THE SNO-EXPERIMENT AND NEUTRINO OSCILLATIONS*

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Using the pre-SNO neutrino oscillation data of the solar and the atmospheric neutrino oscillations including and excluding the LSND (Los Alamos) measurements fits for the three mixing angle of the unitary transformation between the three neutrino mass eigenstates and the weak eigenstates are given. At the same time the differences of the squared masses are fitted to the data. Using an averaged upper value of $\langle m_{\nu e}\rangle = 0.62$ eV from the neutrino double beta decay, one can limit the sum of the three neutrino masses to be less than 2.53 eV. The new data from the Sudbury Neutrino Observatory allow for the first time with the help of the SuperKamiokande data to determine the solar e and $(\mu+\tau)$ neutrino fluxes separately. The sum is within errors identical to the theoretical electron neutrino flux from the sun. We thus see all expected solar neutrinos. They only oscillated partially from electron to $\mu+\tau$ neutrinos. In the last part we report calculations of neutrino masses in the R-parity violating Minimal Supersymmetric Standard Model ($\not{R}$ MSSM). The R-parity violating interaction mixes the three neutrino flavors by R-parity violation with the four neutralinos (photino, zino and the two higgsinos). One finds neutrino masses for the first two neutrinos between 0.001 and 0.04 eV and for the third one between 0.03 and 1 eV.

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1. Introduction

Until now the only indication for finite masses of the neutrinos are the neutrino oscillations. Direct measurements of the neutrino masses gave the following upper limits [1, 2]:

$$m_{\nu e} < 2.3\,; \quad m_{\nu\mu} < 160 \text{ keV}\,; \quad m_{\nu_\tau} < 23 \text{ MeV}\,. \tag{1}$$

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In the standard model neutrinos are massless and left-handed. If they would be massive, a boost can make out of a left-handed neutrino a right-handed one.

In the supersymmetric model we add to the left-handed doublet of a neutrino and a lepton its SUSY partners.

$$\begin{pmatrix} \nu_e & \nu_\mu & \nu_\tau \\ e^- & \mu^- & \tau^- \\ \tilde{\nu}_e & \tilde{\nu}_\mu & \tilde{\nu}_\tau \\ \tilde{e}^- & \tilde{\mu}^- & \tilde{\tau}^- \end{pmatrix}_{\mathrm{left}} . \tag{2}$$

If one now assumes that the neutrinos are massive with the mass eigenstates

$$(\nu_1, \nu_2, \nu_3) \tag{3}$$

which are different form the weak or the production eigenstates

$$(\nu_e, \nu_\mu, \nu_\tau) \tag{4}$$

one obtains automatically neutrino oscillations. The mass eigenstates (3) and the weak or flavour eigenstates (4) are connected by a unitary matrix.

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} . \tag{5}$$

The unitary matrix connecting the mass (3) and the flavour eigenstates (4) can be characterized by three angles and in the case of Dirac neutrinos by one CP phase and in the case of Majorana neutrinos by two CP phases.

$$U\left(\vartheta_{12}, \vartheta_{13}, \vartheta_{23}, \varphi_{12}, \varphi_{23}\right) . \tag{6}$$

In this work we assume that CP symmetry is not violated and thus the CP violating phases can take only two values $\lambda^{CP} = \mathrm{e}^{i\varphi} = \pm 1$.

Neutrino oscillations give no information about the CP phases, but the charmed II results constrain the CP-phases so that they cannot be all the same for all three neutrinos [11]. This means the choice

$$\lambda_{12}^{CP} = \mathrm{e}^{i\varphi_{12}} = +1 ; \quad \lambda_{23}^{CP} = \mathrm{e}^{i\varphi_{23}} = +1 \tag{7}$$

is not allowed.

The neutrino oscillations are often analyzed in a pairwise mixing of only two neutrinos with the orthogonal transformation between the two mass states and the two flavour states.

