THE EFFECTIVE MASS USING SKYRME INTERACTIONS

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The effective mass concept was introduced for small non-local potentials. For Skyrme interactions the effective mass depends on the density of the system. We parameterised the density by a generalised Fermi distribution. The parameters of the distribution are fitted with the charge root mean square radii of spherical nuclei. To test the parameters of the velocity dependent term of the interaction at high density or temperature, we studied the behaviour of the effective mass with density and temperature. The behaviour of the Landau parameter $F1$ was also discussed. We applied this study to most of the well known Skyrme forces. We recommended SKM* and SKS4 as good examples of Skyrme forces as they reveal small non-locality effects and well behaved functions at high density and temperature.

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1. Introduction

In non-relativistic nuclear physics, the nuclear potential is non-local (velocity dependent) and energy dependent. For small non-local effects one might express the velocity dependent potential in a power series in the momentum. If the quadratic approximation for the momentum is considered, the Schrodinger equation for non-local potential has the same form as the local one except for the nucleon mass is replaced by a mass like quantity $m^*(r)$ [1–3]. This quantity is referred to as the effective mass. Thus, the effective mass idea transforms the non-local potential to a local one. Solving Schrodinger equation, the equivalent local potential will yield the same scattering cross section and the single particle energies as the non-local one. The behaviour of the effective mass in space (r) characterises the locality of the potential [3]. Also, the empirical value of the effective mass in the

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central region of the nuclear interior characterises the energy dependence of the potential [4].

Applying Fermi liquid theory to nuclear matter it was shown that [5]: the key quantity to study the thermal properties of a nuclear system is the level density parameter which is directly proportional to the effective mass. So, the thermal properties of a nuclear system are reflected in its effective mass.

The Landau parameters for Fermi liquids represent the effective particle–hole interaction at the Fermi surface. It is instructive to relate these Landau parameters, which are important for the phenomenological description of nuclear properties, to the underlying nucleon–nucleon interaction. Within the framework of the Landau Fermi liquid theory, the Landau parameter $F1$ is directly related to the effective mass. This relation enables us to study the behaviour of $F1$ in space and also its temperature dependence.

The parameters of Skyrme interaction are fitted with the ground state properties of finite nuclei. It is important to test these parameters at higher densities and temperatures. This can be done by studying the behaviour of the effective mass with density and temperature. It is also important to study the locality of the potential. This can be done by studying the behaviour of the effective mass in space.

In the present work we study the dependence of the effective mass (and also the Landau parameter $F1$) on temperature and space using Skyrme interactions. A brief review of the formalism is presented in Section 2. The results are shown and discussed in Section 3. Section 4 is devoted to the summary and conclusions.

2. Formalism

2.1. The effective mass using Skyrme interaction

Skyrme interaction is one of the most convenient and popular interactions. It consists of two-body and three-body zero-range forces. The three-body terms simulate the density dependence and the velocity-dependent two-body terms simulate the finite-range expansion of the force. Its zero-range nature makes the numerical calculations easy and fast. Vautherin and Brink [3] did the Hartree–Fock (HF) calculations with the Skyrme interaction first. They fitted two sets of parameters (SI and SII) that differ mainly in the empirical value of the fitted effective mass, *i.e.* in the strength of the velocity dependent term. Later four more sets of parameters (SIII–SVI) were fitted by Beiner *et al.* [4]. These sets of parameters differ also only in the effective mass value. Rayet *et al.* [12] parameterised Skyrme interaction (RATP) to take into account the available information on neutron-rich nuclei (to be suitable for nuclear and neutron matter). All the above sets of the

Skyrme interactions fit the ground state properties almost equally well for spherical nuclei ranging from ^{16}O to ^{208}Pb. However, there are well-known setbacks namely: high value of the nuclear matter incompressibility [6], too high fission barrier [7] and spin instability [8]. Two sets of parameters (SKM and SKM*) were fitted by Brack *et al.* [7] to give an accepted value of the nuclear matter incompressibility and fitted the fission barrier heights. The Skyrme interaction they used is:

