$ and $\nu_{\alpha_s} \equiv \nu_\alpha^{(s)}$ with $\alpha = e\,,\mu\,,\tau$ and then pass to $\nu_\alpha = \nu_e\,,\nu_\mu\,,\nu_\tau\,,\nu_{e_s}\,,\nu_{\mu_s}\,,\nu_{\tau_s}$, where $\alpha = e\,,\mu\,,\tau\,,e_s\,,\mu_s\,,\tau_s$. Six neutrino mass states will be denoted as $\nu_i = \nu_1\,,\nu_2,\nu_3\,,\nu_4\,,\nu_5\,,\nu_6$, where $i = 1\,,2\,,3\,,4\,,5\,,6$.

2. Proposal of a 6 × 6 neutrino mixing matrix

Starting from the phenomenologically useful bimaximal mixing matrix for three active neutrinos [3,5]

$$U^{(3)} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}, \tag{5}$$

we propose the following form of the 6×6 neutrino mixing matrix:

$$U = \begin{pmatrix} U^{(3)} & 0 \\ 0 & 1^{(3)} \end{pmatrix} \begin{pmatrix} C & S \\ -S & C \end{pmatrix} = \begin{pmatrix} U^{(3)}C & U^{(3)}S \\ -S & C \end{pmatrix}, \tag{6}$$

where

$$C = \begin{pmatrix} c_1 & 0 & 0 \\ 0 & c_2 & 0 \\ 0 & 0 & c_3 \end{pmatrix},\; S = \begin{pmatrix} s_1 & 0 & 0 \\ 0 & s_2 & 0 \\ 0 & 0 & s_3 \end{pmatrix},\; 1^{(3)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{7}$$

with $c_i = \cos\theta_i \geq 0$ and $s_i = \sin\theta_i \geq 0$ $(i = 1,2,3)$. One may also denote $s_1 \equiv s_{14}\,, s_2 \equiv s_{25}\,, s_3 \equiv s_{36}$, while $s_{12} = 1/\sqrt{2}\,, s_{23} = 1/\sqrt{2}\,, s_{13} = 0$. Explicitly,

$$U = (U_{\alpha i}) = \begin{pmatrix} \frac{c_1}{\sqrt{2}} & \frac{c_2}{\sqrt{2}} & 0 & \frac{s_1}{\sqrt{2}} & \frac{s_2}{\sqrt{2}} & 0 \\ -\frac{c_1}{2} & \frac{c_2}{2} & \frac{c_3}{\sqrt{2}} & -\frac{s_1}{2} & \frac{s_2}{2} & \frac{s_3}{\sqrt{2}} \\ \frac{c_1}{2} & -\frac{c_2}{2} & \frac{c_3}{\sqrt{2}} & \frac{s_1}{2} & -\frac{s_2}{2} & \frac{s_3}{\sqrt{2}} \\ -s_1 & 0 & 0 & c_1 & 0 & 0 \\ 0 & -s_2 & 0 & 0 & c_2 & 0 \\ 0 & 0 & -s_3 & 0 & 0 & c_3 \end{pmatrix}, \tag{8}$$

where $\alpha = e\,,\mu\,,\tau\,,e_s\,,\mu_s\,,\tau_s$ and $i = 1\,,2,\,3\,,4\,,5\,,6$. The relation

$$\nu_\alpha = \sum_i U_{\alpha i}\nu_i \tag{9}$$

describes the mixing of six neutrinos.

In the representation where the mass matrix of three charged leptons e^- , μ^- , τ^- is diagonal, the 6×6 neutrino mixing matrix U is at the same time the diagonalizing matrix for the $6{\times}6$ neutrino mass matrix $M = (M_{\alpha\beta})$:

$$U^\dagger M U = \mathrm{diag}(m_1\,, m_2\,, m_3\,, m_4\,, m_5\,, m_6)\,, \tag{10}$$

where we put $m_1^2 \leq m_2^2 \leq m_3^2$ and *either* $m_3^2 \leq m_4^2$ *or* $m_4^2 \leq m_1^2$. Then, evidently, $M_{\alpha\beta} = \sum_i U_{\alpha i} m_i U^*_{i\beta}$. From this formula, we obtain with the use of proposal (8) the following particular form of 6×6 neutrino mass matrix (4):

