

QUADRUPOLE COLLECTIVITY IN THE Si ISOTOPES AROUND $N = 20^*$

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The angular momentum projected generator coordinate method using the quadrupole moment as a collective coordinate and the Gogny force as the effective interaction is used to describe the properties of the ground and low-lying excited states of the neutron rich light nuclei $^{32,34,36}\text{Si}$. It is found that the ground state of the nucleus ^{34}Si is spherical. However, this is not due only to the $N = 20$ shell closure, since the ground state of ^{34}Si contains a significant amount of the intruder $f_{7/2}$ neutron orbital. A rather good agreement with experimental data for many observables is obtained.

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1. Introduction

In nuclear physics the mean field approximation is always the first step to understand the properties of the ground and lowest-lying excited states. The mean field approximation provides the concept of magic numbers as well as the concept of spontaneous symmetry breaking. For nuclei with proton and/or neutron numbers close to the magic ones, one expects symmetry-conserving (*i.e.*, non superconducting and spherical) ground states. On the other hand, for nuclei away from the magic configurations, one expects a strong symmetry breaking and an appearance of deformed ground states that generate, *e.g.*, rotational bands.

The experimental studies of light nuclei away from the stability line $N = Z$ seem to imply that for those (usually neutron rich) nuclei some of the properties associated to magic numbers are not preserved. The most striking example is the experimental evidence towards the existence of quadrupole

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deformed ground states in the neutron-rich nuclei around the magic number $N=20$. In addition, the extra binding energy coming from deformation can help to extend thereby the neutron drip line in this region far beyond what could be expected from spherical ground states. Among the variety of available experimental data, the most convincing evidence for a deformed ground state in the region around $N = 20$ is found in the ^{32}Mg nucleus where both the excitation energy of the lowest lying 2^+ state [1] and the $B(E2, 0^+ \rightarrow 2^+)$ transition probability [2] have been measured. Both quantities are fairly compatible with the expectations for a rotational state. At the mean field level, the ground state of ^{32}Mg is spherical. However, when the Zero Point Rotational Energy (ZPRE) correction is considered, the energy landscape as a function of the quadrupole moment changes dramatically and ^{32}Mg becomes deformed [3–9].

A more careful analysis of the energy landscape including the ZPRE correction reveals that, in fact, there are two coexistent configurations (prolate and oblate) with comparable energy indicating thereby that configuration mixing of states with different quadrupole intrinsic deformation has to be considered. Therefore, an Angular Momentum Projected Generator Coordinate Method (AMPGCM) calculation with the quadrupole moment as collective coordinate is called for. We have applied this method in Ref. [10] to the study of the nuclei $^{30-34}\text{Mg}$. The Gogny force [11] (with the D1S parameterization [12]) has been used in the calculations. We have obtained prolate ground states for $^{32-34}\text{Mg}$ indicating that the $N = 20$ shell closure is not preserved for the Mg isotopes. Moreover, a good agreement with the experimental data for the 2^+ excitation energies and $B(E2)$ transition probabilities was obtained. The purpose of this paper is to extend those calculations to the study of the nuclei $^{32-36}\text{Si}$ which are obtained by adding two protons to the $^{30-34}\text{Mg}$ nuclei. These nuclei were recently studied experimentally [13], and their ground states seem to be spherical and, therefore, preserve the $N = 20$ shell closure. Therefore, our intention is to check whether our method and interaction is not only able to predict a deformed ground state in ^{32}Mg ($Z = 12$) but also to predict a spherical one in ^{34}Si ($Z = 14$).

2. Theoretical framework

The AMPGCM with the mass quadrupole moment as generating coordinate is used as the theoretical framework. As we restrict ourselves to axially symmetric configurations, we use the following Ansatz for the $K = 0$ wave functions of the system

$$|\Phi_\sigma^I\rangle = \int dq_{20} f_\sigma^I(q_{20}) \hat{P}_{00}^I |\varphi(q_{20})\rangle. \quad (1)$$

