NUCLEON–NUCLEON AMPLITUDE PHASE VARIATION

Mohamed Ahmed Hassan

Mathematics Department, Faculty of Science, Ain Shams University
Cairo, Egypt

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A technique to calculate the rate of change of the nucleon–nucleon amplitude phase is given. The results for the nucleon–nucleon potential in the Gaussian form are consistent with the previous values of phase variation parameter γ.

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1. Introduction

At 1 GeV energy, Ahmed and Alvi [1] obtained values of the order of 1 $(\mathrm{GeV}/c)^{-2}$ for the nucleon–nucleon amplitude phase variation parameter γ [2]. The authors used an effective nucleon–nucleon potential in the Gaussian form. They obtained the same results in the case of Yukawa potential form. Thus, the authors concluded that the value of γ must be positive and it was not as large as required for fitting α-particle scattering at 7 GeV/c, where γ was in the range 7.5–11.51 $(\mathrm{GeV}/c)^{-2}$ [3]. Therefore, the phase variation of nucleon–nucleon amplitude alone is not sufficient to bring the Glauber model calculations closer to the α-particle scattering experiment.

In fact, Ahmed and Alvi [1] used incorrect mathematical technique to obtain the values of the parameter γ, see Section 2. Therefore, their results and conclusions cannot be accepted. In this work we try to present a correct mathematical technique to obtain the range of the values of γ. The Gaussian form of nucleon–nucleon effective potential is used to calculate the nucleon–nucleon elastic scattering amplitude. The momentum transfer dependence of the nucleon–nucleon amplitude phase and the dependence of the ratio of the real to the imaginary part of the amplitude are presented and discussed. The obtained model amplitude is compared to the spin independent nucleon–nucleon amplitude obtained from experiments on the nucleon–nucleon scattering.

2. Ahmed and Alvi technique

Comparing the coefficients of expansions in powers of q^2 of the integral representation of nucleon–nucleon amplitude [4]

$$f(\vec{q}) = \frac{ik}{2\pi}\int d^2\,\vec{b}\,\mathrm{e}^{i\vec{q}\cdot\vec{b}}(1-\mathrm{e}^{i\chi(\vec{b})})\,, \tag{1}$$

and these of the form [2]

$$f(\vec{q}) = \frac{k\sigma}{4\pi}(i+\rho)\,\mathrm{e}^{-(\beta^2+i\gamma)q^2/2}\,, \tag{2}$$

Ahmed and Alvi [1] obtained the following relation for γ

$$\gamma = \frac{1}{2}\mathrm{Im}\frac{\int\limits_0^\infty db\,b^3\mathit{\Gamma}(\vec{b})}{\int\limits_0^\infty db\,b\mathit{\Gamma}(\vec{b})}\,, \tag{3}$$

where k is the incident particle momentum, $\vec{q}$ is the momentum transfer vector, σ is the nucleon–nucleon total cross section, ρ is the ratio of the real part to the imaginary part of the forward scattering amplitude and β is the slope parameter. The phase shift function in the high-energy approximation [4] is given by

$$\chi(\vec{b}) = -\frac{1}{\hbar v}\int\limits_{-\infty}^{\infty} V(\vec{r})\,dz\,, \tag{4}$$

where $V(\vec{r})$ is the nucleon–nucleon potential, $\vec{b}$ is the impact parameter vector, $\vec{r} = \vec{b} + \vec{k}z$, where $\vec{k}$ is the unit vector in the z-direction which is usually taken in the incident direction, v is the incident nucleon velocity and $\mathit{\Gamma}(\vec{b}) = 1 - \mathrm{e}^{i\chi(\vec{b})}$.