$$\begin{aligned} M &= \begin{pmatrix} M^{(\mathrm{L})} & M^{(\mathrm{D})} \\ M^{(\mathrm{D})\dagger} & M^{(\mathrm{R})} \end{pmatrix} = (M_{\alpha\beta}) \\ &= \begin{pmatrix} M_{ee} & M_{e\mu} & -M_{e\mu} & M_{ee_s} & M_{e\mu_s} & 0 \\ M_{e\mu} & M_{ee}+M_{\mu\tau} & M_{\mu\tau} & -\frac{M_{ee_s}}{\sqrt{2}} & \frac{M_{e\mu_s}}{\sqrt{2}} & M_{\mu\tau_s} \\ -M_{e\mu} & M_{\mu\tau} & M_{ee}+M_{\mu\tau} & \frac{M_{ee_s}}{\sqrt{2}} & -\frac{M_{e\mu_s}}{\sqrt{2}} & M_{\mu\tau_s} \\ M_{ee_s} & -\frac{M_{ee_s}}{\sqrt{2}} & \frac{M_{ee_s}}{\sqrt{2}} & M_{e_s e_s} & 0 & 0 \\ M_{e\mu_s} & \frac{M_{e\mu_s}}{\sqrt{2}} & -\frac{M_{e\mu_s}}{\sqrt{2}} & 0 & M_{\mu_s\mu_s} & 0 \\ 0 & M_{\mu\tau_s} & M_{\mu\tau_s} & 0 & 0 & M_{\tau_s\tau_s} \end{pmatrix}, \end{aligned} \tag{11}$$

where

$$\begin{aligned} M_{ee} &= \tfrac{1}{2}\left(c_1^2 m_1 + c_2^2 m_2 + s_1^2 m_4 + s_2^2 m_5\right), \\ M_{\mu\mu} &= M_{\tau\tau} = M_{ee} + M_{\mu\tau} \\ &= \tfrac{1}{4}\left(c_1^2 m_1 + c_2^2 m_2 + 2c_3^2 m_3 + s_1^2 m_4 + s_2^2 m_5 + 2s_3^2 m_6\right), \\ M_{e\mu} &= -M_{e\tau} = \tfrac{1}{2\sqrt{2}}\left(-c_1^2 m_1 + c_2^2 m_2 - s_1^2 m_4 + s_2^2 m_5\right), \\ M_{\mu\tau} &= \tfrac{1}{4}\left(-c_1^2 m_1 - c_2^2 m_2 + 2c_3^2 m_3 - s_1^2 m_4 - s_2^2 m_5 + 2s_3^2 m_6\right) \end{aligned} \tag{12}$$

and

$$\begin{aligned} \tfrac{1}{\sqrt{2}} M_{ee_s} &= -M_{\mu e_s} = M_{\tau e_s} = \frac{c_1 s_1}{2}\,(-m_1 + m_4)\,,\; M_{e_s e_s} = s_1^2 m_1 + c_1^2 m_4\,, \\ \tfrac{1}{\sqrt{2}} M_{e\mu_s} &= M_{\mu\mu_s} = -M_{\tau\mu_s} = \frac{c_2 s_2}{2}\,(-m_2 + m_5)\,,\; M_{\mu_s\mu_s} = s_2^2 m_2 + c_2^2 m_5, \\ M_{\mu\tau_s} &= M_{\tau\tau_s} = \frac{c_3 s_3}{\sqrt{2}}\,(-m_3 + m_6)\,,\; M_{\tau_s\tau_s} = s_3^2 m_3 + c_3^2 m_6\,, \end{aligned} \tag{13}$$

while

$$M_{e\tau_s} = 0\,,\quad M_{e_s\mu_s} = M_{e_s\tau_s} = M_{\mu_s\tau_s} = 0 \tag{14}$$

(of course, $M_{\alpha\beta} = M_{\beta\alpha}$ for all α and β). Hence,

$$\begin{aligned} M_{ee} - M_{e\mu}\sqrt{2} \pm M_{e_s e_s} &= \left\{ \begin{array}{l} m_1 + m_4 \\ (c_1^2 - s_1^2)(m_1 - m_4) \end{array} \right. , \\ M_{ee} + M_{e\mu}\sqrt{2} \pm M_{\mu_s \mu_s} &= \left\{ \begin{array}{l} m_2 + m_5 \\ (c_2^2 - s_2^2)(m_2 - m_5) \end{array} \right. , \\ M_{\mu\mu} + M_{\mu\tau} \pm M_{\tau_s \tau_s} &= \left\{ \begin{array}{l} m_3 + m_6 \\ (c_3^2 - s_3^2)(m_3 - m_6) \end{array} \right. , \end{aligned} \tag{15}$$

where $M_{\mu\mu} = M_{ee} + M_{\mu\tau}$. After a simple calculation we get from Eqs. (15)

$$m_{1,4} = \frac{M_{ee} - M_{e\mu}\sqrt{2} + M_{e_s e_s}}{2} \pm \sqrt{\left(\frac{M_{ee} - M_{e\mu}\sqrt{2} - M_{e_s e_s}}{2} \right)^2 + 2M_{ee_s}^2} \tag{16}$$

and analogical formulae for $m_{2,5}$ and $m_{3,6}$ (note that $m_1 > m_4$, but not always $m_4 > 0$, and similarly for $m_{2,5}$ and $m_{3,6}$).

