CRANKING IN ISOSPACE — TOWARDS A CONSISTENT MEAN-FIELD DESCRIPTION OF $N = Z$ NUCLEI*

Wojciech Satuła and Ramon A. Wyss

Institute of Theoretical Physics, Warsaw University
Hoża 69, 00-681 Warsaw, Poland

and

Royal Institute of Technology, Physics Department Frescati
Frescativägen 24, 10405 Stockholm, Sweden

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Excitation spectra ΔE_T of $T = 0,1,2$ states in even–even (e–e) and odd–odd (o–o) $N = Z$ nuclei are analyzed within a mean-field based model involving isovector and isoscalar pairing interactions and the iso-cranking formalism applied to restore approximately isospin symmetry. It is shown that $T = 0$ states in o–o and $T = 1$ states in e–e nuclei correspond to two-quasiparticle, time-reversal symmetry breaking excitations since their angular momenta are $I \neq 0$. On the other hand the lowest $T = 2$ states in e–e and $T = 1$ states in o–o nuclei, which both are similar in structure to their even–even isobaric analogue states, are described as e–e type vacua excited (iso-cranked) in isospace. It appears that in all cases isoscalar pairing plays a crucial role in restoring the proper value of the inertia parameter in isospace *i.e.* ΔE_T.

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1. Introduction

The theoretical treatment of the generalized pairing problem is interesting and clearly nontrivial. Though its foundations in the form of generalized BCS (or HFB) theory were laid down almost thirty years ago in a number of papers by different groups [1] there are still many open problems.

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They cover a broad range of questions starting from the form of *effective* pn-pairing interaction and structure of *effective* pn-Cooper pairs, to problems related to symmetries and symmetry restoration, problems of interplay between the quasiparticle and isospin degrees of freedom to the role played by higher-order effects like quartetting or α-type condensation. It includes also the fundamental question concerning the experimental fingerprints of pn-collectivity.

Thus far, the strongest indications of pn(isoscalar)-collectivity come from: *(i)* shifts of crossing frequencies in ground-state bands of some e–e $N = Z$ nuclei (the best studied case is ^{72}Kr) and *(ii)* the mass defect in $N \sim Z$ nuclei commonly known as the Wigner energy problem. The latter effect can be locally restored by enforcing collective isoscalar pn-pairing [2]. Based on such a model we aim here to look into the impact of collective pn correlations upon the structure of $T = 0, 1, 2$ excitations in $N = Z$ nuclei. The goal is to reveal the interplay between quasiparticle and isospin excitations in $N = Z$ nuclei and role played by isoscalar pn-collectivity. The present paper supplements our previous letters [3, 4].

2. The paradigm of the Wigner energy

Traditional mass models based on the mean-field approach strongly underbind $N \sim Z$ nuclei [5–7]. This additional binding energy which is known as the Wigner energy, is usually parametrized as [8]:

$$E_W = W(A)|N-Z| + d(A)\delta_{NZ}\pi_{pn}\,, \quad \text{where} \quad \pi_{pn} = \begin{cases} 1 & \text{for odd–odd nuclei} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

and $W(A) \sim 47/A\,\text{MeV}$ [7]. A microscopic explanation of the Wigner energy within the <u>*mean-field*</u> approach is still lacking. The "congruence" energy mechanism proposed in [9] is in its nature a geometrical concept (*i.e.* independent on a specific form of nuclear interaction). There one assumes an enhanced particle-hole (ph) interaction at the $N \sim Z$ line which, however, is not at all present in traditional <u>*spherical*</u> Skyrme-HFB calculations, see Fig. 1. This is basically due to the pairing interaction which evenly distributes particles over spherical subshells thus strongly averaging over the properties of neighboring nuclei. One would expect the effect to show up (and it indeed does) in <u>*deformed*</u> microscopic calculations. However, as shown in Fig. 1 for a particular set of SIII+BCS mass calculations of Ref. [10] it is seen only in light nuclei and accounts for at most 20% of the empirical Wigner energy strength $W(A)$. Even in the limit of single-particle (sp) Skyrme-HF calculations one can account for at most 30% of the empirical value.

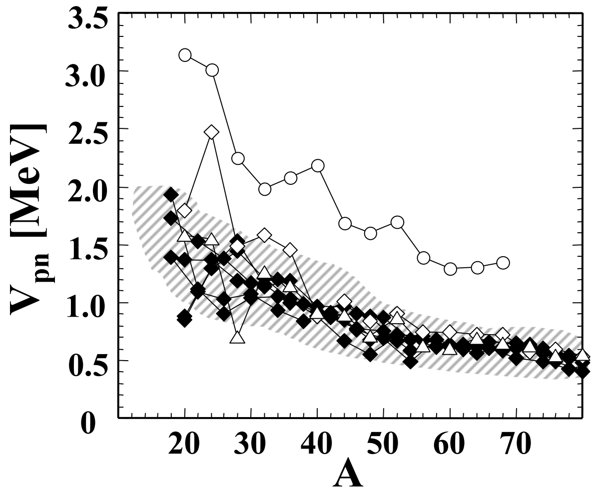

Fig. 1. Empirical and theoretical values of $V_{pn} \approx \frac{\partial^2 B}{\partial N \partial Z}$ (for both spherical and deformed HFB calculations). Solid symbols mark $V_{pn}(A)$ for e–e $N-Z=2,4$. In these cases V_{pn} probes essentially the quadratic term in the nuclear symmetry energy $E_{\rm sym} \sim (N-Z)^2$ and both empirical and theoretical curves do roughly overlay each other. For e–e $N=Z$ nuclei a strong enhancement of V_{pn} is seen in the empirical data ($\circ$). This apparent Wigner energy effect is not at all seen in the spherical calculations ($\triangle$) and only modestly visible in deformed calculations ($\diamond$).