At first, we note that, the nucleon–nucleon amplitude obtained from equation (1) using an effective nucleon–nucleon potential — in general — has a phase of a variable rate of change. At the same time, the phase of the nucleon–nucleon amplitude of equation (2) has a constant rate of change $-\gamma/2$. Therefore, the authors of Ref. [1] compared the expansions of two mathematically different functions which were given by equations (1) and (2). Secondly, the coefficients of the expansion of the right-hand side of equation (1) were calculated at $q^2 = 0$. Therefore, the right-hand side of equation (3), after dividing it by –2, represents the rate of change of nucleon–nucleon amplitude phase with respect to q^2 at $q^2 = 0$ only. Thus, the obtained result which is represented by equation (3), is correct only at

$q^2 = 0$. At the same time, $-\gamma/2$ in equation (2) gives the rate of change of the phase at any q^2. Thus, the obtained value for γ from the relation (3) is not correct.

3. The correct technique

Practically, we can consider the nucleon–nucleon elastic scattering amplitude $f(\vec{q})$ as a complex function of q^2. Thus, the phase $\phi(q^2)$ of this function is

$$\phi(q^2) = \mathrm{Im} \ln f(\vec{q}) \,. \tag{5}$$

Then, the rate of change of $\phi(q^2)$ with respect to q^2 is

$$\frac{d\phi}{dq^2} = \mathrm{Im} \frac{d}{dq^2} \ln f(\vec{q}) = \mathrm{Im} \left(\frac{1}{f(\vec{q})} \frac{df(\vec{q})}{dq^2} \right) \,. \tag{6}$$

To calculate the correct values of the rate of phase variation we must use equation (6). For convenience we define the phase variation function P_v as follows

$$P_v = -2 \frac{d\phi}{dq^2} \,. \tag{7}$$

From (1), integrating with respect to the angular part of $d^2\vec{b}$, we get

$$f(\vec{q})|_{q^2=0} = ik \int_0^\infty db \, b \, \Gamma(\vec{b}) \,. \tag{8}$$

Using the integral representation of zero-order Bessel function of the first kind J_0

$$J_0(qb) = \frac{1}{2\pi} \int_0^{2\pi} d\,\phi \, \mathrm{e}^{iqb\cos\phi} \,,$$

and the expansion

$$J_0(qb) = \sum_{m=0}^{\infty} \frac{(-1)^m \left(\frac{qb}{2}\right)^{2m}}{(m!)^2} \,,$$

we can show that

$$\left. \frac{df(\vec{q})}{dq^2} \right|_{q^2=0} = -\frac{ik}{4} \int_0^\infty db \, b^3 \, \Gamma(\vec{b}) \,. \tag{9}$$

Thus, from (6)–(8) and (9), we obtain

$$P_v|_{q^2=0} = \frac{1}{2}\mathrm{Im}\frac{\int_0^\infty db\, b^3\, \Gamma(\vec{b})}{\int_0^\infty db\, b\, \Gamma(\vec{b})} \tag{10}$$

which is in agreement with our point of view about the right-hand side of equation (3).

4. Gaussian potential calculations

As an example, we will consider the effective nucleon–nucleon potential, as in Ref. [1], in the Gaussian form

$$V(\vec{r}) = (V_0 - iW_0)\mathrm{e}^{-\alpha^2 r^2}\,, \tag{11}$$

where V_0, W_0 and α^2 are the potential parameters. From equation (1), using equation (4), we get

$$\begin{aligned} f(\vec{q}) &= \frac{ik}{2\pi}\int d^2\vec{b}\,\mathrm{e}^{i\vec{q}\cdot\vec{b}}\left(1 - \mathrm{e}^{i\chi_0 \mathrm{e}^{-\alpha^2 b^2}}\right) \\ &= \frac{ik}{2\pi}\int d^2\vec{b}\,\mathrm{e}^{i\vec{q}\cdot\vec{b}}\left(1 - \sum_{n=0}^{\infty}\frac{(i\chi_0)^n\,\mathrm{e}^{-n\alpha^2 b^2}}{n!}\right) \\ &= -\frac{ik}{2\pi}\sum_{n=1}^{\infty}\frac{i^n\chi_0^n}{n!}\int d^2\,\vec{b}\,\mathrm{e}^{-n\alpha^2 b^2 + i\vec{q}\cdot\vec{b}} \\ &= -\frac{ik}{2\alpha^2}\sum_{n=1}^{\infty}\frac{i^n\chi_0^n}{n!n}\,\mathrm{e}^{-q^2/4n\alpha^2}\,, \end{aligned} \tag{12}$$

