NUCLEAR AND ATOMIC STATES OF Σ HYPERONS AND THE ΣN INTERACTION

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It is shown that among four models of the Nijmegen baryon–baryon interaction, only model F is consistent both with the analysis of Σ^- atoms and (K^-, π) reactions. Simple estimates of the strong-interaction shifts and widths of the lowest observed levels of Σ^- atoms are applied for model F with satisfying results. It is concluded that model F is favored as a realistic representation of the ΣN interaction.

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1. Introduction

Observed properties of Σ^- atoms, *i.e.*, strong-interaction shifts ε and widths Γ of the lowest observed levels, provide us with valuable information on the strong interaction between Σ^- and the nucleons, as well as on the nucleon density distribution in the nucleus of the Σ^- atom. In a recent comprehensive phenomenological analysis of the existing Σ^- data Batty, Friedman, and Gal [1] found the following striking property of the single particle (s.p.) strong-interaction potential of Σ^-: it is repulsive inside the nucleus and attractive outside. The need for the repulsion arose when new data were included into the analysis, namely the results of Powers *at al.* [2], especially their precise data on the Σ^-Pb atom.

This behavior of Σ^- s.p. potential found in the analysis of Σ^- atoms is consistent with the analysis of the pion spectra[1] measured in (K^-, π) reactions, which suggests a Σ s.p. potential repulsive inside nuclei [3, 4] (with a substantial positive Lane potential V_τ [5]).

In the present paper we consider the Nijmegen models of the baryon-baryon interaction: models D [6], F [7], Soft-Core (SC) model [8], and the New Soft-Core (NSC) model [9], and want to find out whether any of them leads to the observed properties of Σ^- atoms. In our analysis, we apply the effective $\Sigma^- N$ interaction in nuclear matter, $\mathcal{K}$, obtained within the Low Order Brueckner (LOB) theory with the above interaction models by Yamamoto, Motoba, Himeno, Ikeda, and Nagata [10], and by Rijken, Stoks, and Yamamoto [9] (the so called YNG interactions).

The single-particle (s.p.) potential V of the Σ^- moving with momentum $\hbar k_\Sigma$ in nuclear matter with nucleon density ρ and neutron excess $\alpha = (N - Z)/A$ has the form [5]:

$$V(k_\Sigma, \rho, \alpha) = V_0(k_\Sigma, \rho) + \frac{1}{2}\alpha V_\tau(k_\Sigma, \rho) \, . \tag{1}$$

Here, we ignore terms connected with spin excess, considered in [11], which are usually negligibly small.

Expressions for the isoscalar potential V_0 and for the Lane potential V_τ in terms of the effective ΣN interaction $\mathcal{K}$ are given in [5]. When we apply the expression for V_0 to the YNG effective ΣN interactions, we get the results shown in Fig. 1. (Because of the relatively small magnitude of Σ momenta in Σ^- atoms, the value $k_\Sigma = 0$ is used in Fig. 1.) We see that only model F of the Nijmegen baryon–baryon interaction leads to repulsive V_0 at nucleon densities $\rho \gtrsim 0.05$ fm^{-3} encountered inside nuclei, and to attractive V_0 at lower densities encountered in the nuclear surface. All the remaining models lead to attractive V_0 at all densities. This means that only model F leads to the Σ s.p. potential which is in qualitative agreement with the phenomenological analysis [1] of Σ^- atoms[2] and also with the pion spectra measured in the (K^-, π) reactions.

The important question is whether model F can explain quantitatively the measured properties of Σ^- atoms. It is the purpose of the present paper to show that this is indeed the case. We start with model F of the hyperon–nucleon interaction, estimate the energy shifts ε and widths Γ of the Σ^- atomic levels, and show that they are reasonably close to experimental data.

[1] The shift of the pion spectrum toward higher Σ energies, compared with the quasi-free model, obviously suggests repulsion.

[2] The behavior of V_τ is irrelevant here, because the analysis in [1] was applied also to the Σ^- atoms with $N = Z$ in which V_τ plays a negligible role.