It seems therefore, that a microscopic scenario based on isoscalar pn-pairing [2] is the most promising so far. It requires (within the mean-field model) the isoscalar pairing to be on the average stronger than the isovector. Although there are clear empirical [7] as well as theoretical [7,11,12] arguments that the Wigner energy is indeed due to the isoscalar interaction it is not at all settled whether it is due to a static pairing effect. In spite of that, in the following we will enforce the strength of the isoscalar pairing interaction so as to reproduce the Wigner energy and will look into the impact of these correlations on the structure of isobaric excitations in $N=Z$ nuclei.

3. The model

Our model Hamiltonian contains the Woods–Saxon sp potential and a schematic pairing interaction including isovector ($t=1$) and isoscalar ($t=0$) terms[1] of the form:

$$H_{\rm pair} = -G^{t=1} P^{\dagger}_{1\mu} P_{1\mu} - G^{t=0} P^{\dagger}_{00} P_{00}\,, \tag{2}$$

where $P^{\dagger}_{tt_z}$ create isovector and isoscalar pairs in time-reversed orbits:

[1] Throughout the text small letter t is reserved to label the type of pairing correlations while capital T corresponds to the total nuclear isospin.

$$P^{\dagger}_{1\pm 1} = a^{\dagger}_{\alpha\tau}\bar{a}^{\dagger}_{\alpha\tau}; \qquad P^{\dagger}_{10(00)} = \frac{1}{\sqrt{2}}\,(a^{\dagger}_{\alpha n}\bar{a}^{\dagger}_{\alpha p} \pm a^{\dagger}_{\alpha p}\bar{a}^{\dagger}_{\alpha n})\,. \tag{3}$$

Note that our interaction does not ascribe any specific structure (*e.g.* deuteron like) to the pn-pairs but only counts their effective number. The Bogoliubov transformation which is used (α runs over sp states, τ denotes third component of isospin, and k labels the quasiparticles):

$$\alpha^{\dagger}_{k} = \sum_{\alpha\tau>0} (U_{\alpha\tau,k}\, a^{\dagger}_{\alpha\tau} + V_{\bar{\alpha}\tau,k}\, a_{\bar{\alpha}\tau} + U_{\bar{\alpha}\tau,k}\, a^{\dagger}_{\bar{\alpha}\tau} + V_{\alpha\tau,k}\, a_{\alpha\tau}) \tag{4}$$

is the most general one, allowing for an unconstrained mixing of neutron and proton holes and particles. The problem is solved using the Lipkin–Nogami (LN) approximate number projection technique. In this respect the present model follows rather closely the description of Ref. [13]. The use of the LN model allows for mixing of both $t = 1$ and $t = 0$ phases at the cost of spontaneous isospin symmetry breaking [2, 13]. Generally, the mechanism of spontaneous symmetry breaking is the only mechanism which allows to take into account correlations which are seemingly beyond the mean-field. Obviously, the best example is the spontaneous breaking of spherical symmetry. Without it the *mean-field* would be capable to describe only a few nuclei. In our opinion the spontaneous breaking of isospin symmetry brings our solution closer to reality and allows for simulation of higher order effects like quartetting or α-clustering. These effects are present in exact-model solutions [14] which always do mix $t = 0$ and $t = 1$ phases in contrast to the generalized BCS approximation [15–17]. Moreover, in the following we will approximately restore this symmetry by applying the isospin cranking mechanism to generate isospin $T = 0, 1, 2$ states in $N = Z$ nuclei.

In the following calculations we will freeze the deformation degree of freedom of our Woods–Saxon potential at $\beta_2 = 0.05$. We compute $G^{t=1}(A)$ using the average gap method of Ref. [18] and taking a symmetric cut-off including the lowest $[A/2]$ neutron and $[A/2]$ proton sp states. The strength of isoscalar $G^{t=0}(A)$ pairing is computed by means of a direct fit to the Wigner energy strength $W(A) \sim 47/A\,$MeV, see [3] for details. This method assumes that $W(A)$ is entirely due to the isoscalar pairing field. Since we use here an almost spherical mean-field this assumption seems to be consistent with the conclusions of the preceding section. Deformation effects are expected to result in a rather modest reduction of $G^{t=0}$. Let us mention that the same procedures and parameters are used systematically in all presented calculations and that they exactly follow Refs. [3, 4].