where $\chi_0 = -\frac{1}{\hbar v}\sqrt{\frac{\pi}{\alpha^2}}(V_0 - iW_0)$. It is easily seen that

$$\frac{df}{dq^2} = \frac{ik}{8\alpha^4}\sum_{n=1}^{\infty}\frac{i^n\chi_0^n}{n!n^2}\,\mathrm{e}^{-q^2/4n\alpha^2}\,. \tag{13}$$

In equation (12) the scattering amplitude is a combination of constant phase terms and the interference between these terms leads to a variable rate of change of the scattering amplitude phase.

Using equations (6), (12) and (13) we can, easily, calculate the phase variation function P_v at any q^2 for the Gaussian potential. To calculate the results of Ref. [1], we calculate P_v at $q^2 = 0$. Although the amplitude of equation (2) is a fitting form, and the value of γ is an effective value which gives a good fit with the data, the range of P_v values gives some indication about the possible values of γ. Equation (2) corresponds to the special case, where $P_v(q^2)$=constant=γ.

From equation (12) we can obtain the nucleon–nucleon total cross section $\sigma = (4\pi/k)\ \mathrm{Im}f(0)$ and the ratio of the real part to the imaginary part of the elastic scattering amplitude in the forward direction $\rho = \mathrm{Re}f(0)/\mathrm{Im}f(0)$ in terms of χ_0 and α^2. The mean values of proton–proton and proton–neutron experimental data, at 1.75 GeV/c, for σ and ρ are 44 mb and –0.23, respectively [3]. These values correspond to $\chi_0 = -0.272 + i0.896$ and $\alpha^2 = 0.041\,(\mathrm{GeV}/c)^2$ with χ^2 value (of χ^2 method) equal to 0.15×10^{-3}.

Our values of χ_0 and α^2 are clearly different from the previously obtained values [1], where $\chi_0 = 0.514 + i\,0.71$ and $\alpha^2 = 0.14\,(\mathrm{GeV}/c)^2$ at the same energy and for the same values of σ and ρ. Firstly, we believe that the positive sign of Re χ_0 is a written mistake. Secondly, these values give $\sigma =$ 11.23 mb and $\rho = -0.566$, which are different from the experimental data. Finally, with these values of χ_0 and α^2 we get $P_v(0) = 0.213\,(\mathrm{GeV}/c)^{-2}$, while the given value of $\gamma = P_v(0)$ in [1] is 0.8 $(\mathrm{GeV}/c)^{-2}$.

From equation (7), using equations (12) and (13), the quantity P_v is calculated using our values of χ_0 and α^2 at 1.75 GeV/c. The results are presented in figure 1 (solid curve) in the region $0 < q^2 < 2\,(\mathrm{GeV}/c)^2$, where equation (1) can be used as a good approximation. Firstly, the value of P_v at $q^2 = 0$ is equal to 0.376 $(\mathrm{GeV}/c)^{-2}$. Secondly, the quantity P_v has a general oscillating behaviour with very sharp peak at $q^2 \approx 0.6\,(\mathrm{GeV}/c)^2$, where $P_v = 27.34\,(\mathrm{GeV}/c)^{-2}$. This oscillating character of P_v is consistent with the variation of phase variation effect on nucleon–nucleus and nucleus–nucleus scattering with q^2 values. For example, in p–d elastic scattering at 1.75 GeV/c, this effect can be neglected for $q^2 < 0.2\,(\mathrm{GeV}/c)^2$ [5]. The function P_v has the smaller values in this region. Also, the role of the phase variation in the region of the first minimum, $0.2 < q^2 < 0.6\,(\mathrm{GeV}/c)^2$ is important. In this region the arithmetic mean value of P_v is $7.32\,(\mathrm{GeV}/c)^{-2}$. Taking $\gamma = 8\,(\mathrm{GeV}/c)^{-2}$ a good fit with p–d elastic scattering experimental data at 1.75 GeV/c is obtained [5]. For 1.27 GeV/c incident momentum per nucleon the authors of [3] used approximately the same value ($\gamma = 7.5\,(\mathrm{GeV}/c)^{-2}$) to obtain a good agreement with the α–^{4}He elastic scattering data. Also, they used the value $10\,\mathrm{GeV}/c)^{-2}$ for γ to obtain a good fit with the data of α–^{1}H, α–^{2}H, α–^{3}He, and α–^{4}He at 1.75 GeV/c per nucleon. For the data of α–^{4}He at 1.08 GeV/c per nucleon, the used value of γ is $11.5\,(\mathrm{GeV}/c)^{-2}$ [3]. Finally, the arithmetic mean of P_v in the range