In the 6×6 matrix (11) there are generally nine independent nonzero matrix elements. If $s_2 = 0$ and $s_3 = 0$ (what implies complete decoupling of two sterile neutrinos ν_{μ_s} and ν_{τ_s}), this number is reduced to seven. In this case, Eqs. (13) and (15) give

$$M_{e\mu_s} = M_{\mu\mu_s} = M_{\tau\mu_s} = 0\,, \;\; M_{\mu\tau_s} = M_{\tau\tau_s} = 0\,, \;\; M_{\mu_s\mu_s} = m_5\,, \;\; M_{\tau_s\tau_s} = m_6 \tag{17}$$

and

$$M_{ee} + M_{e\mu}\sqrt{2} = m_2\,, \;\; M_{\mu\mu} + M_{\mu\tau} = m_3\,, \tag{18}$$

but the formulae (16) for m_1 and m_4 are not much simplified (unless $M_{ee_s} = 0$ *i.e.*, $c_1 s_1 = 0$). Then, from Eq. (11)

$$\begin{aligned} M^{(\mathrm{D})} &= \begin{pmatrix} -\frac{c_1 s_1}{\sqrt{2}}(m_1 - m_4) & 0 & 0 \\ \frac{c_1 s_1}{2}(m_1 - m_4) & 0 & 0 \\ -\frac{c_1 s_1}{2}(m_1 - m_4) & 0 & 0 \end{pmatrix}, \\ M^{(\mathrm{R})} &= \begin{pmatrix} s_1^2 m_1 + c_1^2 m_4 & 0 & 0 \\ 0 & m_5 & 0 \\ 0 & 0 & m_6 \end{pmatrix} \end{aligned} \tag{19}$$

and $M_{ee}^{(\mathrm{L})} = M_{ee} = \frac{1}{2}(c_1^2 m_1 + s_1^2 m_4 + m_2)$, *etc.* If $c_1 > s_1\sqrt{2}$ and $s_1^2 m_1 \gg c_1^2 |m_4|$ (*i.e.*, $m_1 \gg |m_4|$) and, in addition, m_5 and m_6 are vanishing, the texture is in a way of a type opposite to the see–saw (now, symbolically (L) > (D) > (R) or even (L) $\gg$ (D) $\gg$ (R) if $c_1^2 \gg s_1^2$ and $m_4 = 0$).

At any rate, the active existence of extra massive neutrino ν_4 (in addition to the massive ν_1 , ν_2 , ν_3) is induced by the sterile neutrino ν_{e_s} mixing with the active ν_e , ν_μ , ν_τ. Of course, two completely decoupled sterile neutrinos ν_{μ_s} and ν_{τ_s} (with $s_2 = 0$ and $s_3 = 0$) induce trivially the passive existence of two massive neutrinos $\nu_5 = \nu_{\mu_s}$ and $\nu_6 = \nu_{\tau_s}$ with masses m_5 nad m_6 which, most naturally, ought to be put zero. However, another point of view is not excluded that there is still a tiny mixing of ν_{μ_s} and ν_{τ_s} with the rest of six neutrino flavors, caused by spontaneously breaking a GUT symmetry at a high mass scale and so, accompanied by large masses $|m_5|$ and $|m_6|$. If instead of $|m_4| \ll m_1$ there is $m_1 \ll |m_4|$, such an inequality may be not so impressive as in the familiar see-saw referring to the GUT mass scale: it may happen, for instance, that $|m_4| \sim 1$ eV and $m_1 \sim 10^{-4}$ eV; *cf.* Eq. (33) as an alternative to the more natural Eq. (35).

3. Effective four-neutrino oscillations

Due to mixing of six neutrino fields described by Eq. (9), neutrino states mix according to the relation

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i} |\nu_i\rangle . \tag{20}$$

This implies the following familiar formulae for probabilities of neutrino oscillations $\nu_\alpha \to \nu_\beta$ on the energy shell:

$$P(\nu_\alpha \to \nu_\beta) = |\langle\beta| e^{iPL} |\alpha\rangle|^2 = \delta_{\beta\alpha} - 4 \sum_{j>i} U^*_{\beta j} U_{\beta i} U_{\alpha j} U^*_{\alpha i} \sin^2 x_{ji} \,, \tag{21}$$

being valid if the quartic product $U^*_{\beta j} U_{\beta i} U_{\alpha j} U^*_{\alpha i}$ is real, what is certainly true when the tiny CP violation is ignored. Here,

$$x_{ji} = 1.27 \frac{\Delta m^2_{ji} L}{E} \,, \quad \Delta m^2_{ji} = m^2_j - m^2_i \tag{22}$$

with Δm^2_{ji}, L and E measured in eV^2, km and GeV, respectively (L and E denote the experimental baseline and neutrino energy, while

$$p_i = \sqrt{E^2 - m^2_i} \simeq E - \frac{m^2_i}{2E}$$

are eigenvalues of the neutrino momentum P).