$$\begin{aligned} H(r) &= \frac{\hbar^2}{2m}\tau + \frac{1}{4}t_0\left[(2+x_0)\rho^2 - (2x_0+1)\left(\rho_n^2+\rho_p^2\right)\right] \\ &+\frac{1}{24}t_3\rho^\alpha\left[(2+x_3)\rho^2 - (1+2x_3)\left(\rho_n^2+\rho_p^2\right)\right] \\ &+\frac{1}{8}\left[t_1(2+x_1)+t_2(2+x_2)\right]\tau\rho \\ &+\frac{1}{8}\left[t_2(2x_2+1)-t_1(2x_1+1)\right](\tau_n\rho_n+\tau_p\rho_p) \\ &+\frac{1}{32}\left[3t_1(2+x_1)-t_2(2+x_2)\right](\nabla\rho)^2 \\ &-\frac{1}{32}\left[3t_1(1+2x_1)+t_2(2+x_2)\right]\left[(\nabla\rho_n)^2+\nabla\rho_p)^2\right] \\ &+\frac{1}{2}w_0\left[\boldsymbol{j}\cdot\nabla\rho+\boldsymbol{j_n}\cdot\nabla\rho_n+\boldsymbol{j_p}\cdot\nabla\rho_p\right]\,. \end{aligned} \tag{1}$$

The nucleon effective mass is given by [7]

$$\frac{h^2}{2m_q^*} = \frac{\partial H(r)}{\partial\tau_q(r)}\,, \tag{2}$$

where q refers to a neutron or a proton. This yields:

$$\frac{m}{m_q^*} = 1 + \frac{2m}{\hbar^2}\frac{1}{8}\left[\rho C_1 + \rho_q C_2\right]\,, \tag{3}$$

where

$$C_1 = t_1(2+x_1)+t_2(2+x_2)$$

and

$$C_2 = t_2(1+2x_2)-t_1(1+2x_1)\,.$$

The asymmetry parameter x is defined by

$$x = \frac{\rho_n-\rho_p}{\rho}\,, \tag{4}$$

where $\rho=\rho_n+\rho_p$.

Using equation (4), we can write

$$\rho_n = \tfrac{1}{2}\rho(1+x)$$

and

$$\rho_p = \tfrac{1}{2}\rho(1-x)\,. \tag{5}$$

The particle densities are related to their numbers by

$$N = \frac{4}{3}\pi R_n^3\rho_n\,, \quad Z = \frac{4}{3}\pi R_p^3\rho_p \quad \text{and} \quad A = \frac{4}{3}\pi R^3\rho\,. \tag{6}$$

If we neglect the neutron skin, which is basic assumption of the liquid drop model [9], we can write the asymmetry parameter x as: $x = (N-Z)/A$. This is the neutron excess parameter. The nucleon effective mass is then:

$$\frac{m}{m_q^*} = 1 + \frac{2m}{h^2}F_1(\rho)\,, \tag{7}$$

where $F_1(\rho) = \rho[c1 + \frac{1}{2}c2(1 \pm x)]$. The $(+)$ and $(-)$ sign corresponds to the neutron and proton effective mass, respectively.

Liu *et al.* [10] proposed an extended Skyrme–Landau force (SL1) which includes velocity-dependent three-body forces and a tensor force to overcome the problems of the high nuclear matter incompressibility and spin instability. The contribution of these extra terms to the effective mass is $F_2(\rho)$. Thus we get

$$\frac{m}{m_q^*} = 1 + \frac{2m}{h^2}\left[F_1(\rho) + F_2(\rho)\right]\,, \tag{8}$$

where

$$\begin{aligned} F_2(\rho) \;=\; & \frac{1}{96}t_{23}\rho^2\left[(1 \pm x)(3 \pm x(x_{13}-1))\right] \\ & + \frac{1}{192}t_{23}\rho^2\left[15 + 12x_{23} \pm 2x(1+2x_{23}) - x^2(5+4x_{23})\right]\,. \end{aligned}$$

The above Skyrme interactions took into account only the long-range part of the nuclear interaction and did not consider the short-range part [11]. This part reflects pairing correlations and is important to explain level density in the low excitation region, which is highly correlated to the effective mass. Gomez *et al.* [11] extended the Skyrme interaction (SKS4) to describe both mean field and pairing properties of the nuclear effective interaction.

The above Skyrme forces did not reproduce very well the breathing mode energies [14]. Recently, Farine *et al.* [14] overcame this problem and proposed a new version of the force SKM* called SKKM with much improved symmetry properties. They introduce a t_4 term which simultaneously depends on density and momentum. The contribution of this term to the effective mass is $F_3(\rho)$ and the effective mass becomes

$$\frac{m}{m_q^*} = 1 + \frac{2m}{h^2}\left[1 + F_1(\rho) + F_2(\rho) + F_3(\rho)\right] , \tag{9}$$

where

$$F_3(\rho) = \frac{1}{16} t_4 \rho^{\beta+1}\left[(2+x_4)(1\pm x) + (1-x_4)(1\pm x)^{\beta+1}\right] .$$

2.2. The dependence of the effective mass on the radial coordinate

For spherical nuclei it is convenient to parameterise the density distributions by a Fermi function to a power [7,13]

$$\rho(r) = \frac{\rho_0}{\left(1 + \exp\left(\frac{r-R}{a}\right)\right)^{\nu}} , \tag{10}$$

where R, a and ν are the half-value radius, the diffuseness parameter and the skewness parameter, respectively. This parameterisation has been found [7,13] to fit a vast amount of experimental data with an excellent accuracy. The density parameters R, a and ν are determined by minimising the energy of the nucleus with respect to these parameters. This is called the restricted variational method. In this method the parameters of the density depend upon the force parameters [7,13].

