# SELECTED ASPECTS OF PHYSICS OF FERMIONIC BUBBLES*

Piotr Magierski

Institute of Physics, Warsaw University of Technology
Koszykowa 75, 00-665 Warsaw, Poland

and Aurel Bulgac

Department of Physics, University of Washington
Seattle WA98195, USA

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We discuss properties of the Fermi system which contain one or more spherical (or almost spherical) objects. The interplay between various effects, such as shell correction and chaotic behavior is considered. We briefly mention the role of the temperature, pairing, and effects associated with bubble dynamics.

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The term "fermionic bubble" is used here to denote an (almost) spherical impurity/inhomogeneity immersed in an otherwise homogeneous fermionic system. There is a number of situations, where such systems can be formed. In particular, halo nuclei [1], bubble nuclei [2], highly charged alkali metal clusters [3], various heterogeneous atomic clusters [4] and neutron star crust [5] are a few scattered examples of systems which may be regarded as containing bubbles. Fermions reside there in a rather smoothly behaving mean-field potential, except for the regions, where bubbles are formed and, where it changes its depth. A simple semiclassical analysis shows that if the difference in depths is large then the amplitude of fermionic wave function inside the bubble is small (in most situations) and consequently a bubble will act approximately like a hard wall.

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Despite the fact that conditions that lead to formation of fermionic bubbles as well as their stability depend on a particular system under consideration, there are many aspects of bubble physics which are generic. For example, in the case of a finite one bubble system the question arises concerning the most favorable position of the bubble inside the system. It is tacitly assumed that bubble position has to be determined according to symmetry considerations. For a Bose condensate one can easily show that a bubble has to be off-center [6] but in the case of a Fermi system the most favorable arrangement is not obvious [2, 7]. The total energy of a many fermion system has the general form:

$$E(N) = e_v N + e_s N^{2/3} + e_c N^{1/3} + E_{\rm sc}(N), \tag{1}$$

where the first three terms represent the smooth liquid drop part of the total energy and $E_{\rm sc}$ is the pure quantum shell correction contribution, the amplitude of which grows in magnitude approximately as $\propto N^{1/6}$, see Ref. [8]. The character of the shell correction is in general strongly correlated with the existence of regular and/or chaotic motion [9, 10]. If a spherical bubble appears in a spherical system and if the bubble is positioned at the center, then for certain "magic" fermion numbers the shell correction energy $E_{\rm sc}(N)$, and hence the total energy $E(N)$, has a very deep minimum. However, if the number of particles is not "magic", in order to become more stable the system will in general tend to deform. This situation is very similar to the celebrated Jahn–Teller effect in molecules. One has to remember however that real deformations lead to an increased liquid drop energy, whereas merely shifting a bubble off-center deforms neither the bubble nor the external surface and therefore the liquid drop part of the total energy for neutral systems remains unchanged. Thus it is expected that the one-bubble system, for "non-magic" number of fermions, will rather exhibit softness toward the off-center displacements of the bubble. On the other hand as the bubble is moved off center, the classical problem becomes more chaotic [11] and one can naively expect that the single particle (s.p.) spectrum would approach that of a random Hamiltonian [12], and that the nearest-neighbor splitting distribution would be given by the Wigner surmise [13]. This is however not the case and one can show [2] that even for extreme displacements large gaps in the s.p. spectrum occur significantly more frequently than in the case of a random (which is closer to an uniform) spectrum. One should however keep in mind that large gaps in the energy spectra can occur if several noninteracting chaotic spectra are superimposed as well [14].

The formation of two or more bubbles at the same time, in finite, infinite or semi-infinite systems opens a plethora of new problems associated with their mutual interactions and the most favorable positional arrangements. For the sake of simplicity let us consider first two spherical, identical bubbles that have been formed in an otherwise homogeneous and electrically neutral fermionic matter. We shall also assume that the bubbles are completely hollow, stable and rigid. According to a liquid drop model approach the energy of the system should be insensitive to the relative positioning of the two bubbles. In the semiclassical approach, which is justified for the "sizeable" bubbles (*i.e.* when the Fermi wavelength is small comparing to the size of the bubble), the shell correction energy is determined by periodic orbits in the system. In the case of two spherical bubbles there exists only one such trajectory (with repetitions) which gives rise to the interaction energy between bubbles [5]. The interaction energy between the two bubbles of radii $R$, due to the existence of the periodic orbit at large separations $d$, reads:

$$E_{\mathrm{shell}} = \frac{1}{8\pi}\frac{\hbar^2 k_{\mathrm{F}}}{m}\frac{R^2}{d^3}\cos(2k_{\mathrm{F}}d), \tag{2}$$

where $k_{\mathrm{F}}$ denotes the Fermi momentum and $m$ the fermion mass. This semiclassical formula approximates the exact result surprisingly well even for relatively small distances [15]. Although the above result resembles the well known Ruderman–Kittel interaction between two point-like impurities, the interaction (2) is much stronger since the large obstacle reflects back more of the incident wave than a point object, which acts like a pure $s$-wave scatterer.