With the use of proposal (8) for the 6×6 neutrino mixing matrix and under the assumption that $s_2 = 0$ and $s_3 = 0$ the oscillation formulae (21) give

$$
\begin{aligned}
P(\nu_e \to \nu_e) &= 1 - c_1^2 \sin^2 x_{21} - (c_1 s_1)^2 \sin^2 x_{41} - s_1^2 \sin^2 x_{42}\,, \\
P(\nu_\mu \to \nu_\mu) &= 1 - \frac{c_1^2}{4} \sin^2 x_{21} - \frac{c_1^2}{2} \sin^2 x_{31} - \frac{(c_1 s_1)^2}{4} \sin^2 x_{41} \\
&\quad -\frac{1}{2} \sin^2 x_{32} - \frac{s_1^2}{4} \sin^2 x_{42} - \frac{s_1^2}{2} \sin^2 x_{43} \\
&= P(\nu_\tau \to \nu_\tau)\,, \\
P(\nu_\mu \to \nu_e) &= \frac{c_1^2}{2} \sin^2 x_{21} - \frac{(c_1 s_1)^2}{2} \sin^2 x_{41} + \frac{s_1^2}{2} \sin^2 x_{42} = P(\nu_\tau \to \nu_e)\,, \\
P(\nu_\mu \to \nu_\tau) &= -\frac{c_1^2}{4} \sin^2 x_{21} + \frac{c_1^2}{2} \sin^2 x_{31} - \frac{(c_1 s_1)^2}{4} \sin^2 x_{41} \\
&\quad +\frac{1}{2} \sin^2 x_{32} - \frac{s_1^2}{4} \sin^2 x_{42} + \frac{s_1^2}{2} \sin^2 x_{43}\,, \\
P(\nu_\mu \to \nu_{e_s}) &= (c_1 s_1)^2 \sin^2 x_{41} = P(\nu_\tau \to \nu_{e_s})\,, \\
P(\nu_e \to \nu_{e_s}) &= 2(c_1 s_1)^2 \sin^2 x_{41}\,, \\
P(\nu_{e_s} \to \nu_{e_s}) &= 1 - 4(c_1 s_1)^2 \sin^2 x_{41}\,.
\end{aligned}
\tag{23}
$$

Hence, the probability summation rules

$$
\begin{aligned}
P(\nu_e \to \nu_e) &+ P(\nu_e \to \nu_\mu) + P(\nu_e \to \nu_\tau) + P(\nu_e \to \nu_{e_s}) = 1\,, \\
P(\nu_\mu \to \nu_e) &+ P(\nu_\mu \to \nu_\mu) + P(\nu_\mu \to \nu_\tau) + P(\nu_\mu \to \nu_{e_s}) = 1\,, \\
P(\nu_\tau \to \nu_e) &+ P(\nu_\tau \to \nu_\mu) + P(\nu_\tau \to \nu_\tau) + P(\nu_\tau \to \nu_{e_s}) = 1\,, \\
P(\nu_{e_s} \to \nu_e) &+ P(\nu_{e_s} \to \nu_\mu) + P(\nu_{e_s} \to \nu_\tau) + P(\nu_{e_s} \to \nu_{e_s}) = 1
\end{aligned}
\tag{24}
$$

hold, as it should be, for two sterile neutrinos ν_{μ_s} and ν_{τ_s} are completely decoupled due to $s_2 = 0$ and $s_3 = 0$.

With the conjecture that $m_1^2 \simeq m_2^2$, implying $\Delta m_{41}^2 \simeq \Delta m_{42}^2$ and $\Delta m_{31}^2 \simeq \Delta m_{32}^2$, the first three Eqs. (23) can be rewritten approximately as

$$
\begin{aligned}
P(\nu_e \to \nu_e) &\simeq 1 - c_1^2 \sin^2 x_{21} - (1 + c_1^2) s_1^2 \sin^2 x_{42}\,, \\
P(\nu_\mu \to \nu_\mu) &\simeq 1 - \frac{1 + c_1^2}{2} \sin^2 x_{32} - \frac{c_1^2}{4} \sin^2 x_{21} \\
&\quad -\frac{(1 + c_1^2) s_1^2}{4} \sin^2 x_{42} - \frac{s_1^2}{2} \sin^2 x_{43}\,, \\
P(\nu_\mu \to \nu_e) &\simeq \frac{c_1^2}{2} \sin^2 x_{21} + \frac{s_1^4}{2} \sin^2 x_{42}\,.
\end{aligned}
\tag{25}
$$