In our calculations the parameters a and ν are determined from fitting the calculated root mean square (r.m.s.) radius using equation (10) with the experimental values of spherical nuclei (see Appendix A). The half-value radius R is determined from the normalisation condition (see Appendix A). Here, the parameters of the density are independent of the force parameters (except for ρ_0) and depend only on the nucleus mass number.

2.3. The temperature dependence of the effective mass

Equation (9) shows that the dependence of the effective mass on temperature is only through the density. The nuclear density at temperature T is related to that at zero temperature by the relation (15):

$$\rho(T) = \rho(T=0)\left[1 - \frac{9\rho(T=0)}{K}\frac{\partial F_T}{\partial \rho}\right] , \tag{11}$$

where K is the incompressibility of nuclear matter and F_T is the free energy part that depends on temperature. The free energy of the system is:

$$F(\rho,T) = E(\rho,T) - TS(\rho,T) , \tag{12}$$

where $S(\rho,T)$ is the entropy of the system and $E(\rho,T)$ is the total energy of the system.

The total energy of the system $E(\rho,T)$ can be written as [5]:

$$E(\rho,T) = E(\rho,T=0) + \int_0^T \left(\frac{\partial S}{\partial T}\right)_{\rho_q} T' dT' \,. \tag{13}$$

Equations (12)and (13) show that the contribution of temperature to the free energy comes only from the entropy term. In the Thomas–Fermi model, the entropy density is given by [7]:

$$\sigma(\rho) = \frac{1}{\rho}\sum_q \rho_q \left(\frac{J_{3/2}(\eta_q)}{J_{1/2}(\eta_q)} - \eta_q\right) , \tag{14}$$

where $J_\alpha(\eta)$ are the usual Fermi integrals and the parameter η is the unique solution (at constant temperature) of the equation:

$$\rho_q = \frac{1}{2\pi^2}\left(\frac{2m}{h^2}\right)^{3/2} T^{3/2} J_{1/2}(\eta_q) \,. \tag{15}$$

At low temperatures we can expand the Fermi integrals in a power series of temperature. This approximation, up to T^4 terms, was proved [5] to be reliable up to temperatures $T = 15$ MeV. The entropy of the system using this approximation is given by [5]:

$$\begin{aligned} S(\rho,T) &= \frac{1}{6}\left(\frac{2m}{h^2}\right)\left(\frac{3\pi^2}{2}\right)^{1/3} \rho^{-2/3} T^2 \left(1 - \frac{x^2}{9}\right) \\ &\quad - \frac{7}{4320}\left(\frac{2m}{h^2}\right)^3 \rho^{-2} T^4 \left(1 + x^2\right) \,. \end{aligned} \tag{16}$$

This gives

$$\begin{aligned} F_T(\rho,T) &= -\frac{1}{12}\left(\frac{2m}{h^2}\right)\left(\frac{3\pi^2}{2}\right)^{1/3} \rho^{-2/3} T^2 \left(1 - \frac{x^2}{9}\right) \\ &\quad + \frac{7}{17280}\left(\frac{2m}{h^2}\right)^3 \rho^{-2} T^4 (1 + x^2) \end{aligned} \tag{17}$$

and

$$\begin{aligned} \rho(T) &= \rho(T=0)\left[1 - \frac{1}{2K}\left(\frac{2m}{h^2}\right)\left(\frac{3\pi^2}{2}\right)^{1/3} \rho^{-2/3} T^2 \left(1 - \frac{x^2}{9}\right)\right. \\ &\quad \left. + \frac{7}{960K}\left(\frac{2m}{h^2}\right)^3 \rho^{-2} T^4 (1 + x^2)\right] . \end{aligned} \tag{18}$$

2.4. The Landau parameters

The Landau parameters of the Fermi liquids represent the effective particle–hole interaction at the Fermi surface. Some of the Landau parameters are related directly to physical quantities [16]. For example, F_1 is related to the effective mass by:

$$\frac{m_q^*}{m} = 1 + \frac{1}{3}F_1^q\,, \tag{19}$$

F_0 is related to the nuclear matter incompressibility K by:

$$K = 6K_f^2\left(\frac{h^2}{2m^*}\right)(1+F_0) \tag{20}$$

and F_0' is related to the asymmetry energy β by:

$$\beta = \frac{1}{3}K_f^2\left(\frac{\hbar^2}{2m^*}\right)(1+F_0')\,, \tag{21}$$

where $K_f = \left(\frac{3\pi^2\rho}{2}\right)^{1/3}$.