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\vartheta & -\sin\vartheta \\ \sin\vartheta & \cos\vartheta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}. \tag{8}$$

In this type of analysis one can construct a contradiction of the LSND result, describing oscillations from $\bar{\nu}_\mu$ to $\bar{\nu}_e$ and the solar neutrino oscillations, if one assumes that the electron neutrinos ν_e oscillate to ν_μ. The first one requests a difference of the masses squared of the order of 0.3 to 1.0 eV^2 and the second a difference of the masses squared between 10^{-5} and 7×10^{-5} eV^2, depending if one takes a Small (SMA) or a Large (LMA) Mixing Angle solution of oscillations in matter (MSW effect). But an analysis according to equation (9) with the mixing of only two neutrinos is too restrictive. This is similar as when you want to fly from Muenchen to Milano in a two dimensional plane in a straight line. The Alps make this impossible. But as soon as you go in a space of three dimensions, you have no difficulty having a straight line in the first two dimensions, but going up in altitude and crossing the Alps with the help of the third dimension.

The results which we have about neutrino oscillations indicate that the mass eigenstates are strongly mixed in the flavour or production eigenstates. So we expect that an oscillation analysis in two dimensions including only the difference of the masses squared of two neutrinos and their mixing angle θ will be not sufficient and leads to totally wrong conclusions. We therefore use in this work always a three neutrino analysis of the solar, the atmospheric and the LSND neutrino experiments. Due to the large mixing angles it seems that mostly all three neutrinos are involved in the oscillations.

2. Neutrino oscillations and neutrino masses

The unitary matrix which transforms from the neutrino mass eigenstates to the flavour eigenstates (5) can be parametrized by three angles and two phases (6), if one assumes Majorana neutrinos. The CP violating phases can be reduced to one in the case of Dirac neutrinos. In the literature one finds several fits to the solar and the atmospheric neutrino oscillation data, where the SNO data from June 17, 2001 are not yet included. These three neutrino analyses of the solar and the atmospheric neutrino data include or exclude the LSND data from Los Alamos. Below it will always be indicated if the LSND data are included or if they are excluded.

The flavour neutrino mass matrix has the form

$$m_{\alpha\beta} = \sum_{i=1}^{3} U_{\alpha i}\, m_i\, U^{+}_{\beta i}\,(\vartheta_{12}, \vartheta_{13}, \vartheta_{23})\,, \tag{9}$$

where we assumed that the CP phase factors are real and assume the values $\lambda_{12}^{CP} = \pm 1$ and $\lambda_{13}^{CP} = \pm 1$ (7).

The three neutrino analysis of the oscillation data [4–8] yield the three mixing angles and two differences of the squares of neutrino masses.

$$\begin{aligned}
&\vartheta_{12}, \vartheta_{13}, \vartheta_{23} \\
\Delta m_{21}^2 &= m_2^2 - m_1^2\,, \\
\Delta m_{23}^2 &= m_3^2 - m_2^2\,, \\
m_1 &= m_1 \text{ (assumption)}\,, \\
m_2 &= \left[m_1^2 + \Delta m_{21}^2\right]^{1/2}, \\
m_3 &= \left[m_1^2 + \Delta m_{21}^2 + \Delta m_{32}^2\right]^{1/2}. \qquad (10)
\end{aligned}$$

Under the assumption that the phases are real (6), one can calculate the transformation U of equation (5), which is under these assumptions an orthogonal transformation. It is not determined uniquely, since the phases can still have the values $(+1,-1)$ and $(-1,-1)$ (7). The combination $(+1,+1)$ is excluded by CHARME II [3] and the combination $(-1,+1)$ can be reduced to the combination $(+1,-1)$ by relabelling. Table I gives the results of these fits from the literature.

TABLE I

Fits for $\Delta m_{21}^2, \Delta m_{32}^2$ and $\vartheta_{12}, \vartheta_{23}, \vartheta_{13}$ as defined in Eqs. (6) and (11) from references 4 to 8 without or with the inclusion of LSND.