$0.2 < q^2 < 1.75\,(\mathrm{GeV}/c)^2$, where the most experimental nucleon–nucleus and nucleus–nucleus elastic scattering differential cross sections are available, is $4.44\,(\mathrm{GeV}/c)^{-2}$. Approximately the same value for γ is used by [6] to obtain an agreement with p–^{4}He elastic scattering experimental data at 1.75 GeV/c.

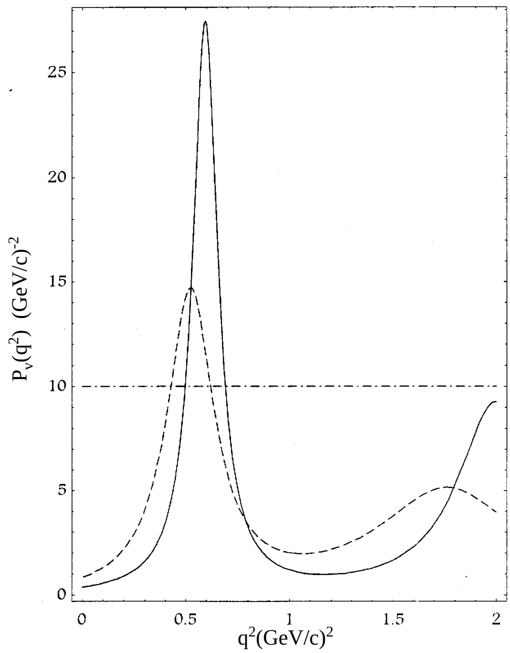

Fig. 1. $P_v(q^2)$ function. Solid and dashed curves correspond to the two sets of potential parameters ($\chi_0 = -0.272 + i0.896$, $\alpha^2 = 0.041\,(\mathrm{GeV}/c)^2$) and ($\chi_0 = -0.514 + i0.71$, $\alpha^2 = 0.0357\,(\mathrm{GeV}/c)^2$), respectively. The dot-dashed curve corresponds to $P_v(q^2) = \gamma = 10\,(\mathrm{GeV}/c)^{-2}$ [3].

At 1 GeV energy there is an uncertainty in the values of the parameters ρ, see for example [2,7,8]. The different values of ρ lead to different values of the potential parameters χ_0 and α^2. The values of $P_v(q^2)$ are different for different values of the parameters χ_0 and α^2. However, in any case we have the same oscillating behaviour. Also, the range of $P_v(q^2)$ values leads to the same conclusion. For example, see figure 1 dashed curve, where $\chi_0 = -0.514 + i\,0.71$ and $\alpha^2 = 0.0357\,(\mathrm{GeV}/c)^2$. These values correspond to $\sigma = 44$mb and $\rho = -0.566$. In this case $P_v(0) = 0.84\,(\mathrm{GeV}/c)^{-2}$ and the maximum value is equal to 14 $(\mathrm{GeV}/c)^{-2}$ at $q^2 = 0.5\,(\mathrm{GeV}/c)^2$.

Thus, all used values for γ in the previous works are in the range of P_v values at the used energy. Consequently, the values of the phase variation parameter γ are consistent with the range of P_v values for the nucleon–nucleon potential in the Gaussian form. These results contradict with those obtained in [1].