Thus the effective mass is an important quantity for the Landau parameters. In our study, we will concentrate on the Landau parameter F_1. We can write

$$F_1^q = 3\left(\frac{m_q^*}{m} - 1\right)\,. \tag{22}$$

3. Results and discussions

3.1. Density distribution

Equation (10) for the density distribution contains four parameters. The half-value radius (R) is determined from the normalisation condition as given in Appendix A. The nuclear matter density (ρ_0) is considered as a free parameter constrained by the Skyrme force type. The skewness parameter (ν) and the diffuseness parameter (a) are determined from fitting the r.m.s. radii (see equation (A.6)) with the experimental values of spherical nuclei ranging from ^{16}O to ^{208}Pb. We have three sets of fitting parameters corresponding to three different values of ρ_0 namely:

For $\rho_0 = 0.15$ fm^{-3}, we got

$$a = 0.45 + \frac{A}{4000} \quad \text{and} \quad \nu = 1.1 + \frac{A}{72}$$

we refer to this set as *pw*1.

For $\rho_0 = 0.16$ fm^{-3}, we got

$$a = 0.5 + \frac{A}{4000} \quad \text{and} \quad \nu = 1.0 + \frac{A}{93}$$

we refer to this set as $pw2$.

For $\rho_0 = 0.17$ fm^{-3}, we got

$$a = 0.6 + \frac{A}{4000} \quad \text{and} \quad \nu = 1.3 + \frac{A}{180}$$

we refer to this set as $pw3$.

In our calculations we neglect the neutron skin and this makes us to consider the matter r.m.s. radius equal to the charge r.m.s. radius [7]. Table I contains our fitting r.m.s. radii, using the above three sets, together with the experimental values. For the sake of comparison we presented in Table I the results of HF calculations using SI and SII of Ref. [3], SKSC4 of Ref. [19], SLI of Ref. [10] and SKKM of Ref. [14]. We also presented the results of the Extended Thomas–Fermi model [7] using SKM*.

TABLE I

R.m.s. charge radii (fm). The experimental data all come from de Varies *et al.* [17]. See text for notations and references for Skyrme interactions.

	Exp.	$pw1$	$pw2$	$pw3$	SI	SII	SKSC4	SL1	SKKM	SKM*
^{16}O	2.73	2.80	2.79	2.80	2.68	2.75	—	2.68	2.78	2.78
^{40}Ca	3.45	3.44	3.43	3.45	3.41	3.49	3.45	3.44	3.50	3.47
^{48}Ca	3.45	3.60	3.59	3.60	3.46	3.54	3.54	3.48	3.50	3.54
^{58}Ni	3.77	3.79	3.77	3.78	—	—	3.82	—	—	3.81
^{90}Zr	4.26	4.28	4.27	4.26	4.22	4.31	4.28	4.28	4.30	4.29
^{112}Sn	4.59	4.56	4.56	4.56	—	—	4.59	—	4.57	—
^{117}Sn	4.62	4.62	4.62	4.61	—	—	4.62	—	—	4.61
^{120}Sn	4.65	4.65	4.65	4.64	—	—	4.64	—	4.64	—
^{124}Sn	4.67	4.70	4.69	4.68	—	—	4.67	—	—	—
^{144}Sm	4.95	4.92	4.91	4.91	—	—	4.93	—	4.95	—
^{148}Sm	4.99	4.96	4.95	4.95	—	—	5.00	—	—	—
^{156}Gd	5.07	5.04	5.04	5.03	—	—	5.12	—	—	—
^{166}Er	5.24	5.13	5.13	5.13	—	—	5.23	—	—	—
^{196}Pt	5.38	5.39	5.39	5.40	—	—	5.41	—	—	—
^{196}Pb	5.50	5.49	5.49	5.50	5.44	5.55	5.50	5.55	5.49	5.53

The density distribution Eq. (10) for ^{16}O and ^{208}Pb is presented in figure 1. We note that the distribution is smooth inside the nucleus where shell corrections are not included.