In this expression $|\varphi(q_{20})\rangle$ is the set of axially symmetric (*i.e.* $K = 0$) Hartree–Fock–Bogoliubov (HFB) wave functions generated by constraining the mass quadrupole moment to the desired values $q_{20} = \langle \varphi(q_{20}) | z^2 - 1/2(x^2 + y^2) | \varphi(q_{20}) \rangle$. The quasiparticle operators associated to the intrinsic wave functions $|\varphi(q_{20})\rangle$ have been expanded in the Harmonic Oscillator (HO) basis containing 10 major shells and with equal oscillator lengths to make the basis closed under rotations [14]. The operator

$$\hat{P}_{00}^I = \frac{(2I+1)}{8\pi^2} \int d\Omega d_{00}^I(\beta) e^{-i\alpha\hat{J}_z} e^{-i\beta\hat{J}_y} e^{-i\gamma\hat{J}_z} \quad (2)$$

is the usual angular momentum projector with the $K = 0$ restriction [15] and $f_{\sigma}^I(q_{20})$ are the “collective wave functions” solution of the Hill–Wheeler (HW) equation

$$\int dq'_{20} \mathcal{H}^I(q_{20}, q'_{20}) f_{\sigma}^I(q'_{20}) = E_{\sigma}^I \int dq'_{20} \mathcal{N}^I(q_{20}, q'_{20}) f_{\sigma}^I(q'_{20}). \quad (3)$$

In the equation above we have introduced the projected norm

$$\mathcal{N}^I(q_{20}, q'_{20}) = \langle \varphi(q_{20}) | \hat{P}_{00}^I | \varphi(q'_{20}) \rangle \quad (4)$$

and the projected Hamiltonian kernel

$$\mathcal{H}^I(q_{20}, q'_{20}) = \langle \varphi(q_{20}) | \hat{H} \hat{P}_{00}^I | \varphi(q'_{20}) \rangle. \quad (5)$$

As the generating states $\hat{P}_{00}^I | \varphi(q_{20}) \rangle$ are not orthogonal, the “collective amplitudes” $f_{\sigma}^I(q_{20})$ cannot be easily interpreted. Instead, one usually introduces [16] the “collective” amplitudes

$$g_{\sigma}^I(q_{20}) = \int dq'_{20} f_{\sigma}^I(q'_{20}) \mathcal{N}^I(q_{20}, q'_{20})^{1/2*} \quad (6)$$

which are orthonormal, $\int dq_{20} (g_{\sigma}^I)^*(q_{20}) g_{\sigma'}^I(q_{20}) = \delta_{I,I'} \delta_{\sigma,\sigma'}$, and, therefore, their modules squared have the meaning of probabilities.

The $B(E2)$ transition probabilities are computed using the AMPGCM wave functions as

$$B(E2, I_i \rightarrow I_f) = \frac{e^2}{2I_i + 1} \left| \int dq_{20}^i dq_{20}^f f_{\sigma_f}^{I_f*}(q_{20}^f) \langle I_f q_{20}^f || \hat{Q}_2 || I_i q_{20}^i \rangle f_{\sigma_i}^{I_i}(q_{20}^i) \right|^2. \quad (7)$$

As we are using the full configuration space no effective charges are needed. Further details of the computational procedure can be found in Ref. [10].

3. Discussion of the results

In Fig. 1 we show the HFB Potential Energy Surfaces (PES) for the three nuclei considered (dashed line) along with the projected energies $E^I = \mathcal{H}^I(q_2, q_2)/\mathcal{N}^I(q_2, q_2)$ for $I = 0, 2, 4, 6$ and 8 (full lines). The HFB PES shows a very pronounced spherical minimum in ^{34}Si , a shallow spherical minimum in ^{36}Si , and, finally, a very shallow oblate minimum in ^{32}Si . These results are rather different from the ones obtained for the corresponding Mg isotopes with the same neutron number [10]. The reason for such differences is that the Si isotopes have the proton $d_{5/2}$ shell completely occupied and, therefore, the up-slopping $K = 5/2$ level of the proton $d_{5/2}$ orbital prevents the appearance of the prolate deformation. At the mean field level, we can say that the Si isotopes around $N = 20$ retain the closed shell properties associated to the $N = 20$ shell closure. However, as it has already been recognized, the effect of the restoration of the rotational symmetry in the HFB states can be very important, and it can substantially modify the conclusions extracted from the mean field results. In the three nuclei considered here, the Angular Momentum Projected (AMP) PES (Fig. 1) show two minima, prolate and oblate, for all the angular momenta considered. However, the prolate and oblate minima lie rather close in energy in almost all the cases and, therefore, we expect strong mixing of the two configurations. Taking into account this fact, and also the fact that the amount of mixing is not only determined by the AMP PES but also by the “collective inertia”, the amount of mixing can only be disentangled when the configuration-mixing