4. Isobaric excitations in $N = Z$ nuclei

The isoscalar pairing field naturally takes into account the mass-excess in $N = Z$ nuclei. The aim of this section is to show that it is also of vital importance to understand the excitation energy pattern, ΔE_T, of the elementary isospin $T = 0, 1, 2$ excitations in even–even (e–e) and odd–odd (o–o) $N = Z$ nuclei. To compute these excitations, which follow a $\Delta E_T \sim T(T+1)$ pattern, we invoke the technique of cranking in isospace in analogy to spatial rotations. The discussion is organized as follows: *(i)* the pure single-particle model will be succeeded by *(ii)* the standard, isovector *nn*- and *pp*-paired model, and eventually *(iii)* by the isovector and isoscalar paired model.

4.1. The extreme single-particle model

Let us consider the isospin-cranked sp model: $\hat{H}^{\omega_\tau} = \hat{H}_{\rm sp} - \hbar\omega_\tau \hat{t}_x$. Let us further assume for simplicity that $\hat{H}_{\rm sp}$ generates a fixed, equidistant spectrum of 4-fold degenerate levels (including isospin and Kramers degeneracy) $e_i = \delta e i$. The isospin-cranking removes the isospin degeneracy. The quartets of sp-states split into two pairs of Kramers degenerate routhians $|\pm\rangle = 1/\sqrt{2}\,(|n\rangle \pm |p\rangle)$. The slope of the pair of $|+\rangle$ ($|-\rangle$) routhians is determined by their sp alignment in iso-space $\langle\pm|\hat{t}_x|\pm\rangle = \pm 1/2$, respectively.

Fig. 2 shows the sp routhians for the lowest sp configurations in even–even (upper part) and odd–odd (lower part) $N = Z$ nuclei. For the case of the even–even vacuum Fig. 2(a) one obtains configuration changes at iso-frequencies: $\omega_\tau^{(c)} = \delta e, 3\delta e, ..., (2n-1)\delta e$. At each crossing a pair of upsloping routhians become empty and a pair of downsloping routhians become occupied. This re-occupation process gives rise to stepwise changes in iso-alignment in units of $\Delta T_x{=}2$. Since simultaneously $T_y = T_z = 0$, one obtains a sequence of even-isospin states $T = 0, 2, 4, ..., 2n$. The odd-T sequence of states in even–even $N = Z$ nuclei can therefore be reached only for excited states. The lowest particle-hole excitation leading to odd-T states is shown in Fig. 2(b). Indeed, the initial iso-alignment of this state is $T_x = 1$ and, as in the case of the e–e vacuum discussed above, it is changed in steps of $\Delta T_x{=}2$ but at iso-frequencies $\omega_\tau^{(c)} = 2\delta e, 4\delta e, ..., 2n\delta e$. Simple calculations give the following expression for the total energy

$$E \equiv E^{\omega} + \omega_\tau T_x = \frac{1}{4}\delta e[1 - (-1)^{T_x}] + \frac{1}{2}\delta e T_x^2 \,. \tag{5}$$

Both odd- and even-T bands have therefore the same inertia parameter (reciprocal of the moment of inertia $a = \Im^{-1}$) $a = \delta e$ which is proportional to the average level spacing at the Fermi energy. The bands are shifted by $\Delta E = E_{T=1} - E_{T=0} = \delta e$ *i.e.* by a ph excitation energy.

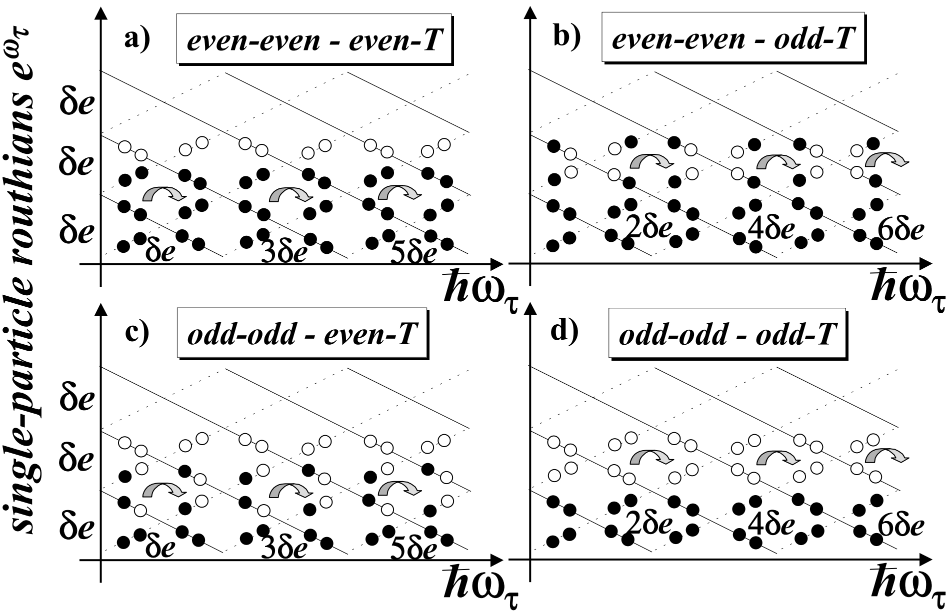

Fig. 2. Single-particle routhians representative for the following cases: (a) e–e nucleus even-T, (b) e–e nucleus odd-T case, (c) o–o nucleus even-T case, and (d) o–o nucleus odd-T case. Filled circles mark occupied states. Arrows indicate configuration changes *versus* iso-frequency.