If $|\Delta m_{21}^2| \ll |\Delta m_{42}^2|$ and [1,6]

$$
|\Delta m_{21}^2| = \Delta m_{\rm sol}^2 \sim (10^{-5} \;\; \text{or} \;\; 10^{-7} \;\; \text{or} \;\; 10^{-10})\ \text{eV}^2 \tag{26}
$$

(for LMA or LOW or VAC solar solution, respectively), then under the conditions of solar experiments the first Eq. (25) gives

$$P(\nu_e \to \nu_e)_{\rm sol} \simeq 1 - c_1^2 \sin^2(x_{21})_{\rm sol} - \frac{(1+c_1^2)s_1^2}{2}\,, \;\; c_1^2 = \sin^2 2\theta_{\rm sol} \lesssim 1\,. \quad (27)$$

If $|\Delta m_{21}^2| \ll |\Delta m_{32}^2| \ll |\Delta m_{42}^2|\,, |\Delta m_{43}^2|$ and [1]

$$|\Delta m_{32}^2| = \Delta m_{\rm atm}^2 \sim 3 \times 10^{-3}\ {\rm eV}^2\,, \quad (28)$$

then for atmospheric experiments the second Eq. (25) leads to

$$\begin{aligned} P(\nu_\mu \to \nu_\mu)_{\rm atm} &\simeq 1 - \frac{1+c_1^2}{2}\sin^2(x_{32})_{\rm atm} - \frac{(3+c_1^2)s_1^2}{8}\,, \\ \frac{1+c_1^2}{2} &= \sin^2 2\theta_{\rm atm} \lesssim 1\,. \end{aligned} \quad (29)$$

Eventually, if $|\Delta m_{21}^2| \ll |\Delta m_{42}^2|$ and [2]

$$|\Delta m_{42}^2| = \Delta m_{\rm LSND}^2 > 0.1\ {\rm eV}^2\,, \quad e.g.\ \sim 1\ {\rm eV}^2\,, \quad (30)$$

then in the LSND experiment the third Eq. (25) implies

$$\begin{aligned} P(\nu_\mu \to \nu_e)_{\rm LSND} &\simeq \frac{s_1^4}{2}\sin^2(x_{42})_{\rm LSND}\,, \\ \frac{s_1^4}{2} &= \sin^2 2\theta_{\rm LSND} > 8 \times 10^{-4}\,, \quad e.g. \sim 5 \times 10^{-3}\,. \end{aligned} \quad (31)$$

Thus,

$$\begin{aligned} &s_1^2 \sim 0.1\,, \quad c_1^2 \sim 0.9\,, \quad \frac{1+c_1^2}{2} \sim 0.95\,, \\ &\frac{(1+c_1^2)s_1^2}{2} \sim 0.095\,, \quad \frac{(3+c_1^2)s_1^2}{8} \sim 0.049\,, \end{aligned} \quad (32)$$

if the LNSD effect really exists and gets the amplitude $s_1^4/2 \sim 5 \times 10^{-3}$.

If the value $c_1^2 = \sin^2 2\theta_{\rm sol} \sim 0.66$ or 0.97 or 0.80 (corresponding to the recent estimation [6] for LMA or LOW or VAC solar solution, respectively) is *accepted*, then the amplitudes $\sin^2 2\theta_{\rm atm} = (1+c_1^2)/2 \sim 0.83$ or 0.99 or 0.90 and $\sin^2 2\theta_{\rm LSND} = s_1^4/2 \sim$ (5.8 or 0.045 or 2.0)$\times 10^{-2}$ are *predicted* for atmospheric and LSND experiments. The value $\sin^2 2\theta_{\rm LSND} = s_1^4/2 \sim 5 \times 10^{-3}$, written in Eq. (31) as an example, corresponds to older estimations for LOW solar solution: $c_1^2 = \sin^2 2\theta_{\rm sol} \sim 0.9$ [1].

Concluding, we can say that Eqs. (27), (29) and (31) are not inconsistent with solar, atmospheric and LSND experiments, respectively. Note that in

Eqs. (27) and (29) there are constant terms that modify moderately the usual two-flavor formulae (of course, the larger c_1^2, the smaller these terms are; in particular, for the LOW solution the constant terms $(1+c_1^2)s_1^2/2 \sim 0.030$ and $(3+c_1^2)s_1^2/8 \sim 0.015$ are minimal of those for three solar neutrino solutions considered here [6]). The above equations, valid for $s_2 = 0$ and $s_3 = 0$, follow from the first three oscillation formulae (23), if *either*

$$m_1^2 \simeq m_2^2 \ll m_3^2 \ll m_4^2 \tag{33}$$

with

$$\begin{aligned} &m_3^2 \ll 1 \text{ eV}^2\,, \quad m_4^2 \sim 1 \text{ eV}^2\,, \\ &\Delta m_{21}^2 \sim (10^{-5} - 10^{-10}) \text{ eV}^2 \ll \Delta m_{32}^2 \sim 10^{-3} \text{ eV}^2 \end{aligned} \tag{34}$$

or

$$m_4^2 \ll m_1^2 \simeq m_2^2 \simeq m_3^2 \tag{35}$$

with

$$\begin{aligned} &m_3^2 \sim 1 \text{ eV}^2\,, \quad m_4^2 \ll 1 \text{ eV}^2\,, \\ &\Delta m_{21}^2 \sim (10^{-5} - 10^{-10}) \text{ eV}^2 \ll \Delta m_{32}^2 \sim 10^{-3} \text{ eV}^2\,. \end{aligned} \tag{36}$$