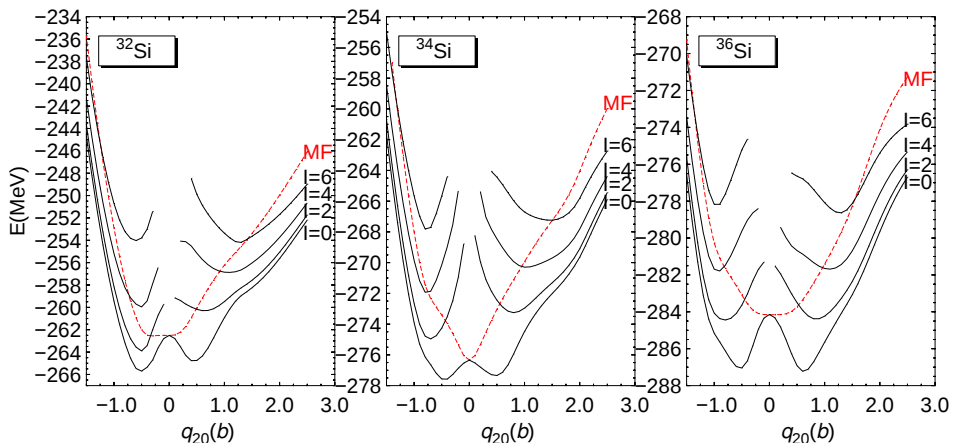


Fig. 1. The HFB potential energy surfaces for the three nuclei considered in the present study (dashed lines) together with the projected energies for $I = 0, 2, 4, 6$ (full lines).

calculation is carried out. We have also carried out configuration mixing (in the framework of the GCM) calculations taking into account angular momentum projected wave functions and the quadrupole constrained HFB wave functions as generating intrinsic states. The solution of the HW equation provides us with energies E_σ^I and collective wave functions $f_\sigma^I(q_{20})$. As mentioned in Sec. 2, the amplitudes f_σ^I are not well suited for a physical interpretation and, therefore, one considers the collective amplitudes $g_\sigma^I(q_{20})$ instead. In terms of these amplitudes one can define an “averaged” intrinsic quadrupole moment $(\bar{q}_{20})_\sigma^I = \int dq_{20} q_{20} |g_\sigma^I(q_{20})|^2$ for each of the AMPGCM solutions.

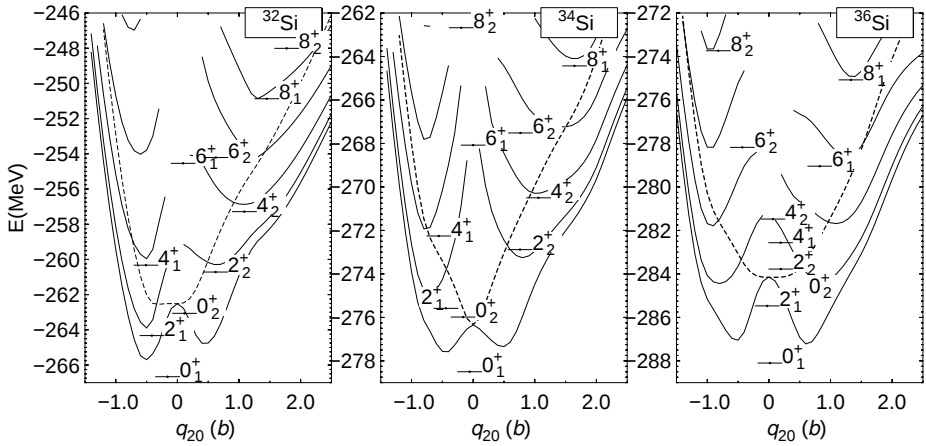


Fig. 2. Same as in Fig. 1, but with added solutions of the HW equation, see text.