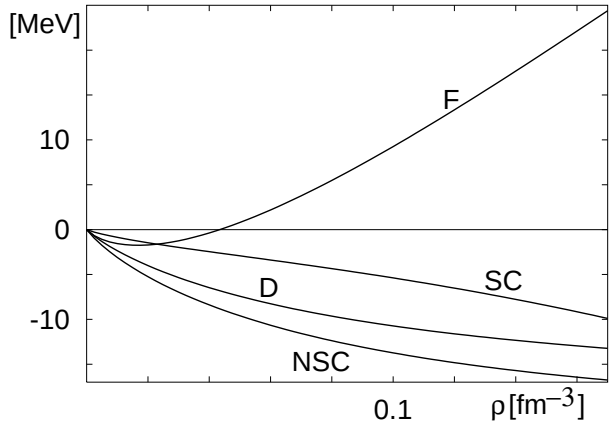

Fig. 1. The isoscalar Σ potential in nuclear matter V_0 as function of nucleon density ρ for $k_\Sigma = 0$.

The paper is organized as follows. Our computational procedure is presented in Sec. 2. In Sec. 2.1 the expression for Γ is derived. In Sec. 2.2 our approximate expression for ε is presented. Our choice of the proton and the neutron densities is discussed in Sec. 2.3. Our results are presented, compared with experimental data, and discussed in Sec. 3.

2. The computational procedure

In our calculations, we apply the local density approximation: the Σ^- atom is treated at each point as Σ^- moving in nuclear matter with the local nuclear density of the Σ^- atom. Since the strong interaction of Σ^- occurs in the tail of the nuclear density distribution (where the derivative of the density tends to zero), gradient corrections should not be important.

2.1. Expression for Γ

Here we follow the procedure applied in [12] in explaining the early data on Σ atomic widths. Our expression (2) for Γ in terms of the $\Sigma\Lambda$ conversion cross section was used before in [13]. In the context of Σ nuclear interaction it was first discussed by Gal and Dover [14].

We consider a Σ^- hyperon moving with momentum $\hbar k_\Sigma$ in nuclear matter with proton density ρ_p. The probability in unit time, w, that the hyperon collides with a proton and undergoes the conversion $\Sigma^- p \to \Lambda n$ is $w = \rho_p v \sigma$, where σ is the total cross section for the conversion process and v is the $\Sigma^- p$ relative velocity. The Σ^- life time $\tau = 1/w$ is connected with Γ through

the relation $\tau\Gamma = \hbar$. Thus we get:

$$\Gamma = \Gamma(\rho_p, k_\Sigma) = \rho_p \hbar \langle v\sigma \rangle = \rho_p \frac{\hbar^2}{\mu_{\Sigma p}} \langle k_{\Sigma p}\sigma \rangle \,, \tag{2}$$

where $\langle\ \rangle$ denotes averaging over proton momenta, $\hbar k_{\Sigma p}$ is the $\Sigma^- p$ relative momentum, and $\mu_{\Sigma p}$ is the $\Sigma^- p$ reduced mass.

A justification of semi-classical expression (2) may be found in [12] and [13], where a more precise expression for Γ was obtained by applying the optical theorem to the Brueckner reaction matrix $\mathcal{K}$. This more precise expression differs from expression (2) by containing corrections caused by the exclusion principle and dispersive effects. The crucial point is that at the very small nucleon densities relevant in Σ^- atoms, both these corrections are negligibly small.

When applying expression (2) to Σ^- atoms, we insert for ρ_p the average proton density $\bar{\rho}_p$ in Σ^- atom:

$$\bar{\rho}_p = \int d\boldsymbol{r}\, \rho_p(r) |\, \Psi_\Sigma(\boldsymbol{r})|^2 \,, \tag{3}$$

where $\Psi_\Sigma(\boldsymbol{r})$ is the wave function of Σ^- and $\rho_p(r)$ is the proton density distribution in the Σ^- atom.

Similarly, we insert for k_Σ in (2) the average Σ momentum of Σ^- in the rest frame of nuclear medium, $\bar{k}_\Sigma$, connected with the average relative Σ^--nucleus momentum $k_{\Sigma A}$ by: $\bar{k}_\Sigma = M_\Sigma \bar{k}_{\Sigma A}/\mu_{\Sigma A}$, where $\mu_{\Sigma A} = M_\Sigma M_A/(M_\Sigma + M_A)$ is the Σ^--nucleus (of mass M_A) reduced mass. We determine $\bar{k}_{\Sigma A}$ from:

$$\frac{\hbar^2 \bar{k}^2_{\Sigma A}}{2\mu_{\Sigma A}} = \langle\, \Psi_\Sigma | T |\, \Psi_\Sigma \rangle \,, \tag{4}$$

where T is the operator of the kinetic energy of the relative Σ^--nucleus motion.