Here, we must have $m_2^2 \ll m_3^2 \ll m_4^2 \sim 1 \text{ eV}^2$ or $m_4^2 \ll m_2^2 \simeq m_3^2 \sim 1 \text{ eV}^2$, since $\Delta m_{32}^2 \sim 10^{-3} \text{ eV}^2 \ll |\Delta m_{42}^2| \sim 1 \text{ eV}^2$. The second case $m_4^2 \ll m_1^2 \simeq m_2^2 \simeq m_3^2 \sim 1 \text{ eV}^2$, where the neutrino mass state ν_4 induced by the sterile neutrino ν_{e_s} gets a vanishing mass, seems to be more natural than the first case $m_1^2 \simeq m_2^2 \ll m_3^2 \ll m_4^2 \sim 1 \text{ eV}^2$, where such a state gains a considerable amount of Majorana righthanded mass "for nothing". (This is so, unless one believes in the liberal maxim "whatever is not forbidden is allowed": the Majorana righthanded mass is not forbidden by the electroweak SU(2)×U(1) symmetry, in contrast to Majorana lefthanded and Dirac masses requiring this symmetry to be broken, say, by a combined Higgs mechanism that becomes then the origin of these masses.) In the second case if, in addition, the masses m_5 and m_6 connected with two decoupled sterile neutrinos are vanishing, *the specific Majorana lefthanded mass matrix $M^{(\mathrm{L})}$ dominates over the whole neutrino mass matrix M.* Such a Majorana lefthanded dominance may be the reason, why neutrino mixing appears to be qualitatively different from the more familiar down-quark mixing implied by the interplay of up- and down-quark Dirac mass matrices. Note also that, when looking for too close analogies between textures of neutrinos and charged leptons, one fails to describe adequately the observed neutrino oscillation effects (including the possible LSND effect) [7].

If $s_2 = s_3 = 0$ and $m_4 = m_5 = m_6 = 0$, then writing $m_1 = m\,,\, m_2 = m + \delta m_{21}\,,\, m_3 = m + \delta m_{21} + \delta m_{32}$ we can present the neutrino mass matrix (11) in the form $M = M^{(0)} + \delta M$, where

$$M^{(0)} = m \begin{pmatrix} \frac{1}{2}(1+c_1^2) & \frac{1}{2\sqrt{2}}s_1^2 & -\frac{1}{2\sqrt{2}}s_1^2 & -\frac{1}{\sqrt{2}}c_1 s_1 & 0 & 0 \\ \frac{1}{2\sqrt{2}}s_1^2 & \frac{1}{4}(3+c_1^2) & \frac{1}{4}s_1^2 & \frac{1}{2}c_1 s_1 & 0 & 0 \\ -\frac{1}{2\sqrt{2}}s_1^2 & \frac{1}{4}s_1^2 & \frac{1}{4}(3+c_1^2) & -\frac{1}{2}c_1 s_1 & 0 & 0 \\ -\frac{1}{\sqrt{2}}c_1 s_1 & \frac{1}{2}c_1 s_1 & -\frac{1}{2}c_1 s_1 & s_1^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{37}$$

is slightly modified by

$$\delta M = \begin{pmatrix} \frac{1}{2}\,\delta m_{21} & \frac{1}{2\sqrt{2}}\,\delta m_{21} & -\frac{1}{2\sqrt{2}}\,\delta m_{21} & 0 & 0 & 0 \\ \frac{1}{2\sqrt{2}}\,\delta m_{21} & \frac{1}{4}(3\,\delta m_{21} + 2\,\delta m_{32}) & \frac{1}{4}(\,\delta m_{21} + 2\,\delta m_{32}) & 0 & 0 & 0 \\ -\frac{1}{2\sqrt{2}}\,\delta m_{21} & \frac{1}{4}(\,\delta m_{21} + 2\,\delta m_{32}) & \frac{1}{4}(3\,\delta m_{21} + 2\,\delta m_{32}) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{38}$$