Δm_{21}^2 [eV^2]	Δm_{32}^2 [eV^2]	ϑ_{12}^0	ϑ_{23}^0	ϑ_{13}^0	Ref.
no LSND		no LSND			
$(3 \div 70) \times 10^{-6}$	0.01	53–62	28–37	≤ 13	[8]
$(4 \div 70) \times 10^{-6}$	1	51–72	27–32	< 4	[8]
$(4 \div 70) \times 10^{-6}$	0.1	51–72	28–33	< 4	[8]
10^{-4}	8×10^{-4}	39	45	27	[6]
with LSND		with LSND			
$(1 \div 10) \times 10^{-4}$	0.3	35	27	13	[4]
$(1 \div 10) \times 10^{-4}$	0.3	54	27	13	[4]
3×10^{-4}	1	45	29	4	[7]
$(1 \div 10) \times 10^{-4}$	0.4	38	26	10	[5]
$(4 \div 70) \times 10^{-6}$	1	51–72	27–32	3–4	[8]

From Eq. (10) one can now calculate the neutrino mass matrix in flavour or weak interaction space, if one makes an assumption about the value of one neutrino mass, for example m_1. Since the phases (6) do not depend on the neutrino oscillations, they cannot be determined. In the following we are choosing all possible different neutrino phases for Majorana neutrinos and give for the neutrino mass matrix (9) element by element always the largest value. So the neutrino mass matrices given in Table II are element by element upper limits for the different assumptions of the mass m_1 of the lowest mass eigenstate.

TABLE II

Neutrino mass matrix in flavour space $(\nu_e, \nu_\mu, \nu_\tau)$ defined in Eq. (10) calculated for the different fits to the oscillations given in Table I. The CP phases $\lambda^{CP}_{12/13} = \pm 1$ are assumed to be real and are varied to all possible combinations. Always the largest value, element by element, is given. The four matrices are therefore upper limits for the assumptions $m_1 = 0$ eV and $m_1 = 0.1$ eV without and with the inclusion of LSND.

no LSND included	
$m_1 = 0$ eV	$m_1 = 0.1$ eV
$\begin{pmatrix} 0.00 & 0.01 & 0.00 \\ 0.01 & 0.11 & 0.10 \\ 0.01 & 0.10 & 0.11 \end{pmatrix}$	$\begin{pmatrix} 0.10 & 0.01 & 0.01 \\ 0.01 & 0.18 & 0.08 \\ 0.01 & 0.08 & 0.19 \end{pmatrix}$
with LSND included	
$m_1 = 0$ eV	$m_1 = 0.1$ eV
$\begin{pmatrix} 0.03 & 0.06 & 0.07 \\ 0.06 & 0.29 & 0.34 \\ 0.07 & 0.34 & 0.46 \end{pmatrix}$	$\begin{pmatrix} 0.11 & 0.05 & 0.07 \\ 0.05 & 0.35 & 0.30 \\ 0.07 & 0.30 & 0.50 \end{pmatrix}$

One sees clearly, that the inclusion of LSND is increasing the upper limits of the matrix elements of the neutrino mass matrix in flavor space. This means that the masses of the neutrinos get appreciably larger with the inclusion of LSND. Without LSND the average neutrino mass for $m_1 = 0.1$ eV has an average value of 0.16 eV and with the inclusion of LSND an average mass of 0.32 eV. In table two we have included an absolute mass scale by assuming without further justification a mass for the lowest mass eigenstate. One can also obtain an absolute mass scale also from experiment: The triton decay and the neutrinoless double beta decay give both such limits. The more restricted limit one obtains from the neutrinoless double beta decay.

$$
\begin{aligned}
|\langle m_{\nu_e}\rangle| &= \left|\sum_{i=1}^{3} m_i \xi_i(CP)\right| U_{ei}|^2| \leq 0.62\ [\mathrm{eV}]\,, \\
\xi_1 &= 1\,; \quad \xi_2 = \lambda_{12}^{CP}\,; \quad \xi_3 = \lambda_{13}^{CP}\,.
\end{aligned}
\tag{11}
$$

In Eq. (11) we have used the double beta decay matrix elements of the Tuebingen group [9, 10] and the ^{76}Ge neutrinoless double beta decay experiment of Baudis *et al.* We build the transformation U in Eq. (5) from the mixing angles of Table I and choose all possible phase combinations (6) and increase the mass of the lightest neutrino m_1 in Eq. (10), so that we reach the maximal allowed averaged neutrino mass of 0.62 eV (11). This yields now element for element an other limit of the neutrino mass matrix in flavor space.