In the observed states of Σ^- atoms the Σ^- hyperon moves on a circular orbit. We describe the circular orbit ($l = n-1$, where l and n are the orbital and principal quantum numbers) around a nucleus (A nucleons, Z protons) by the hydrogen wave function:

$$\begin{aligned} \Psi_\Sigma(\mathbf{r}) &= \frac{R(r)}{r} Y_{lm}(\hat{r}) \,, \\ R(r) &= \left(\frac{2nr}{a_n}\right)^n \frac{\exp(-nr/a_n)}{[(2n-1)!a_n]^{1/2}} \,, \end{aligned} \tag{5}$$

where $a_n = (n^2/Ze^2)\hbar^2/\mu_{\Sigma A}$ is the radius of the orbit.

With form (5) of Ψ_Σ the integration in expression (3) for $\bar{\rho}_p$ for a general form of $\rho_p(r)$ has to be performed numerically. On the other hand, expression (4) does not require any computation, since the virial theorem leads to the known result: $\langle \Psi_\Sigma | T | \Psi_\Sigma \rangle = Ze^2/2a_n$.

For the total $\Sigma\Lambda$ conversion cross section σ we shall use two parametrizations. The first one, adjusted by Gal, Toker, and Alexander [15] to the Σ^- low energy regime up to 300 MeV/c in the laboratory frame, has the form

$$\frac{v}{c}\,\sigma = \left(1 + 13\frac{v}{c}\right)^{-1} 5.1\ \mathrm{fm}^2\,. \tag{6}$$

Expression (6) gives for $(v/c)\sigma$ results very close to the results obtained with model F (see [7]). Consequently, using expression (2) with $v\sigma$ given by expression (6) is equivalent to (and much simpler than) calculating Γ starting with model F of the hyperon–nucleon interaction[3].

The second one, suggested by Oset *et al.* [16] and adjusted to the Σ^- low energy regime up to 160 MeV/c, has the form:

$$\frac{v}{c}\,\sigma \simeq 1.7\ \mathrm{fm}^2\,. \tag{7}$$

This form follows from the assumption that the transition matrix for the $\Sigma^- p \to \Lambda n$ process is constant, and only the phase space factor introduces the energy dependence of σ. The effect of this factor on $(v/c)\sigma$ is negligible in the low energy range relevant in Σ^- atoms and is not indicated in Eq. (7).

The two parametrizations differ. This is possible because the experimental points to which both of them are adjusted have big error bars. Furthermore, we need in expression (2) for Γ the cross section σ at the average Σ^- momentum $\bar{p}_\Sigma = \hbar\bar{k}_\Sigma$ which varies from 13 MeV/c for the upper level in ^{12}C to 80 MeV/c for the lower level in ^{208}Pb. Now, the experimental points start at 110 MeV/c, and thus we use the two parametrizations to extrapolate the values of σ to Σ^- momenta smaller than 110 MeV/c. This leads to a big uncertainty in σ and consequently in our calculated values of Γ.

Whereas the extrapolation of σ to small Σ momenta with parametrization (6) is consistent with model F, this is not the case with parametrization (7). Thus the results for Γ obtained with parametrization (7) go beyond the discussion of model F. Nevertheless we include this parametrization in our estimate of Γ to indicate that a more precise measurement of the $\Sigma\Lambda$ conversion cross section is essential for discussing widths of Σ^- atoms and ΣN interaction.

[3] Notice that at very small nucleon densities the K matrix is identical with the free scattering matrix whose imaginary part — via the optical theorem — is proportional to $v\sigma$.

2.2. Estimate of ε

Let us consider a Σ^- atom with proton and neutron density distributions $\rho_p(r)$ and $\rho_n(r)$ respectively. At any distance r, we treat the system as nuclear matter with total nucleon density $\rho(r) = \rho_p(r) + \rho_n(r)$ and with neutron excess $\alpha(r) = [\rho_n(r) - \rho_p(r)]/\rho(r)$, and with a Σ^- hyperon with momentum $k_\Sigma \approx 0$. [The last approximation is justified by the very weak dependence of the Σ s.p. potential in nuclear matter on k_Σ found in [5], and by the relatively small magnitude of Σ momenta in Σ^- atoms (see values of $\bar{k}_\Sigma$ presented in Sec. 3).] To get the value of the Σ^- s.p. potential in Σ^- atom at a distance r, we calculate $V_{0,\tau}(k_\Sigma, \rho(r))$ at $k_\Sigma = 0$ by applying the expressions given in [5] with the YNG effective interactions of [10] (and [9]). In this way we obtain the isoscalar and the Lane potentials in Σ^- atom at a distance r,