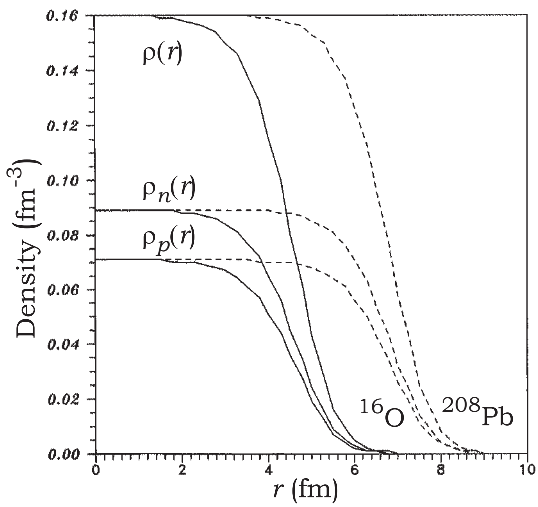

Fig. 1. The dependence of the proton density ρ_p, the neutron density ρ_n and the total density ρ on the ^{16}O and ^{208}Pb nuclei radius.

3.2. The radial dependence of the effective mass

The nucleon effective mass is given by equation (7), for most of Skyrme interactions. For symmetric nuclei ($N = Z$) we get the so called isoscalar effective mass m^*. Fig. 2 shows the neutron m_n^*, the proton m_p^* and the isoscalar m^* effective masses (in units of m) as a function of the ^{208}Pb radius using SKM* interaction. We note that m^* represents the average between m_n^* and m_p^*. This result is fulfilled for any nucleus except for self-conjugate nuclei (such as ^{16}O and ^{40}Ca) where $X = 0$ and consequently $m_n^* = m_p^* = m^*$. Since the density inside the ^{208}Pb nucleus is very close to the nuclear matter density, the behaviour of the effective mass inside the nucleus determines accurately the non-locality of the interaction (3). Figure 3 shows the (isoscalar) effective mass m^* as a function of the ^{208}Pb radius. We have important non-locality effects for SLI and RATP (with $m^* < 0.75m$). Skyrme forces SKM* and SKS4 (with $0.75 < m < 1$) are nearly local. Skyrme force SKSC4 (with $m^* = m$) is local (independent of velocity). Skyrme forces SKKM and SK220 (with $m^* > m$) are irregular. The behaviour of SK220 (also SK180 and SK200) is unexpected as seen from Fig. 3 (see also Fig. 6). The behaviour of the Landau parameter $F1$ inside

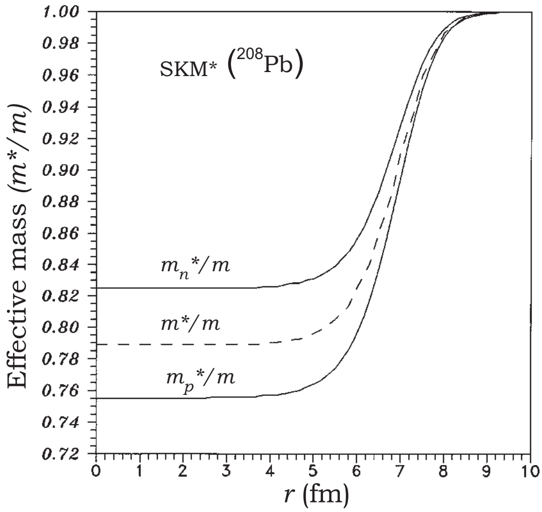

Fig. 2. The dependence of the neutron effective mass m_n^*/m, proton effective mass m^*/m on the radius of the ^{208}Pb nucleus using SKM* interaction.

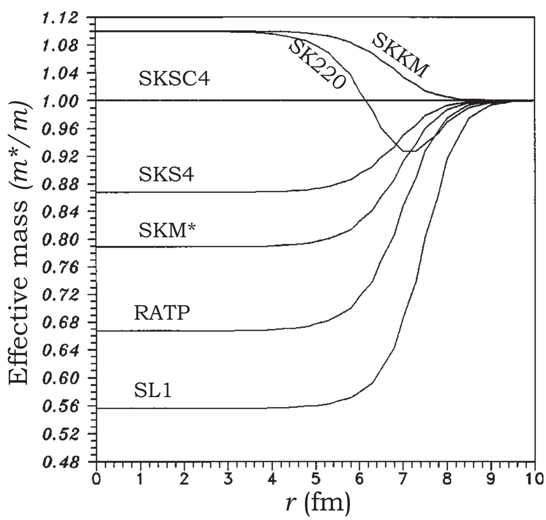

Fig. 3. The isoscalar effective mass m^*/m as a function of the ^{208}Pb radius for different Skyrme interactions.

the nucleus is presented in figure 4. It has exactly the same behaviour as the effective mass. There is a constraint on $F1$, given by Backman *et al.* [8], namely $-0.75 < F1 < 0$ with the sum rule favouring the low value. The lower value of $F1$ corresponds to $m^* = 0.75m$. This value of m^* was confirmed by Mahaux and Sartor [20] in their analyses of neutron ^{208}Pb mean field at the Fermi surface. This suggests Skyrme forces to be nearly local (*e.g.* SKM* and SKS4).