In Fig. 2 we show, along with the angular-momentum projected PES (plotted again to guide the eye), the energies E_σ^I obtained by solving the HW equation for $\sigma = 1$ and 2 and angular momenta $I = 0, 2, 4, 6$, and 8. Each energy has been placed at the value of the quadrupole moment corresponding to the “averaged” intrinsic quadrupole moment $(\bar{q}_{20})_\sigma^I$. From Fig. 2 we see that in ^{32}Si the two 0^+ states shown are spherical, the 2_1^+ and 4_1^+ are oblate whereas the 2_2^+ , 4_2^+ , 6_2^+ and 8_1^+ are prolate deformed states. The 6_2^+ state is almost degenerate in energy with the nearly spherical 6_1^+ state and this could explain the anomalous quadrupole moment of the former state. A nearly identical pattern is also seen in the nucleus ^{34}Si , except for the position of the 0_2^+ state that lies below the 2_1^+ . In the nucleus ^{36}Si , the ground state is spherical and a prolate deformed 0_2^+ state is obtained. The 2^+ and 4^+ states are all of them almost spherical whereas the 6_1^+ and 8_1^+ are prolate and the 6_2^+ and 8_2^+ states are oblate. We observe that in the three nuclei considered, the ground state is spherical due to the configuration mixing between the prolate and oblate minima. The rest of the spectra shows rather unclear

patterns that can only be elucidated by looking at the “collective amplitudes” $g_\sigma^I(q_{20})$. In Fig. 3, the collective wave functions squared $|g_\sigma^I(q_{20})|^2$ for the two lowest solutions $\sigma = 1$ and 2 obtained in the AMPGCM calculations are depicted. We also show in each panel the projected energy for the

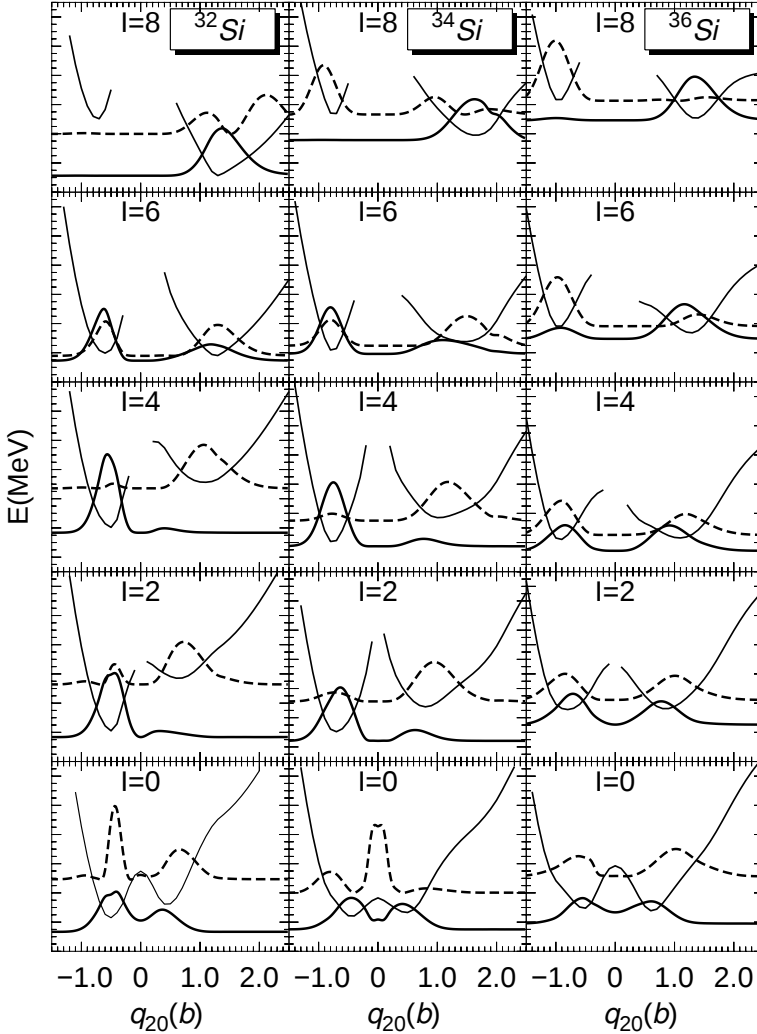


Fig. 3. The collective amplitudes $|g_\sigma^I(q_{20})|^2$ (thick lines) for $\sigma = 1$ (full) and 2 (dashed) and spin values of $I = 0\hbar, \dots, 8\hbar$ for the nuclei ^{32}Si , ^{34}Si and ^{36}Si . The projected energy curve for each spin is also plotted (thin line). The y -axis scales are in energy units and span an energy interval of 13 MeV. The collective amplitudes have also been plotted against the energy scale after a proper scaling and shifting, that is, the quantity $E_\sigma^I + 15 \times |g_\sigma^I(q_{20})|^2$ is the one actually plotted.