$$V_0(r) = V_0(k_\Sigma = 0, \rho(r)), \quad V_\tau(r) = V_\tau(k_\Sigma = 0, \rho(r)), \tag{8}$$

and the total nuclear s.p. Σ^- potential,

$$V(r) = V_0(r) + \frac{1}{2}\alpha(r)V_\tau(r)\,. \tag{9}$$

With this $V(r)$, we estimate ε in the first order perturbation approximation:

$$\varepsilon = -\langle \Psi_\Sigma | V | \Psi_\Sigma \rangle = -\int_0^\infty dr V(r) R(r)^2\,. \tag{10}$$

Notice the negative sign which makes ε positive for downward shift of the level. The measured energy of γ transition to the level is then increased by ε. Thus ε defined in (10) is equal to this increase in γ energy.

2.3. Proton and neutron density distributions

The proton and neutron density distributions, $\rho_p(r)$ and $\rho_n(r)$ used in our calculation have been obtained from the Isomorphic Shell Model (ISM).

The ISM model differs from the conventional shell model by the state dependence of the s.p. Hamiltonian: the s.p. potential is different in each shell — in each of them it is assumed to have the shape of a harmonic oscillator [17]. The way of determining the parameters of these harmonic oscillator potentials is explained in [18] (see also [19] and references therein). The important point is that the ISM model reproduces reasonably well the total nuclear binding, and — what is particularly important in our calculations — the proton and neutron separation energies and the empirical charge distributions.

The final version of the ISM neutron densities in the case of ^{184}W and ^{208}Pb are not available, and in these two cases we assumed that the neutron density has the same shape as the proton density, *i.e.*, we put $\rho_n(r) = (N/Z)\rho_p(r)$. We checked that this procedure when applied in cases of all other nuclei considered here would have only a very small effect on the calculated values of ε.

3. Results and discussion

Our results are presented in Table I together with the existing experimental data. The two calculated values of Γ were obtained by applying the two parametrizations of the $\Sigma\Lambda$ conversion cross section σ: the greater value was obtained with parametrization (6) of Gal *et al.* [15], and the lower value with parametrization (7) of Oset *et al.* [16].

TABLE I

Energy shifts ε of the lower level calculated with model F of the ΣN interaction, and widths Γ and Γ^u calculated respectively for the lower and upper level of the indicated Σ^- atoms together with the corresponding experimental results. All energies are in eV.

Nucl.	$n+1\to n$	ε	$\varepsilon_{\rm exp}$	Γ	$\Gamma_{\rm exp}$	Γ^u	$\Gamma^u_{\rm exp}$
^{12}C	$4\to3$	8.65	—	17–43	—	0.007–0.019	0.031 ± 0.012[a]
^{16}O	$4\to3$	52.0	320 ± 230[b]	178–425	—	0.14–0.36	1.0 ± 0.7[b]
^{24}Mg	$5\to4$	32.1	25 ± 40[b]	28–65	<70[b]	0.05–0.13	0.11 ± 0.09[b]
^{27}Al	$5\to4$	66.7	68 ± 28[b]	65–149	43 ± 75[b]	0.15–0.36	0.24 ± 0.06[b]
^{28}Si	$5\to4$	138.6	159 ± 36[b]	136–306	220 ± 110[b]	0.37–0.88	0.41 ± 0.10[b]
^{32}S	$5\to4$	440.6	360 ± 220[b]	501–1086	870 ± 700[b]	1.87–4.30	1.5 ± 0.8[b]
^{40}Ca	$6\to5$	26.6	—	24–52	—	0.08–0.18	0.41 ± 0.22[a]
^{48}Ti	$6\to5$	47.2	—	74–155	—	0.31–0.68	0.65 ± 0.42[a]
^{138}Ba	$9\to8$	33.3	—	55–99	—	0.88–1.65	2.9 ± 3.5[a]
^{184}W	$10\to9$	126.2	214 ± 60[c]	121–203	18 ± 149[c]	2.66–4.67	2 ± 2[c]
^{208}Pb	$10\to9$	457.0	422 ± 56[c]	539–903	430 ± 160[c]	15.9–26.7	17 ± 3[c]

[a] Data taken from Ref. [20].
[b] Data taken from Ref. [21].
[c] Data taken from Ref. [2].