For the case of o–o nuclei the last quartet is only half-filled. Hence, two different sp-configurations can be obtained: *(i)* an aligned one of $T_x{=}1$ Fig. 2(d) and *(ii)* a non-aligned one of $T_x{=}0$ Fig. 2(c). Since the reoccupation taking place at high iso-frequency always gives rise to a stepwise change in total isospin in units of $\Delta T_x{=}2$ the aligned configuration gives rise to an odd-T sequence of states while the non-aligned one builds up the even-T sequence of states. Observing further that crossings take place at $\hbar\omega_\tau^{(c)} = 2n\delta e$ (case *(i)*) and $\hbar\omega_\tau^{(c)} = (2n-1)\delta e$ (case*(ii)*) it is straightforward to compute the total energy:

$$E = -\frac{1}{4}\delta e\left[1-(-1)^{T_x}\right] + \frac{1}{2}\delta e T_x^2\,. \tag{6}$$

Two very interesting conclusions arise from this discussion. First of all, the *docoupling effect* is so strong in this case that it gives rise to a complete degeneracy of $T{=}0$ and $T{=}1$ states. This is in nice qualitative agreement with the data since $T{=}0$ and $T{=}1$ states are indeed nearly degenerate in o–o but not in e–e nuclei [19, 20]. Secondly, let us observe that the aligned configuration *does not break time-reversal invariance* while the non-aligned configuration *does break it*. Again this is in agreement with empirical data since all $T=0$ states have non-zero angular momentum $I \neq 0$ while $T=1$ states have $I=0$. In other words the theoretical treatment of $T=0$ states requires the explicit breaking of time-reversal invariance. In the presence of pairing correlations it means that $T=0$ states should be treated as two-quasiparticle (2qp) excitations. Treating them on the same footing as the neighboring e–e vacua would correspond to what is usually known in the literature as the *filling approximation*. Within the *filling approximation* pairs of $\boldsymbol{\alpha\alpha}$ type are always accompanied (with the same occupation probability) by pairs of $\bar{\boldsymbol{\alpha}}\bar{\boldsymbol{\alpha}}$ type forming a time-reversal invariant many-body state.

Note also, that the same situation applies for the $T=1$ states in e–e nuclei. These states, within the sp-model are ph excitations. Therefore, they do break time-reversal invariance and, in the presence of pairing correlations, correspond to 2qp configurations.

4.2. Influence of T=1 pairing

Let us investigate next the influence of standard *nn* and *pp* isovector pairing correlations on the sp iso-alignment processes and iso-inertias discussed in the preceding subsection. Let us still consider the equidistant level model and assume that the pairing gaps $\Delta_{pp} = \Delta_{nn} = \Delta$ are constant as a function of $\hbar\omega_\tau$. In the gap-non-self-consistent regime the BCS equations can be solved analytically. The eigenenergies (positive) are ($\tilde{e}_\alpha \equiv e_\alpha - \lambda$):

$$E_{\alpha,\pm}[\equiv E_{\bar{\alpha},\pm}] = \sqrt{\left[\tilde{e}_\alpha \pm \tfrac{1}{2}\hbar\omega_\tau\right]^2 + \Delta^2}, \tag{7}$$

and the associated eigenvectors:

$$\begin{bmatrix} U_{\alpha,n} \\ U_{\alpha,p} \\ U_{\bar{\alpha},n} \\ U_{\bar{\alpha},p} \\ V_{\alpha,n} \\ V_{\alpha,p} \\ V_{\bar{\alpha},n} \\ V_{\bar{\alpha},p} \end{bmatrix} : \longrightarrow \begin{bmatrix} U_\alpha^{(+)} \\ -U_\alpha^{(+)} \\ 0 \\ 0 \\ 0 \\ 0 \\ -V_\alpha^{(+)} \\ V_\alpha^{(+)} \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ U_\alpha^{(+)} \\ -U_\alpha^{(+)} \\ V_\alpha^{(+)} \\ -V_\alpha^{(+)} \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} U_\alpha^{(-)} \\ U_\alpha^{(-)} \\ 0 \\ 0 \\ 0 \\ 0 \\ -V_\alpha^{(-)} \\ -V_\alpha^{(-)} \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ U_\alpha^{(-)} \\ U_\alpha^{(-)} \\ V_\alpha^{(-)} \\ V_\alpha^{(-)} \\ 0 \\ 0 \end{bmatrix}, \tag{8}$$

where:

$$U_\alpha^{(\pm)} = \frac{1}{2}\sqrt{1 + \frac{\tilde{e}_\alpha \pm \frac{1}{2}\hbar\omega_\tau}{E_{\alpha,\pm}}}, \quad \text{and} \quad V_\alpha^{(\pm)} = \frac{1}{2}\sqrt{1 - \frac{\tilde{e}_\alpha \pm \frac{1}{2}\hbar\omega_\tau}{E_{\alpha,\pm}}}\,. \tag{9}$$

It is interesting to observe that the solutions of the sp model $|\alpha;\pm\rangle$ form, in this case, the canonical basis. Indeed, see (8), the quasiparticle operators take the following structure:

$$\alpha^\dagger_{\alpha,\pm} = \sqrt{2}U^{(\pm)}a^\dagger_{\alpha,\mp} - \sqrt{2}V^{(\pm)}\bar{a}_{\alpha,\mp}\,, \quad \text{where} \quad a^\dagger_{\alpha,\pm} = \frac{1}{\sqrt{2}}(a^\dagger_{\alpha,n} \pm a^\dagger_{\alpha,p})\,. \tag{10}$$

Let us further observe that this solution does also preserve iso-signature ($R_\tau = \exp^{-i\pi t_x}$) as a self-consistent symmetry.