$$
\begin{aligned}
|m_{\alpha\beta}|\ [\mathrm{eV}] &\leq \begin{pmatrix} 0.60 & 0.97 & 0.85 \\ 0.97 & 0.76 & 0.80 \\ 0.85 & 0.80 & 1.17 \end{pmatrix} \\
\sum_{i=1}^{3} m_i &\leq \mathrm{trace}\{|m_{\alpha\beta}|\mathrm{max}\} \\
&= 0.60 + 0.86 + 1.17 = 2.53\ [\mathrm{eV}]\,.
\end{aligned}
\tag{12}
$$

One obtains therefore for the sum of all three neutrino masses an upper limit of 2.53 eV.

From the measurement of the triton decay, where the Minsk and the Troisk group give values of 2.8 to 2.3 eV one can extract also an upper limit for the sum of the three neutrino masses. But this limit lies higher.

3. New results from SNO

The Sudbury Neutrino Observatory announced on June 17, 2001 results which make it now highly probable, that one sees all the neutrinos [20] which are produced by the sun [21]. The results indicate that about 2/3 of the ^{8}B electron neutrinos were oscillating into muon or tauon neutrinos. The Sudbury Neutrino Observatory (SNO) is located in Canada in the Creighton mine in Ontario, 2000 m under ground. The detector consists of 1 000 tons of heavy water D_2O in an acrylic spherical vessel with the diameter of 12 m and 9 456 photomultipliers on an outer sphere of a diameter of 17 m. Everything is inside a vessel of about a cylindrical form with a diameter of 22 m. In principle the following reactions are possible:

$$\begin{array}{lll} \nu_e + d & \longrightarrow p + p + e^- & \text{(CC)}, \\ \nu_x + e^- & \longrightarrow \nu_x + e^- & \text{(ES)}, \\ \nu_x + d & \longrightarrow p + n + \nu_x & \text{(NC)}. \end{array} \tag{13}$$

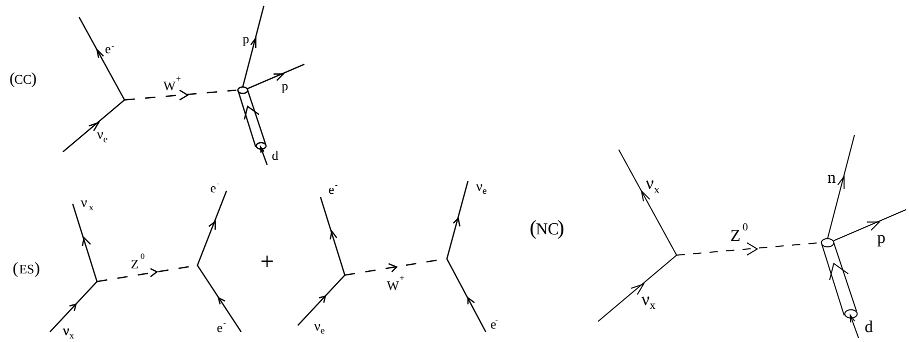

Fig. 1. Charge current (CC), electron scattering (EC) and neutral current (NC) diagrams for neutrino scattering and reactions in SNO.

Figure 1 shows the diagrams of the Charge Current (CC) the elastic electron scattering (ES) and the Neutral Current (NC) diagrams which can be measured in principle by SNO. Presently only (CC) and (ES) have been measured. The charge current (CC) can only be measured because they are using heavy water with deuterium replacing the hydrogen. The ^{8}B solar neutrinos go up to an energy of 14.6 MeV. They would have a too small energy to change a neutron in ^{16}O into a proton. But this is possible for the deuteron which is only bound by 2.2 MeV. The lower threshold of the SNO detector for the ^{8}B electron neutrinos from the sun is 6.5 MeV. The charge current reaction (CC) is measured by looking to the Cherenkov radiation from the electron. But at the same time one gets also the Cherenkov radiation from the electron, which is elastically scattered by the incoming solar neutrinos (ES). One can now separate the two by studying the angular correlations. In the elastic neutrino–electron scattering (ES) a neutrino of about 10 MeV is hitting an electron, which has a rest mass of about 0.5 MeV and thus its angular distribution is peaked forward. In the charge current reaction (CC) one is hitting with the electron neutrino the deuteron and one obtains roughly an isotropic distribution. The charged current measures only the electron neutrino flux from the sun. It is with the help of the angular distribution determined to be:

$$
\begin{aligned}
\Phi^{\rm CC}_{\rm SNO}(\nu_e) &= (1.75 \pm 0.23) \times 10^6\ [\mathrm{cm}^{-2}\mathrm{s}^{-1}]\,, \\
\Phi^{\rm EC}_{\rm SNO}(\nu_x) &= (2.39 \pm 0.49) \times 10^6\ [\mathrm{cm}^{-2}\mathrm{s}^{-1}]\,, \\
\Phi^{\rm ES}_{\rm SK}(\nu_x) &= (2.32 \pm 0.10) \times 10^6\ [\mathrm{cm}^{-2}\mathrm{s}^{-1}]\,, \\
\sigma(\nu_e e^-) &\approx 6 \cdot \sigma(\nu_\mu e^-;\ \nu_\tau e^-)\,.
\end{aligned} \tag{14}
$$

In principle one could determine from the two SNO measurements of the neutrino fluxes for the charge current and the elastic scattering reaction with the relation between the electron–neutrino electron and the muon–neutrino electron cross section the flux of muon and tauon neutrinos reaching the SNO detector. But the errors of the SNO measurement for the elastic electron neutrino–electron scattering is too large. One can here use the more accurate result of SuperKamiokande (14). The values given in (15) are neutrino fluxes from the sun, calculated without neutrino oscillations. From the elastic electron–neutrino electron scattering data from SK (SuperKamiokande) and the electron neutrino flux determined by the charged current in SNO, one sees that one must have muon– and tauon–neutrinos, which are detected in the elastic electron–neutrino electron scattering (ES) in SuperKamiokande (SK). If one now takes into account the different values for the cross sections of the neutrino electron scattering one obtains a muon– and tauon–neutrino flux from the sun of:

$$
\begin{aligned}
\Phi(\nu_\mu + \nu_\tau) &= (3.69 \pm 1.13) \times 10^6\ [\mathrm{cm}^{-2}\mathrm{s}^{-1}]\,, \\
\Phi_{\rm total}\ (\nu_e + \nu_\mu + \nu_\tau) &= (5.44 \pm 0.99) \times\ 10^6\ [\mathrm{cm}^{-2}\mathrm{s}^{-1}]\,, \\
\Phi_{\rm SUN}\ (\nu_e) &\approx 5.1 \times 10^6\ [\mathrm{cm}^{-2}\mathrm{s}^{-1}]\,.
\end{aligned} \tag{15}
$$

The sum of the muon– and tauon–neutrino fluxes and the electron–neutrino flux determined by the charge current reaction (15) agrees very nicely with the theoretical result of Bahcall and Pinsonneault [21], who predict a total electron neutrino current produced by the sun of about 5.1×10^6 cm^{-2}s^{-1} thus we seem to see all electron neutrinos from the sun. But two third of the electron neutrinos have been oscillating into other neutrinos.

The analysis using the charge current electron neutrino flux measurement from SNO and the elastic scattering neutrino flux measurement from SuperKamiokande is shown in figure 2.

The results seem to indicate within the error bars that there is now oscillation of the electron neutrinos from the sun to sterile neutrinos.