Now let us discuss the accuracy of our results.

Our estimate of Γ is based on the assumption that Γ is equal to the width of Σ moving with an average momentum in nuclear matter of an average density. This procedure was applied successfully in nuclear physics

(in estimating the width of nucleon hole states) a long time ago by Köhler [22]. We introduce an additional approximation by calculating the average momentum and density with the help of the hydrogenlike function. In most of the atoms, the calculated widths Γ and Γ^u consistent with model F, *i.e.*, the greater values listed in Table I, are in reasonable agreement with experimental data. However in some cases, including the case of Pb, the agreement is better for our results obtained with parametrization (7) of the conversion cross section σ. A definite conclusion is difficult because of the insufficient knowledge of the cross section σ and the poor accuracy of the measured widths, $\Gamma_{\rm exp}$ and $\Gamma^u_{\rm exp}$.

In our estimate of ε, we apply the first order perturbation approximation in the nuclear s.p. Σ^- potential V, and the problem arises whether perturbative treatment of the nuclear interaction in Σ^- atoms is justified. This problem in other hadronic atoms has been investigated in a number of publications (see, *e.g.*, [1, 23–25]). It appears that the Zel'dovich effect [26, 27] plays a crucial role here. When V alone is sufficiently attractive to provide binding, then just at the onset of such binding (when the nuclear binding energy is comparable to to the atomic binding energy), a drastic change of the atomic spectrum occurs, and perturbation theory no longer holds. Now, our V derived from model F of the ΣN interaction is repulsive inside of the nucleus and has only a relatively shallow attractive pocket in the nuclear periphery (see Fig. 2). This interaction is too weak to provide binding, and the argument against the applicability of perturbation theory does not apply.

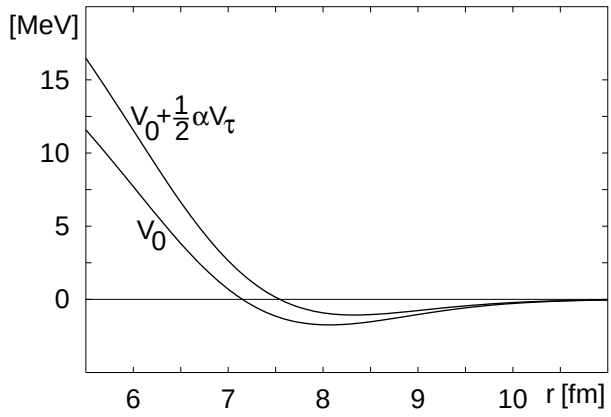

Fig. 2. Potential V_{Σ^-} in ^{208}Pb.

To get an idea about the accuracy of the first order perturbation approximation in our estimate of ε, we have applied our approximation in the case of the Σ^-Pb atom, and the best fit Σ^- nuclear potential obtained in [1]

with the macroscopic (MAC) parametrization of nuclear densities. In this case our result for ε is smaller[4] than the exact result obtained in [1] (by solving the wave equation with the Σ^- nuclear potential) and the error is equal to 22% of the exact result. This error includes also the effect of the finite size of the nuclear charge distribution, neglected in our estimate.

Even if we take into account the possibility of an error in ε of an order of 20%, our results collected in Table I appear reasonably close to the experimental results. Thus our simple estimate of ε and Γ demonstrates the consistency of model F with Σ^- atomic data.

The potential $V_{\Sigma^-}(r)$ in the Pb atom with and without the Lane term is shown in Fig. 2. If we calculated ε with V_0 only, we would get for ε the value of 689 eV, much bigger than the experimental result. This demonstrates that a substantial Lane potential is essential in the description of Σ^- atoms.

The YNG ΣN interaction was applied before in the theory of Σ^- atoms by Yamada and Yamamoto [28] in an attempt to explain the early Σ^- atomic data. These authors calculated the energy shift ε and the width Γ of the lowest state in ^{16}O, ^{24}Mg, ^{28}Si and ^{32}S Σ^- atoms. They went beyond our local density approximation and solved the Schrödinger equation for Σ^- with a complex s.p. potential obtained with the YNG interaction with the help of the Hartree–Fock nuclear wave functions calculated with the Skyrme interaction. Their F model results for both ε and Γ are much bigger than our results, and clearly disagree with the experimental data. The reason might be the nuclear wave functions used in [28] which do not reproduce the empirical charge distribution.