It is relatively simple to derive an analytical expression for the kinematical iso-moment of inertia $\Im = T_x/\omega_\tau$. In case of the symmetric basis cut-off it reads:

$$\Im = \frac{1}{2\omega_\tau}\sum_{i=1}^{A/4}\left\{\frac{(2i-1)\delta e + \hbar\omega_\tau}{\sqrt{\frac{1}{4}[(2i-1)\delta e + \hbar\omega_\tau]^2 + \Delta^2}} - \frac{(2i-1)\delta e - \hbar\omega_\tau}{\sqrt{\frac{1}{4}[(2i-1)\delta e - \hbar\omega_\tau]^2 + \Delta^2}}\right\}. \tag{11}$$

In the low-frequency limit one obtains:

$$\Im \sim \sum_{i=1}^{A/4}\frac{\Delta^2}{\left[\frac{1}{4}\left[(2i-1)\delta e\right]^2 + \Delta^2\right]^{3/2}} \sim \frac{1}{\delta e}\left\{\begin{array}{llll} 0.80 & \text{for} & \Delta & = \frac{1}{2}\delta e \\ 0.96 & \text{for} & \Delta & = \delta e \\ 0.90 & \text{for} & \Delta & = 2\delta e \end{array}\right. \tag{12}$$

i.e. $\Im \sim \Im_{\rm sp}$ almost independently, within reasonable range, on the magnitude of pairing correlations, see also Fig. 3 showing the numerical results of the exact formula (11). Taking into account the self-consistency of the pair gap does not change this conclusion as it was demonstrated within the LN approximation in [3].

The isovector *nn*- and *pp*-pairing introduces a kind of *collectivity* on top of the discrete sp solutions. In order to take into account quantum fluctuations in a similar way as for spatial rotations, the cranking constraint in our calculations corresponds to $T_x = \langle \hat{t}_x\rangle = \sqrt{T(T+1)}$.

Following the results of the sp model we have calculated $T=2$ states in e–e nuclei by means of iso-cranking the quasiparticle vacuum to $T_x = \sqrt{6}$. The results of the calculations are shown in Fig. 4. In these calculations we used the static sp spectrum of the weakly ($\beta_2 = 0.05$) deformed Woods–Saxon potential plus standard isovector *pp*- and *nn*-pairing interaction. The results of the calculations are far below the empirical data. It is rather obvious that this level of disagreement must be related to a physics mechanisms which is beyond our model. First of all the ph isovector terms are missing.

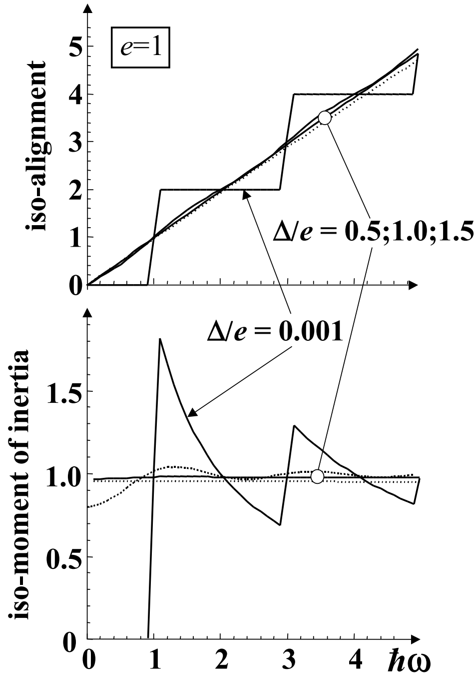

Fig. 3. Iso-alignment (a) and iso-MoI (b) calculated using Eq. (11) for very weak $\Delta = 0.001\delta e$, weak $\Delta = 0.5\delta e$, intermediate $\Delta = \delta e$ and strong $\Delta = 1.5\delta e$ pairing as a function of iso-frequency.

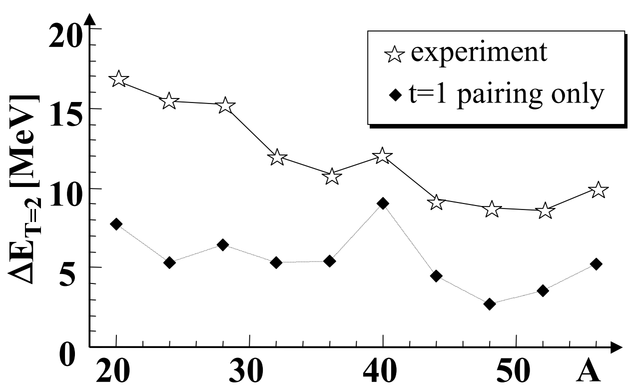

Fig. 4. Empirical (stars) and calculated (squares) excitation energies of $T = 2$ states in e–e $N = Z$. These calculations do include only $t = 1$ pairing correlations which give rise to large discrepancies between theory and experiment.