In fact, $\delta m_{21} \sim m\,\delta m_{21}/\mathrm{eV} \simeq \Delta m^2_{21}/2\mathrm{eV} \sim 0.5 \times (10^{-5}$ or 10^{-7} or $10^{-10})$ eV and $\delta m_{32} \sim m\,\delta m_{32}/\mathrm{eV} \simeq \Delta m^2_{32}/2\mathrm{eV} \sim 1.5 \times 10^{-3}$ eV, while $m \sim 1$ eV. In the formal limit of $s_1 \to 0$, we obtain $M^{(0)}$ diagonal and degenerated in active and sterile neutrinos separately,

$$M^{(0)} \to \mathrm{diag}\,(m\,,\, m\,,\, m\,,\, 0\,,\, 0\,,\, 0)\,, \tag{39}$$

and so, from Eqs. (10) and (8) we infer that

$$U^\dagger\,\delta M U \to \mathrm{diag}\,(0\,,\, \delta m_{21}\,,\, \delta m_{21} + \delta m_{32}\,,\, 0\,,\, 0\,,\, 0) \tag{40}$$

as $U^\dagger M^{(0)} U \to \mathrm{diag}(m\,,\, m\,,\, m\,,\, 0\,,\, 0\,,\, 0)$. Note from Eq. (8) that (with $s_2 = s_3 = 0$) in this limit *we get bimaximal mixing matrix U in spite of the fact that in a good approximation the mass matrix $M \simeq M^{(0)}$ is diagonal* (here, of course, the degeneracy of $\lim_{s_1 \to 0} M^{(0)}$ in active neutrinos works).

In the approximation used before to derive Eqs. (27), (29) and (31) there are true also the relations

$$\begin{aligned} P(\nu_e \to \nu_e)_{\mathrm{sol}} &\simeq 1 - P(\nu_e \to \nu_\mu)_{\mathrm{sol}} - P(\nu_e \to \nu_\tau)_{\mathrm{sol}} - (c_1 s_1)^2, \quad (c_1 s_1)^2 \sim 0.09\,, \\ P(\nu_\mu \to \nu_\mu)_{\mathrm{atm}} &\simeq 1 - P(\nu_\mu \to \nu_\tau)_{\mathrm{atm}} - \frac{(1+c_1^2)s_1^2}{4}\,, \quad \frac{(1+c_1^2)s_1^2}{4} \sim 0.048\,, \end{aligned} \tag{41}$$

as well as

$$P(\nu_\mu \to \nu_e)_{\rm LSND} \simeq \frac{1}{2}\left(\frac{s_1}{c_1}\right)^2 P(\nu_\mu \to \nu_{e_s})_{\rm LSND}\,,\; \frac{1}{2}\left(\frac{s_1}{c_1}\right)^2 \sim 0.056\,. \quad (42)$$

Here, $s_1^4/2 \sim 5\times 10^{-3}$ as given in Eq. (31). The second relation (41) demonstrates a leading role of the appearance mode $\nu_\mu \to \nu_\tau$ in the disappearance process of atmospheric ν_μ's, while the relation (42) indicates a direct interplay of the appearance modes $\nu_\mu \to \nu_e$ and $\nu_\mu \to \nu_{e_s}$. In the case of the first relation (41), both appearance modes $\nu_e \to \nu_\mu$ and $\nu_e \to \nu_\tau$ contribute equally to the disappearance process of solar ν_e's, and the role of the appearance mode $\nu_e \to \nu_{e_s}$ (responsible for the constant term) is also considerable.

Finally, for the Chooz experiment [8], where $(x_{ji})_{\rm Chooz} \simeq (x_{ji})_{\rm atm}$ for any Δm^2_{ji}, the first Eq. (25) predicts

$$P(\bar{\nu}_e \to \bar{\nu}_e)_{\rm Chooz} \simeq P(\bar{\nu}_e \to \bar{\nu}_e)_{\rm atm} \simeq 1 - \frac{(1+c_1^2)s_1^2}{2}\,,\;\; \frac{(1+c_1^2)s_1^2}{2} \sim 0.095\,, \quad (43)$$