corresponding spin. We observe that the 0_1^+ ground state wave functions of the three nuclei contain significant admixtures of both the prolate and oblate configurations. Looking into the single particle diagram of ^{34}Si (not shown here) it is observed that the neutron $f_{7/2}$ shell, that is empty at $q_{20} = 0$ and 5.78 MeV higher than the $d_{3/2}$, crosses the Fermi surface at the deformation of $q_{20} = -0.5$ b on the oblate side (the $K = 7/2$ level) and $q_{20} = 0.5$ b on the prolate side (the $K = 1/2$ level). Therefore, the intrinsic states with $q_{20} > 0.5$ b and $q_{20} < -0.5$ b have (at least) two particles in the intruder $f_{7/2}$ orbital. As the ground state collective amplitude in ^{34}Si (and in the other two nuclei) is not negligible in the intervals $q_{20} > 0.5$ b and $q_{20} < -0.5$ b we conclude that the ground states of the three nuclei contain significant amounts of the intruder $f_{7/2}$ neutron configuration in spite of being spherical. To make the argument more quantitative we have computed the quantity $1 - \int_{-0.5}^{0.5} dq_{20} |g_{\sigma}^{I=0}(q_{20})|^2$ that gives us an idea of the percentage of the “collective” amplitude outside the q_{20} interval between -0.5 and 0.5 b (*i.e.*, the interval where the intrinsic states contain the intruder $f_{7/2}$ configuration). This quantity is roughly 0.35 for $\sigma = 1$ in the three nuclei considered and it corresponds roughly to 0.7 particles in the neutron $f_{7/2}$ sub-shell. Therefore, we conclude that the ground state of ^{34}Si does not correspond to a pure configuration with $N = 20$ acting as a closed shell. Similar conclusions have been obtained in shell model calculations [17] and also in the Shell Model Monte Carlo calculations of [18].

The collective amplitude of the 0_2^+ excited state of ^{34}Si is rather unusual, being concentrated around $q_{20} = 0$. This unusual behavior is probably responsible for the low excitation energy of this state (see below). In the ^{32}Si and ^{34}Si nuclei and $I = 2$ and 4 the collective amplitudes are well located inside the corresponding prolate and oblate minima. For $I = 6$ we have accidentally coexisting prolate and oblate minima, and for $I = 8$ the prolate minimum becomes the lowest one. In the nucleus ^{36}Si , at $I = 2$ and 4 we have shape coexistence between the prolate and oblate minima and, therefore, the collective amplitudes are equally distributed among them; the net result being that these states are, on the average, spherical. For $I = 6$ and 8 the prolate minimum becomes dominant and the collective amplitudes are localized around the minima.

Now, we want to compare our results with the available experimental data. The first piece of information we can compare to is the two neutron and two proton separation energies. The two neutron separation energies ($S(2n)$) for ^{34}Si and ^{36}Si are 11.83 MeV and 9.6 MeV, respectively. These numbers have to be compared with the experimental data [19]: 12.018 MeV and 8.587 MeV, respectively. We observe a reasonably good agreement with experiment for both nuclei. Concerning the two proton separation energies

($S(2p)$) we have used the previous results [10] for the magnesium isotopes to obtain 28.70, 32.72, and 36.26 MeV for $^{32-36}\text{Si}$, respectively. The corresponding experimental values [19] are 29.777, 33.737, and 35.430 MeV. The agreement between theory and experiment is again rather reasonable.

TABLE I

Calculated and experimental results for excitation energies (in MeV) and $B(E2, 0_{\sigma_1}^+ \rightarrow 2_{\sigma_2}^+)$ transition probabilities (in $e^2\text{fm}^4$). The columns marked (a), (b) and (c) correspond to $0_1^+ - 2_1^+$, $0_1^+ - 0_2^+$ and $0_1^+ - 2_2^+$, respectively. The ones marked (d), (e) and (f) correspond to $0_1^+ - 2_1^+$, $0_2^+ - 2_1^+$ and $0_1^+ - 2_2^+$, respectively.