At present we do not have any control over these effects. However, it is very unlikely that they can fully cure the problem since the self-consistent mean-field models which do include the isovector ph channel yield:

$$E_{\rm sym}(A,T) \approx \frac{1}{2}\, a_{\rm sym}\frac{T^2}{A} \equiv \frac{1}{2}\left[\frac{T^2}{T(T+1)}\, a_{\rm sym}\right]\frac{T(T+1)}{A}\,, \qquad (13)$$

i.e. provide only $T^2/(T(T+1))$ fraction of the empirical symmetry energy[2].

4.3. The influence of $t=0$ pairing

The condensate of *pn* Cooper pairs with isospins coupled antiparallel is expected to lower considerably the iso-Moment-of-Inertia (iso-MoI) in a similar fashion as *nuclear superfluidity* influences the spatial MoI. Furthermore, in response to iso-rotations, the isoscalar paired nucleus is expected to undergo a phase transition due to the iso-Coriolis Anti-Pairing effect (CAP) similar to the standard CAP effect which is so well documented in high-spin physics. The situation here is probably even simpler as compared to the case of spatial rotations. There, the phase transition is always heavily modified due to the strong dependence of CAP on the orbital angular momentum (Stephens–Simon effect [21]). In isospace one would expect a phase transition to be of bulk type resembling much closer the Meissner effect known in macroscopic superconductors [22].

4.3.1. The $T=2$ states in even–even nuclei

To verify these ideas we have performed a set of microscopic calculations for $T=2$ states in $8<N=Z<30$ e–e nuclei using the LN model which includes both isovector and isoscalar pairing correlations. The $T=2$ states were computed by iso-cranking the e–e vacuum to iso-spin $T_x=\sqrt{6}$, see Fig. 5(a). These calculations nicely reveal all features of $t=0$ pairing expected from our intuitive considerations. The iso-MoI are indeed considerably lowered by $t=0$ pairing. At large iso-frequencies, but systematically below $T_x=\sqrt{6}$, we observe a phase transition from the $t=1$ and $t=0$ paired system to the $t=1$ paired system. The transition is indeed sharp, indicating its bulk character although one can always argue that fluctuations can smear it out, as it is usually the case in finite many-body system. In spite of that one can rather safely state that $T=2$ states in e–e $N=Z$ nuclei are $t=0$ unpaired, or at most vibrational. This conclusion is in nice agreement with direct calculations of the ground-states in $N-Z=4$ nuclei. These isobaric analogue states are predicted systematically to be *pn*-unpaired by various types of models [12,15,16]. The results of the $\Delta E_{T=2}$

[2] In fact, the empirical data in $N\sim Z$ nuclei are consistent with $T(T+\lambda)$ dependence for the symmetry energy where $\lambda\sim 1.25$ [19].

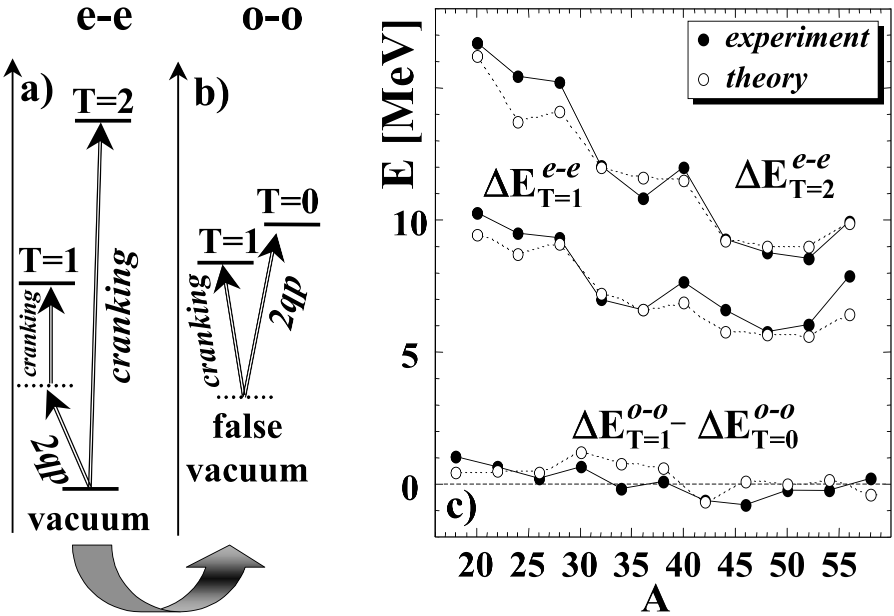

Fig. 5. Schematic illustration of the calculation scheme applied to compute $T = 1, 2$ states in e–e $N = Z$ nuclei (a) and $T = 0, 1$ states in o–o $N = Z$ nuclei (b). Part (c) shows the calculated and experimental excitation energies of these states.

excitation energy calculations are summarized in Fig. 5(c). Provided the simplicity of the model the agreement is rather amazing. Let us also bring the reader's attention to certain details. For example the shell-structure, reflecting proportionality of the sp iso-inertia to the sp energy splitting at the Fermi surface, is quite clearly seen. Simultaneously, let us also note that these shell-structure effects are smeared out quite substantially by $t = 0$ correlations as compared to calculations which include only $t = 1$ pairing, see Fig 4. For more details showing the basic features of these calculations we refer the reader to [3].