if there is the LSND effect with the amplitude $s_1^4/2 \sim 5\times 10^{-3}$ as written in Eq. (31). Here, $\sin^2(x_{42})_{\rm Chooz} = 1/2$ since $|(x_{42})_{\rm Chooz}| \simeq |(x_{42})_{\rm atm}| \gg (x_{32})_{\rm atm} \sim 1$ with $|\Delta m^2_{42}| \gg \Delta m^2_{32}$. In terms of the usual two–flavor formula, the negative result of Chooz experiment excludes the disappearance process of reactor $\bar{\nu}_e$'s for moving $(1+c_1^2)s_1^2 \equiv \sin^2 2\theta_{\rm Chooz} \gtrsim 0.1$, when the range of moving $|\Delta m^2_{42}| \equiv \Delta m^2_{\rm Chooz} \gtrsim 0.1$ eV^2 is considered ($\Delta m^2_{\rm Chooz} \gg \Delta m^2_{\rm atm} \sim 3\times 10^{-3}$ eV^2, implying $\sin^2(x_{42})_{\rm Chooz} = 1/2$). Thus, the nonobservation of Chooz effect for reactor $\bar{\nu}_e$'s in the above parameter ranges leads to $(1+c_1^2)s_1^2 \lesssim 0.1$ and hence, to the upper bound $\sin^2 2\theta_{\rm LSND} \equiv s_1^4/2 \lesssim 1.3\times 10^{-3}$, *when* $\Delta m^2_{\rm LSND} \equiv |\Delta m^2_{42}| \equiv \Delta m^2_{\rm Chooz} \gtrsim 0.1$ eV^2. It means that $\sin^2 2\theta_{\rm LSND}$, constrained by Chooz (in our four-neutrino texture), lies outside the parameter region suggested at 90% CL by the *existing* LSND data [2], *if* the KARMEN2 results [2] excluding a large part of this region are taken into account (in fact, in the corrected region $\sin^2 2\theta_{\rm LSND} \gtrsim 2\times 10^{-3}$). But, at 99% CL, this may be not true, allowing for $\Delta m^2_{\rm LSND} \gtrsim 1$ eV^2 (as, then, in the *existing* LSND parameter region $\sin^2 2\theta_{\rm LSND} \gtrsim 8\times 10^{-4}$). At any rate, among three solar neutrino solutions considered here [6], only the LOW solution is consistent with the Chooz bound [*cf.* the paragraph following Eq. (32)]. Also the value $\sin^2 2\theta_{\rm LSND} \sim 5\times 10^{-3}$, written in Eq. (31) as an example, is eliminated by this bound. Unfortunately, the Chooz-allowed, LOW-induced value $\sin^2 2\theta_{\rm LSND} \sim 4.5\times 10^{-4}$ (in contrast to $\sin^2 2\theta_{\rm LSND} \sim 5\times 10^{-3}$) is situated outside the parameter range implied by the *existing* LSND data [2] (even at 99% CL).

Of course, the existence of Chooz bound for the LSND effect and of the relation of this bound to solar neutrino solutions is caused by the *correlations* between different neutrino oscillation modes connected through the parameter s_1^2 appearing in our four-neutrino texture [*cf.* Eqs. (25)]. In fact, the identities

$$\frac{s_1^4}{2} = \frac{(1-c_1^2)^2}{2}\,, \quad (1+c_1^2)s_1^2 = 1 - c_1^4 \quad \text{and} \quad c_1^2 + \frac{(1+c_1^2)s_1^2}{2} + \frac{s_1^4}{2} = 1$$

can be translated into the correlations

$$\sin^2 2\theta_{\rm LSND} = \frac{(1-\sin^2 2\theta_{\rm sol})^2}{2}\,, \;\; \sin^2 2\theta_{\rm Chooz} = 1 - \sin^4 2\theta_{\rm sol} \tag{44}$$

and the sum rule

$$\sin^2 2\theta_{\rm sol} + \frac{\sin^2 2\theta_{\rm Chooz}}{2} + \sin^2 2\theta_{\rm LSND} = 1 \tag{45}$$

for three neutrino oscillation amplitudes (each in the reasonable two-flavor approximation). The sum rule (45) follows also from the first probability summation relation (24) considered with the assumption of $m_1^2 \simeq m_2^2$ for solar ν_e's (when $|(x_{42})_{\rm sol}| \gg (x_{21})_{\rm sol} \simeq \pi/2$).

We can see that, *when* accepting the present Chooz results, we stand with our four-neutrino texture before the alternative: *either* there is no LSND effect at all (then $\sin^2 2\theta_{\rm LSND} \equiv s_1^4/2 = 0$ and we are left with the three-neutrino bimaximal texture [3,5]), *or* this effect exists all right, but at a point in parameter space, where the oscillation amplitude $\sin^2 2\theta_{\rm LSND}$ is shifted (*versus* the existing LSND data) towards a smaller value $\sin^2 2\theta_{\rm LSND} \equiv s_1^4/2 \lesssim 1.3 \times 10^{-3}$ (though >0). Note that, if $\sin^2 2\theta_{\rm LSND} = s_1^4/2$ was at the Chooz bound value 1.3×10^{-3}, then $\sin^2 2\theta_{\rm sol} = c_1^2$ would be at 0.95 and s_1^2 at 0.051. If, rather, $\sin^2 2\theta_{\rm LSND} = s_1^4/2$ was at the value 8×10^{-4} equal to its *existing* LSND lower limit at 99% CL, then $\sin^2 2\theta_{\rm sol} = c_1^2$ would be at 0.96 and s_1^2 at 0.04. Of course, we should keep in mind the fact that the present estimations for $\sin^2 2\theta_{\rm sol}$ (and even more for $\sin^2 2\theta_{\rm LSND}$) are preliminary and, in fact, very fragile.

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