We conclude that among the Nijmegen baryon–baryon interactions, model F is best suited to represent the ΣN interaction both in Σ hypernuclear states and in Σ^- atoms.

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[4] In the case of point Coulomb potential and pure real strong interaction, our expression (10), would give a lower bound for ε.

REFERENCES

[1] C.J. Batty, E. Friedman, A. Gal, *Phys. Rep.* **287**, 385 (1997).

[2] R.J. Powers *et al.*, *Phys. Rev.* **C47**, 1263 (1993).

[3] J. Dąbrowski, J. Rożynek, *Acta Phys. Pol.* **B29**, 2147 (1998).

[4] Y. Shimizu, Ph.D. thesis, University of Tokyo, 1996 (unpublished).

[5] J. Dąbrowski, *Phys. Rev.* **C60**, 025205 (1999).

[6] N.M. Nagels, T.A. Rijken, J.J. de Swart, *Phys. Rev.* **D12**, 744 (1975); **D15**, 2547 (1977).

[7] N.M. Nagels, T.A. Rijken, J.J. de Swart, *Phys. Rev.* **D20**, 1663 (1979).

[8] P.M.M. Maessen, T.A. Rijken, J.J. de Swart, *Phys. Rev.* **C40**, 2226 (1989); *Nucl. Phys.* **A547**, 245c (1992).

[9] T.A. Rijken, V.G.J. Stoks, Y. Yamamoto, *Phys. Rev.* **C59**, 21 (1999).

[10] Y. Yamamoto, T. Motoba, H. Himeno, K. Ikeda, S. Nagata, *Progr. Theor. Phys. Suppl.* **117**, 361 (1994).

[11] J. Dąbrowski, *Acta Phys. Pol.* **B30**, 2783 (1999).

[12] J. Dąbrowski, J. Rożynek, *Acta Phys. Pol.* **B14** 439 (1983).

[13] J. Dąbrowski, J. Rożynek, *Phys. Rev.* **C23**, 1706 (1981).

[14] A. Gal, C.B. Dover, *Phys. Rev. Lett.* **44**, 379 (1980).

[15] A. Gal, G. Toker, Y. Alexander, *Ann. Phys.* **137**, 341 (1981).

[16] E. Oset, L.L. Salcedo, R. Brockmann, *Phys. Rep.* **188**, 79 (1990).

[17] G.S. Anagnostatos, *Canad. J. Phys.* **70**, 361 (1992).

[18] G.S. Anagnostatos, *Int. J. Theor. Phys.* **24**, 579 (1985).

[19] G.S. Anagnostatos, P. Ginis, J. Giapitzakis, *Phys. Rev.* **C58**, 3305 (1998).

[20] G. Backenstoss, T. Bunacin, J. Egger, H. Koch, A. Schwitter, L. Tauscher, *Z. Phys.* **A273**, 137 (1975).

[21] C.J. Batty, S.F. Biagi, M. Blecher, S.D. Hoath, R.A.J. Riddle, B.L. Roberts, J.D. Davies, G.J. Pyle, G.T.A. Squier, D.M. Asbury, *Phys. Lett.* **74B**, 27 (1978).

[22] H.S. Köhler, *Nucl. Phys.* **88**, 529 (1966).

[23] M. Krell, *Phys. Rev. Lett.* **26**, 584 (1971).

[24] C.J. Batty, E. Friedman, A. Gal, G. Kalbermann, *Nucl. Phys.* **A535**, 548 (1991).

[25] A. Gal, E. Friedman, C.J. Batty, *Nucl. Phys.* **A606**, 283 (1996).

[26] Ya.B. Zel'dovich, *Fiz. Tverd. Tela* **1**, 1637 (1959) [English translation: *Sov. Phys. Solid State* **1**, 1497 (1960).

[27] B.M. Karnakov, A.E. Kudryavtsev, V.D. Mur, V.S. Popov, *Nuovo Cim.* **98B**, 63 (1987).

[28] T. Yamada, Y. Yamamoto, *Progr. Theor. Phys. Suppl.* **117**, 241 (1994).