4.3.2. The $T = 1$ states in even–even nuclei

Let us now turn our attention to $T = 1$ states in e–e nuclei. Guided by the sp-model we will treat these states as the lowest 2qp configurations. We block the two lowest quasiparticles in the original Woods–Saxon basis by exchanging:

$$\begin{pmatrix} \mathrm{U}_K \\ \mathrm{V}_K \end{pmatrix} \longrightarrow \begin{pmatrix} \mathrm{V}^*_K \\ \mathrm{U}^*_K \end{pmatrix} \tag{14}$$

for $K = 1, 2$ while solving the HFB(LN) equations. Since at iso-frequency $\omega_\tau = 0$ the lowest 2qp state carries zero iso-alignment we impose the iso-cranking condition $T_x = 1$, as shown in Fig. 5(a).

At $\omega_\tau = 0$ the 2qp state mixes both $t = 0$ and $t = 1$ pairing phases (both in BCS as well as BCSLN approximations). However, different to the e–e iso-ground-state-band, subsequent iso-cranking of the 2qp state does not affect strongly isoscalar but quenches isovector pairing correlations. The *pp*- and *nn*-pairing correlations disappear completely, exactly when the iso-alignment reaches unity. At this point the system becomes trapped and the state of the nucleus does not change till the very high frequency — high enough to destroy isoscalar correlations. Note that, Fig. 5(c), the excitation energies of these states $\Delta E_{T=1}$ again agree surprisingly well with empirical data.

Certain clues explaining this seemingly counterintuitive picture can be obtained by going to the canonical basis in which the density matrix is diagonal. One observes that:

- For mixed-phases solution (below $T_x = 1$) the blocking of the original Woods–Saxon states is not equivalent to the blocking of the canonical states *i.e.* all canonical quasiparticles have fractional occupation numbers.

- With increasing frequency the occupation probability of the two lowest canonical quasiparticles increases gradually and, at $T_x = 1$, reaches exactly unity. In this state one pair decouples from the purely $t = 0$ *pn*-paired core and the isospins are aligned along cranking axis.

- At high frequency these canonical quasiparticles are built either on symmetric $|+\rangle \sim |n\rangle + |p\rangle$ or asymmetric $|-\rangle \sim |n\rangle - |p\rangle$ combinations of the initial Woods–Saxon sp states.

These observations are illustrated in Fig. 6. The upper panel of the figure clearly shows that the e–e core is inert. Indeed, the entire iso-alignment can be traced back to four canonical quasiparticles emerging from the degenerate (at $\omega_\tau = 0$) quartet of sp states. At low frequencies all quasiparticles forming this quartet contribute to the iso-alignment *i.e.* they all have fractional occupations, see open and black dots in the lower part of Fig. 6. With increasing ω_τ, the occupation probability of the energetically favored pair of canonical quasiparticles increase until they reach unity while the energetically unfavored pair becomes empty. Simultaneously, the iso-alignments of the sp canonical states building these quasiparticles approach exactly $\pm 1/2$ *i.e.* they are built on $\sim |n\rangle \pm |p\rangle$ combinations of the original basis, see the discussion in Sec. 4.1. Clearly, the process of iso-cranking restores the isospin

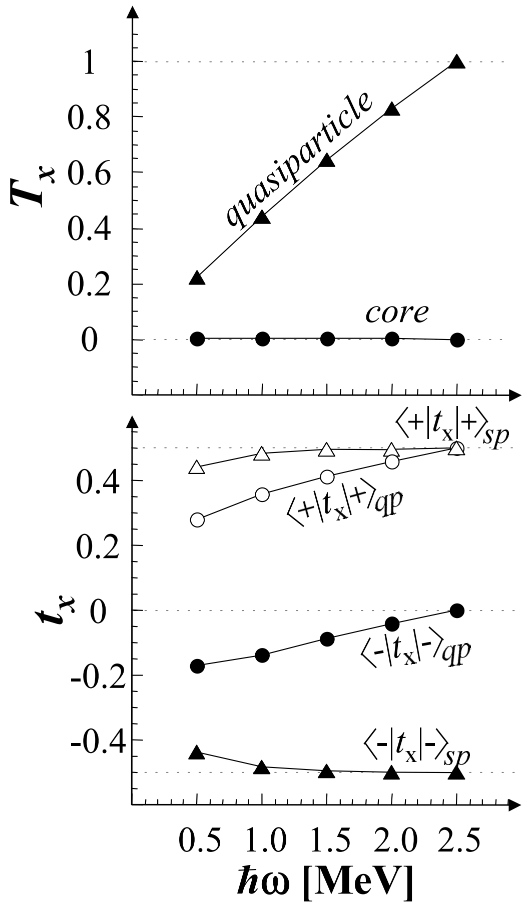

Fig. 6. Contributions of the core and the quartet of canonical quasiparticles to the total iso-alignment T_x for the case of an e–e nucleus with isospin $T = 1$. The lower part illustrates the contributions of the individual canonical sp and qp states to the total iso-alignment as a function of ω_τ. See text for details.

symmetry, $T_x = 1$ of the blocked 2qp state and, in fact, the symmetry of the whole system which is composed by a $t = 0$ paired o–o core and a single, properly coupled pair of quasiparticles.