The interaction between many bubbles in the Fermi system may look, at the first glance, quite complicated since three-, four-, and other many-body terms will appear as a result of multiple fermion scattering between bubbles. One can show, however, that many-body terms are quite small and give merely a small corrections to the dominant pairwise interaction [15]. If the bubble density in the system becomes sufficiently large then a new form of matter can be created, foam. One might argue that sometimes a "misty" state could be more likely. As in the case of percolation, whether a "foamy" or a "misty" state would be formed, should strongly depend on the average matter density. At very low average densities, formation of droplets is more likely, while at higher average densities (lower than the equilibrium density however) the formation of a foam is more probable.

The best example of the many-bubble system is the neutron star where various inhomegeneities in the neutron matter may be formed. Apparently, an agreement has been reached concerning the existence of the following chain of phase changes as the density is increasing: nuclei $\rightarrow$ rods $\rightarrow$ plates $\rightarrow$ tubes $\rightarrow$ bubbles $\rightarrow$ uniform matter. The density range for these phase transitions is $0.04 - 0.1\mathrm{fm}^{-3}$ [16,17]. At densities of the order of several nuclear densities the quark degrees of freedom get unlocked and the formation of various quark matter droplets embedded in nuclear matter becomes then energetically favorable [18]. The appearance of different phases is attributed to the interplay between the Coulomb and surface energies. However, in the phase transition region the relative energies between phases coming from the liquid drop model are small and the shell effects coming from the interaction of the type (2) starts to play a dominant role [5]. Our results suggest that the inhomogeneous phase has perhaps an extremely complicated structure, maybe even completely disordered, with several types of shapes present at the same time.

All the fermionic system possessing bubbles are usually characterized by large pairing interaction which we neglected so far in our considerations. One should remember however that pairing correlations will be significant when the Fermi level occurs in a region of high s.p. level density and ultimately lead to a leveling of the potential energy surface. Also with increasing temperature the shell correction energy decreases. Certainly for large temperatures the positional entropy will determine the most favourable positions of the bubbles which for the one-bubble system will be still off-center [2]. At intermediate temperature $T$ the free energy associated with the bubble-bubble interaction is given in the form:

$$F_{\text{shell}} = -T \int\limits_0^\infty [g(\varepsilon, \mathbf{l}) - g_0(\varepsilon)] \log \left[1 + \exp\left(-\frac{\varepsilon - \mu}{T}\right)\right] d\varepsilon, \qquad (3)$$

where $g_0(\varepsilon)$ is the density of states with objects infinitely separated, $g(\varepsilon, \mathbf{l})$ is the density of states in the presence of bubbles and $\mathbf{l}$ is an ensemble of geometrical parameters describing these objects and their relative geometrical arrangement. One can show that it leads to an exponential suppression of the interaction strength as compared to Eq. (2) [19].

The energetics of two or more bubbles, their relative placements and positions with respect to boundaries, their collisions and bound state formation, their impact on the role played by periodic or chaotic trajectories, and their temperature dependence, are but a few in a long list of challenging questions. A plethora of new, extremely soft collective modes is thus generated. The character of the response of such systems to various external fields is an extremely intricate issue. Since the energy of the system changes only very little while the bubble(s) is being moved, a slight change in energy can result in large scale bubble motion. However, still the problem of moving bubbles is far from being fully understood. One expects that a bare bubble mass $M$ should be renormalized in the same way as a bare mass of a particle is renormalized in the quantum field theory, since the impurity travels in the fermionic medium in a cloud of medium excitations. Since the mass is large comparing to fermion masses the motion of the bubble will be governed by two main physical processes: an infrared divergence leading to Anderson's orthogonality catastrophe and recoil of the bubble. The former effect dominates for $M \to \infty$ and the latter for $M \to 0$ (see *e.g.* [20] and references therein). However, there is no agreement so far on the relative importance of these two effects in the general case.

Hopefully, the few results highlighted here will convince the reader of the richness of these systems where both static and dynamic properties are challenging to describe.

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