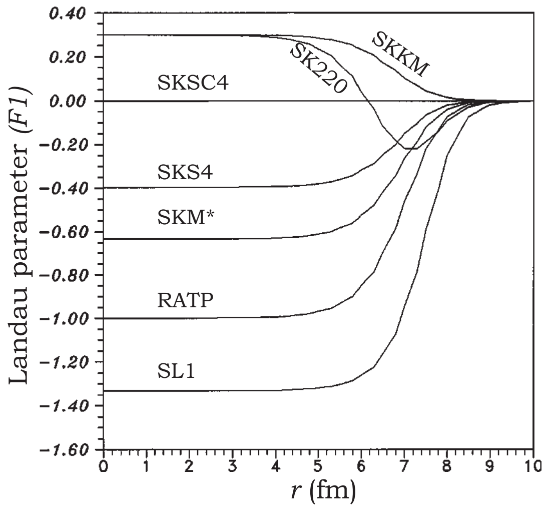

Fig. 4. The same as for figure 3 but for the Landau parameter $F1$.

3.3. The dependence of the effective mass on the density

In figure 5 we presented the effective mass m^*/m as a function of the relative density ρ/ρ_0. We notice that the effective mass decreases with density for Skyrme forces RATP, SKM* and SKS4. This can be interpreted

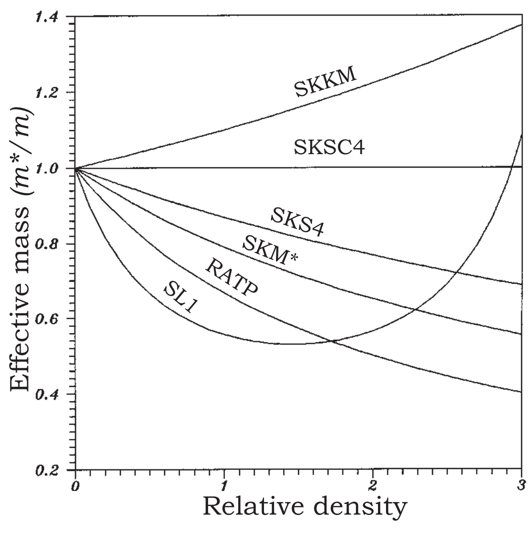

Fig. 5. The dependence of the isoscalar effective mass m^*/m for different Skyrme interactions on the relative density ρ/ρ_0.

as being due to the function $F_1(\rho)$ (see Eq. (7)) which has a ρ term only. This decreasing is controlled by the force parameters included in C1 an C2 (Eq. (3)). The Skyrme force SKSC4 (with $m^* = m$) is independent of the density. For the Skyrme force SLI the effective mass decreases for densities $\rho/\rho_0 < 1.5$ and then increases. This can be seen from Eq. (8), where an extra term, $F_2(\rho)$, contains ρ^2 is added to F_1. For Skyrme forces SKKM and SK220 (also SK180 and SK200) the effective mass increases with density. The parameters of these forces are adjusted to produce $m^*/m = 1.1$ at $\rho = \rho_0$ and naturally $m^*/m = 0$ at $\rho = 0$. This implies an increasing behaviour of the effective mass with density. We found unexpected behaviour for SK220 and SK200 forces as seen in figure 6.

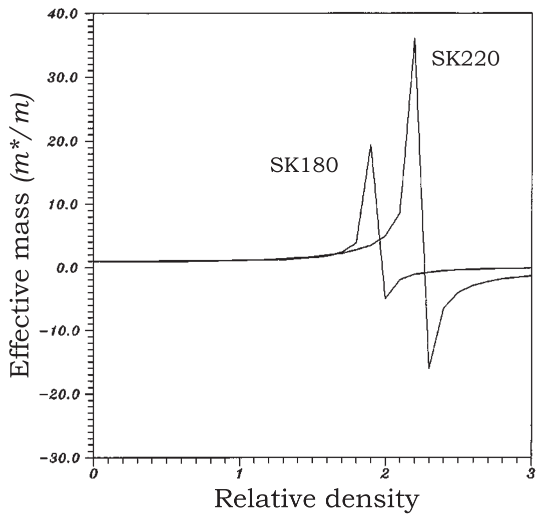

Fig. 6. The same as figure 5 using SK200 and SK220 potentials.