Let us further note that the $t = 1$ and $t = 0$ pairing correspond to an entirely different scattering processes. This becomes evident after transforming the pair operators to $|\pm\rangle$ basis:

$$P^\dagger_{n\bar{n}} + P^\dagger_{p\bar{p}} \longrightarrow a^\dagger_+ \bar{a}^\dagger_+ + a^\dagger_- \bar{a}^\dagger_- \quad \text{and} \quad P^\dagger_{t=0} \longrightarrow a^\dagger_+ \bar{a}^\dagger_- - a^\dagger_- \bar{a}^\dagger_+ \,. \tag{15}$$

Since with increasing ω_τ the canonical basis approach $|\pm\rangle$ states, the contribution to the $t = 1$ pair field from the lowest qp states is $\propto U_+V_+^* \to 0$. In other words, with increasing ω_τ blocking is more and more effective in the $t = 1$ channel. Simultaneously, contributions of the blocked qp state to the $t = 0$ pair field are: $\propto U_-V_+^* + U_+V_-^* \to 1$. It means that $t = 0$ pairing is rather stable with increasing ω_τ. Only at very high ω_τ, when the cranking energy will overcome the coherence of $t = 0$ pairing, the phase transition to the $t = 1$ paired system will take place (see discussion in Sec. 4.3.1).

4.3.3. The $T = 0$ and $T = 1$ states in odd–odd nuclei

Let us finally discuss briefly $N = Z$ o–o nuclei. The lowest $T = 0$ state ($T = 0$ ground state) in o–o nuclei cannot be treated on the same footing as the ground state of the neighboring $N = Z$ e–e nuclei, see also [23]. The major argument stems from the simple empirical fact that all $T = 0$ *ground-states* in o–o nuclei have $I \neq 0$. Therefore, their treatment requires the explicit breaking of time-reversal symmetry as it was already discussed within the sp-model. This can be achieved only by treating this state as a 2qp configuration. In contrast, the lowest $T = 1$ states in o–o nuclei are expected to be seniority-zero states. The basic argument comes from the isobaric symmetry. These $I = 0$ states form a triplet of isobaric analogue states together with the ground states of e–e $N - Z = \pm 2$ nuclei. Hence, they all should have similar structure. A simple calculation scheme (see Fig. 5(b)) emerges from these considerations: *(i)* The $T = 0$ states should be treated as 2qp configurations *i.e.* seniority-two states typical for all o–o nuclei. *(ii)* The $T = 1$ states are HFB(LN) e–e like vacua, false vacua, excited in isospace by means of iso-cranking. Therefore they are seniority-zero states similar to e–e nuclei.

The relative excitation energy $\Delta E = E_{T=1} - E_{T=0}$ resulting from our calculations is shown in Fig. 5(c). Since 2qp excitations are almost as costly in energy as iso-cranking to $T = 1$, therefore, it is not surprising that both states stay nearly degenerate. What surprises is that the model predicts not only the near-degeneracy but also certain details like the inversion of $T = 1$ and $T = 0$ excitations taking place around $f_{7/2}$ ($A \sim 40$) sub-shell nuclei. Let us mention that in our approach the isoscalar and isovector phases are present both in $T = 1$ and $T = 0$ states. It is therefore evident that the near-degeneracy of $T = 1$ and $T = 0$ states cannot be used as an argument to rule out isoscalar pairing as it was done by the Berkeley group [24]. The key lays in the understanding of the underlying structure, which can be achieved by means of a microscopic model only.

5. Summary

We have shown that *mean-field* based models which incorporate both $t = 0$ and $t = 1$ pairing correlations and, at least in approximate manner, number and isospin projection are in principle capable to treat consistently $N \sim Z$ nuclei. The number projection is treated here at the level of the Lipkin–Nogami approximation [13, 25] while isospin is restored using the isospin cranking formalism [3, 26]. Within the model the e–e vacuum is a mixed $t = 0$ and $t = 1$ phase state due to the spontaneous symmetry breaking introduced by number projection [2, 13]. The $T = 2$ state in e–e nuclei is an e–e vacuum calculated at the iso-cranking frequency ω_τ corresponding to $T_x = \sqrt{6}$ while $T = 1$ state in e–e nuclei is described as a 2qp state at which $T_x = 1$. The $T = 0$ states in o–o nuclei are 2qp excitations while $T = 1$ states in o–o nuclei are e–e-like vacua (*false vacua*) computed at ω_τ corresponding to $T_x = \sqrt{2}$. In all cases $t = 0$ superfluidity plays a crucial role in restoring the correct iso-MoI and in turn, the excitation energies ΔE_T. In fact, thanks to the simplicity of the iso-cranking approximation, both the role and response of the $t = 0$ phase against iso-rotations can be very simply and intuitively understood by a number of beautiful analogies to well studied phenomena of high-spin physics.

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