The momentum transfer dependence of the phase $\phi(q^2)$ of the model amplitude (12) is presented in figure 2. Also, the results for the amplitude (2) are presented in two cases, where $\gamma = 0$ and $\gamma = 10\,(\mathrm{GeV}/c)^{-2}$. For $q^2 < 0.5\,(\mathrm{GeV/c})^2$, the results of the model amplitude (12) are close to that of the amplitude (2) with $\gamma = 0$. This amplitude with $\gamma = 0$ was obtained from nucleon–nucleon scattering experiments. With $\gamma = 10\,(\mathrm{GeV}/c)^{-2}$ the results of $\phi(q^2)$ is relatively small. However, we obtained the same order of values of $\phi(q^2)$ for both amplitudes (12) and (2) using $\gamma = 4.4\,(\mathrm{GeV}/c)^{-2}$ in the last amplitude. This value of γ is equal to the mean value of $P_v(q^2)$ in the range $0.2 \leq q^2 \leq 1.75\,(\mathrm{GeV}/c)^2$ and it was used before in [6] to obtain an agreement with p–^{4}He scattering data at 1.75 GeV/c.

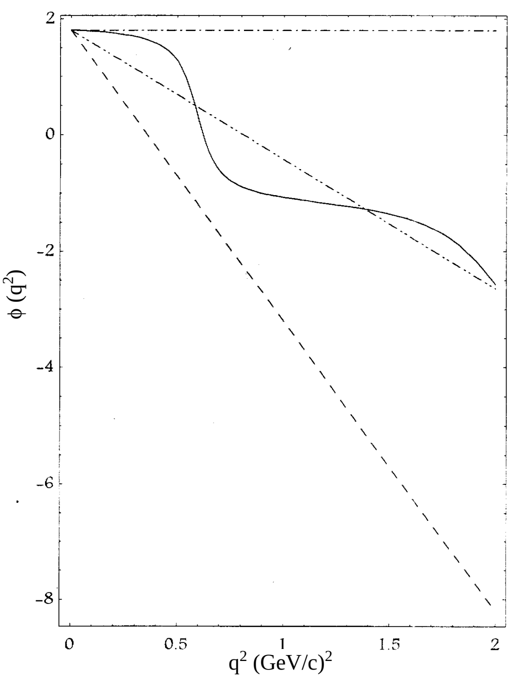

Fig. 2. The phase $\phi(q^2)$ of nucleon–nucleon amplitude. The solid, dashed, dot-dashed and double dot-dashed curves correspond to the amplitude (12) with $\chi_0 = -0.272 + i\,0.896$ and $\alpha^2 = 0.041(\mathrm{GeV}/c)^2$, amplitude (2) with $\gamma = 10, 0$, and 4.44 $(\mathrm{GeV}/c)^{-2}$, respectively.

The ratio $\rho(q^2) = \mathrm{Re}f(\vec{q})/\mathrm{Im}f(\vec{q})$ is presented in figure 3. In the range $0 \leq q^2 \leq 1.8\,(\mathrm{GeV}/c)^2$ except at the singular point, where $q^2 \approx 0.6\,(\mathrm{GeV}/c)^2$ the results of the model amplitude (12) and of the amplitude (2) with $\gamma = 0$ is approximately of the same order. With $\gamma = 10\,(\mathrm{GeV}/c)^{-2}$ the ratio $\rho(q^2)$ for the amplitude (2) takes absolutely different values and different behaviours. For the model amplitude (12), the singular point of $\rho(q^2)$ at $q^2 \approx 0.6\,(\mathrm{GeV}/c)^2$ is related to the maximum point of $P_v(q^2)$ at the same value of q^2, see figures 1 and 3.