3.4. The temperature dependence of the effective mass

As we have a little decreasing effect of density with temperatures (see figure 7), the effective mass has a little increasing effect with temperatures (see figure 8). The same behaviour was obtained by Friedman and Pandharipande [21] using variational calculations of ν_{14} + TNI model. This means that as the temperature of the nucleus increases the strength of the non-local term of the interaction increases. This result is fulfilled for all Skyrme interactions except for SKKM and SK220 as seen from figure 9. We obtain unexpected behaviour of the effective mass (again) for SKKM and SK220 with temperatures.

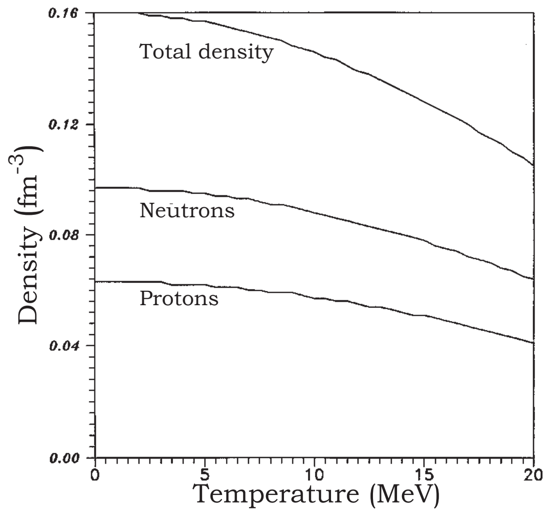

Fig. 7. The temperature dependence of the protons density ρ_p, neutrons density ρ_n and the total density ρ of ^{208}Pb using SKM*.

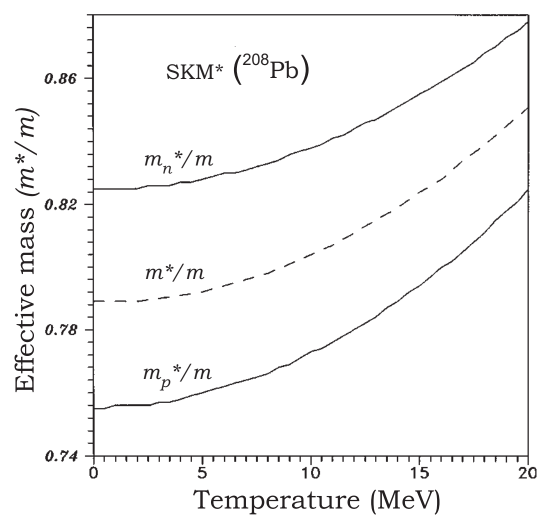

Fig. 8. The same as figure 7 but for the effective masses.

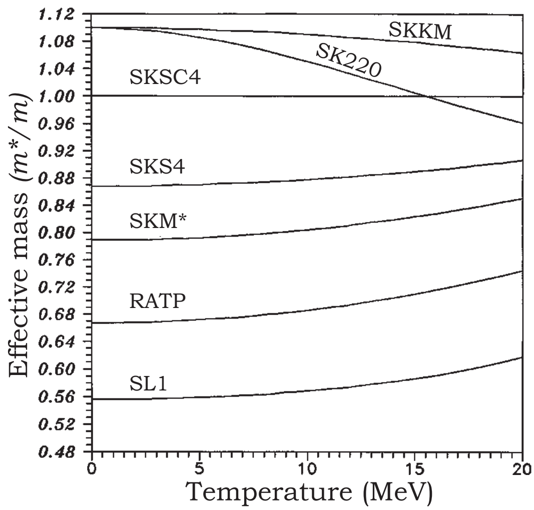

Fig. 9. The temperature dependence of the isoscalar effective mass m^*/m of the ^{208}Pb nucleus using different Skyrme interactions.