	Energies (MeV)			Exp.	$B(E2) e^2\text{fm}^4$			Exp.
	(a)	(b)	(c)	(a)	(d)	(e)	(f)	(d)
^{32}Si	2.34	3.60	5.95	1.941	82.18	0.01	28.00	113 ± 33
^{34}Si	2.92	2.52	5.62	3.327	108.99	50.41	76.92	85 ± 33
^{36}Si	2.63	3.24	4.32	1.399	211.55	65.44	48.81	193 ± 59

In Table I the energy splittings between different states and the $B(E2)$ transition probabilities among some of them are compared with the available experimental data. Concerning the $B(E2, 0_1^+ \rightarrow 2_1^+)$ transition probabilities we find a quite good agreement with the experiment. In fact, our calculation reproduces the increase of the $B(E2)$ values in going from ^{34}Si to ^{36}Si . The other transition probabilities are much smaller than the $0_1^+ \rightarrow 2_1^+$ ones except in ^{34}Si . On the other hand, the theoretical values for the 2_1^+ excitation energies agree reasonably well with the experimental data for ^{32}Si and ^{34}Si but show a discrepancy of 1.2 MeV in the case of ^{36}Si . This discrepancy can be attributed to many sources, like not taking into account triaxial intrinsic configurations or not dealing with pairing correlations in a “beyond mean field” framework. Another source of the discrepancy could be related to the fact that our calculation is of the Projection After Variation (PAV) type instead of a more complete Projection Before Variation (PBV). Usually, the PBV method yields the rotational bands with moments of inertia that are larger than those in the PAV method [20,21]. In Ref. [10] we estimated the effect of considering PBV in calculations similar to the ones performed here but for some deformed magnesium isotopes. The conclusion was that the moment of inertia gets enhanced by a factor 1.4 and, therefore, the excitation energies have to be quenched by a factor 0.7. Unfortunately, the method

used in [10] for the mentioned estimation cannot be applied here as we are dealing with spherical ground states as well as with nearly spherical excited states.

On the other hand, shell model calculations [13,17,22] predict the second excited 0_2^+ state in ^{34}Si with the excitation energy (2 MeV in [13], 3 MeV in [17] and 2.6 MeV in [22]) which is below the excitation energy of the 2_1^+ state. In the experimental work of Ref. [13], such excited state was not observed but it was argued that, with the given experimental setup, it was quite difficult to detect it. However, in the experiment reported in [22], the mentioned excited 0^+ state was observed at a tentative excitation energy of 2.1 MeV. In our calculations we also observe a second 0^+ state in ^{34}Si (spherical in character) which lies at the excitation energy of 2.52 MeV. Our predicted $B(E2)$ transition probability $0_2^+ \rightarrow 2_1^+$ is $50.41 e^2\text{fm}^2$ which is a factor four smaller than the one obtained in the shell model calculations of [13,17] and a factor of six smaller than the predicted value of [22]. In Ref. [22], an experimental indirect estimate of the $B(E2)$ transition probability of $444(210) e^2\text{fm}^2$ is given. At this point it has to be recalled that the $B(E2)$ transition probabilities are more sensitive to the collective wave functions than the excitation energies. Therefore, a relatively small change in the collective wave functions can alter drastically the transition probabilities. In our case the collective amplitude of the 0_2^+ state (see Fig. 3) is concentrated around the spherical intrinsic configuration (result which is not consistent with the shell model expectations of [17,22]). This spherical intrinsic configuration has zero pairing correlations in our HFB calculations and, therefore, it can be expected that the inclusion of pairing correlations in a “beyond mean field” framework can modify the properties of the quadrupole dynamics around this point. To elucidate this and other deficiencies, a simultaneous projection on angular momentum and particle number (like the one performed in [8] with the Skyrme interaction) is called for.

4. Conclusions

In conclusion, we have performed angular momentum projected generator coordinate method calculations with the Gogny interaction D1S and the mass quadrupole moment as the generating coordinate in order to describe quadrupole collectivity in the nuclei ^{32}Si , ^{34}Si and ^{36}Si . We obtain a spherical ground state in ^{34}Si (in opposition to the nucleus ^{32}Mg) but it contains significant admixtures of the intruder configuration with two neutrons in the $f_{7/2}$ orbital. We also find in ^{34}Si the first excited 0^+ state with the excitation energy of 2.5 MeV. In all the nuclei considered the excitation energy of the lowest 2^+ state is rather well reproduced. Moreover, the $B(E2)$ transition

probability from the ground state to the 2^+ state is also well reproduced in the three cases. Other quantities like two neutron and two proton separation energies also compare well with experimental data.

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