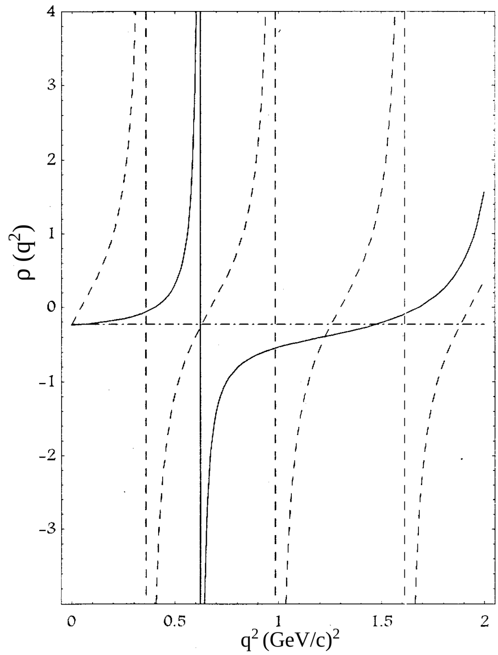

Fig. 3. The ratio $\rho(q^2) = \mathrm{Re}f(\vec{q})/\mathrm{Im}f(\vec{q})$. The solid curve represents the results of amplitude (12) with $\chi_0 = -0.272 + i0.896$ and $\alpha^2 = 0.041\,(\mathrm{GeV}/c)^2$. Dashed and dot-dashed curves represent the results of amplitude (2) with $\gamma = 10$ and 0 $(\mathrm{GeV}/c)^{-2}$, respectively.

Finally, the quantity $|f(\vec{q})|^2$ in the two cases of equations (12) and (2) is presented in figure 4. The absolute value of $f(\vec{q})$ means that the phase variation of amplitude (2) is meaningless in the nucleon–nucleon elastic scattering differential cross section calculations. The model amplitude (12) gives a correct representation of the diffractive character of high-energy nucleon–nucleon elastic scattering differential cross section. The quantity $\ln|f(\vec{q})|^2$, for the amplitude (2), represents a straight line with constant slope $-\beta^2$. For the model amplitude (12) the combinations of different Gaussian terms lead to sequence of maxima and minima.

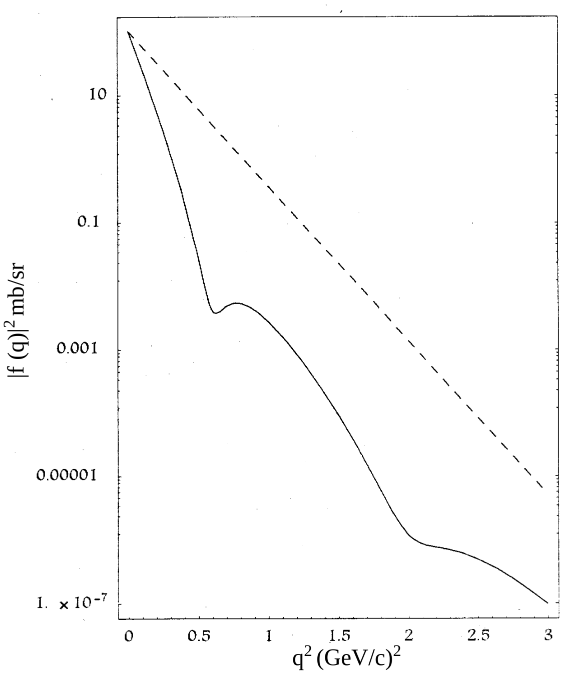

Fig. 4. Nucleon–nucleon elastic scattering differential cross section $|f(\vec{q})|^2$. The solid and dashed curves represent the results of amplitude (12) with $\chi_0 = -0.272 + i0.896$, $\alpha^2 = 0.041\,(\mathrm{GeV}/c)^2$ and amplitude (2), respectively. $\beta^2 = 5.6\,(\mathrm{GeV}/c)^{-2}$ [3].

Thus, using nucleon–nucleon potential in Gaussian form, the nucleon–nucleon elastic scattering amplitude is obtained. For this amplitude, the phase $\phi(q^2)$, the phase variation rate, the ratio $\rho(q^2)$ and $|f(\vec{q})|^2$ are calculated at 1.75 GeV/c. The results are compared to the case of empirical form of nucleon–nucleon amplitude. Some consistence can be observed between the results in the two cases.

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