4. Summary and conclusions

The parameters of Skyrme interactions were fitted with the ground state properties of finite nuclei. The effective mass characterises the velocity-dependent terms of the interaction. The thermal properties of a nuclear system are summarised in its effective mass. Thus, we can test the parameters of the velocity-dependent terms of the interactions by studying the behaviour of the effective mass with density and temperature. The effective mass depends on the density of the system. We took a general form of Fermi distribution for the density and fitted the distribution parameters with the charge r.m.s. radii of spherical nuclei. This fitting gives a density distribution which rests on an experimental ground rather than on force parameters. Defining the density distribution (and expanding it in terms of temperatures), we can study the effective mass and its co-related Landau parameter $F1$. We applied this study for most of the well-known Skyrme forces. Some Skyrme forces (*e.g.* SLI and RATP) have high non-locality effects, some forces (*e.g.* SKM* and SKS4) have a small non-locality effects and some forces (*e.g.* SKSC4) are local. Skyrme forces SKKM, SK200 and SK220 have unexpected behaviour.

Since the effective mass concept itself was introduced for small non-local effects, we recommended SKM* and SKS4 as a good examples of Skyrme forces. However, SKS4 is more suitable since it takes into account beside the finite size of nuclei and realistic incompressibility also the pairing correlations, which is highly correlated to the effective mass, and fits the Landau parameters.

Appendix A

For the density distribution given by equation (10) we need, to calculate the r.m.s. radius, to evaluate integrals of the form $\int \rho^p dr$. Following the method adopted by Srivastava [17], these integrals can be approximated in the form

$$\int \rho^p(r) d\bar{r} = \frac{4\pi}{3}\rho_0^p \left[R^3 - aR^2 A_1(p) + 6a^2 R A_2(p) + 6a^3 A_3(p)\right], \quad \text{(A.1)}$$

where the coefficients $A_n(p)$ are given by:

$$A_n(p) = \frac{1}{(n-1)!}\int \left[1 - (1+\mathrm{e}^{-x})^{-p} + (-1)^n (1+\mathrm{e}^{x})^{-p}\right] x^{n-1} dx\,. \quad \text{(A.2)}$$

The normalisation condition

$$A = \int \rho(r) d\bar{r} = \frac{4\pi}{3}\rho\left(R^3 - aR^2 A_1(\nu) + 6a^2 A_2(\nu) - 6a^3 A_3(\nu)\right) \quad \text{(A.3)}$$

is inverted to obtain the half-value radius R as:

$$R = r_0 A^{1/3} + aA_1(\nu) + \frac{a^2}{r_0}\left[A_1(\nu) - 2A_2(\nu)\right] A^{-1/3} \quad \text{(A.4)}$$

with $r_0^3 = \frac{3}{4\pi\rho_0}$. The r.m.s. radius is given by :

$$\langle r^2 \rangle = \frac{\int \rho(r) r^2 d\bar{r}}{\int \rho(r) d\bar{r}}.$$

Using equations (A.1) we get

$$\langle r^2 \rangle = \frac{3}{5}\frac{R^5\left(1 - 5\lambda A_1(\nu) + 20\lambda^2 A_2(\nu) - 60\lambda^3 A_3(\nu)\right)}{R^3\left(1 - 3\lambda A_1(\nu) + 6\lambda^2 A_2(\nu) - 6\lambda^3 A_3(\nu)\right)} \quad \text{(A.5)}$$

with $\lambda = a/R$.

Using equation (A.4) we get

$$
\begin{aligned}
\text{r.m.s.} \;=\;& \frac{3}{5} r_0^2 A^{2/3} + r_0 a \left(2A_1(\nu) + \frac{3}{5} C1 \right) A^{1/3} \\
&+ a^2 \left(\frac{3}{5} (C1 A_1(\nu) + C2) + 3A_1^2(\nu) - 4A_2(\nu) \right) \\
&+ \frac{3a^3}{5r_0} \left(C3 + C1 \left(A_1^2(\nu) - 2A_2(\nu) \right) \right) A^{-1/3} \\
&+ \frac{3a^4}{5r_0^2} \left(C4 - C5 A_1(\nu) \right) A^{-2/3} + \frac{3a^5}{5r_0^3} \left(C5 - 2C4 A_1(\nu) \right) A^{-1}, \quad \text{(A.6)}
\end{aligned}
$$

where

$$
\begin{aligned}
C1 &= -2A_1(\nu)\,, \\
C2 &= 14A_2(\nu) - 10.5A_1^2(\nu)\,, \\
C3 &= 72A_1(\nu)A_2(\nu) - 54A_3(\nu) - 22.5A_1^3(\nu)\,, \\
C4 &= 180A_1^2(\nu)A_2(\nu) - 192A_1(\nu)A_3(\nu) - 102A_2^2(\nu)
\end{aligned}
$$

and

$$
C5 = 444A_2(\nu)A_3(\nu) - 450A_1(\nu)A_2^2(\nu) - 270A_1^2(\nu)A_3(\nu)\,.
$$

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