The next step is the measurement of the neutral current reaction (NC) (14). Figure 1 shows that for the neutral current reaction one has only neutrinos, neutrons and protons in the final state. All three do not produce

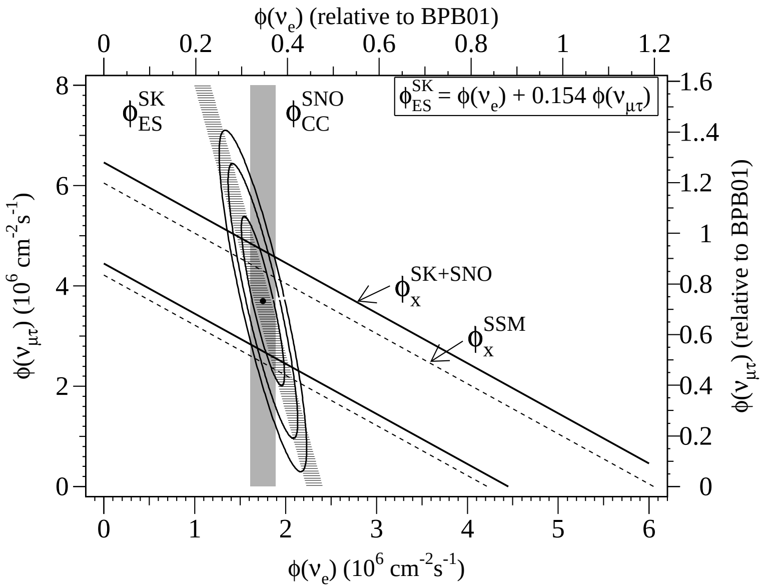

Fig. 2. The figure shows the charge current (CC) measurement of SNO $\Phi^{CC}_{SNO}(\nu_e) = (1.75 \pm 0.23) \times 10^6\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$ on the abscissa and the muon and tauon neutrino flux derived with help of the SuperKamiokande (SK) elastic neutrino–electron measurement at the ordinate (from Ref. [20]).

Cherenkov radiation in water. To detect this Neutral Current reaction (NC) (14) the SNO collaboration has given salt NaCl into the heavy water of the detector. This now allows to see the capture γ-rays of the neutrons cascading down in Na or Cl. The neutral current reaction (NC) (14) will measure the total neutrino flux from the sun without distinguishing between electron, muon and tauon neutrinos. If everything is consistent and correct, it should give within error bars the same number of the total flux already given in Eq. (16).

4. Neutrino masses and supersymmetry

In the minimal supersymmetric model (MSSM) one has for every boson and for every fermion an supersymmetric partner. Each fermion has a SUSY boson and each boson has a SUSY fermion. In the standard model (SM) the fermions are the matter particles like the quarks and the leptons, while the bosons are the carrier of the forces like the photon γ, the charged $W^{\pm}$ and the neutral Z^0 vector bosons and the gluons. To each multiplet of particles in the SM we add in the MSSM the corresponding superfields.

$$L_e = \begin{pmatrix} \nu_e \\ e^- \\ \tilde{\nu}_e \\ \tilde{e}^- \end{pmatrix}_{\text{left}} \; ; \; L_u = \begin{pmatrix} \nu_\mu \\ \mu^- \\ \tilde{\nu}_\mu \\ \tilde{\mu}^- \end{pmatrix}_{\text{left}} \; ; \; L_\tau = \begin{pmatrix} \nu_\tau \\ \tau^- \\ \tilde{\nu}_\tau \\ \tilde{\tau}^- \end{pmatrix}_{\text{left}} ,$$

$$Q_u = \begin{pmatrix} u \\ d \\ \tilde{u} \\ \tilde{d} \end{pmatrix}_{\text{left}} \; ; \; Q_c = \begin{pmatrix} c \\ s \\ \tilde{c} \\ \tilde{s} \end{pmatrix}_{\text{left}} \; ; \; Q_t = \begin{pmatrix} t \\ b \\ \tilde{t} \\ \tilde{b} \end{pmatrix}_{\text{left}} ,$$

$$H_1 = \begin{pmatrix} h_1^\circ \\ h_1^- \\ \tilde{h}_1^\circ \\ \tilde{h}_1^+ \end{pmatrix}_{Y=1} \; ; \qquad H_2 = \begin{pmatrix} h_2^+ \\ h_2^\circ \\ \tilde{h}_2^+ \\ \tilde{h}_2^\circ \end{pmatrix}_{Y=-1} . \tag{16}$$

The interactions in the MSM are the same as in the SM, one only replaces the fields by the superfields. Figure 1 shows that one has to replace always zero or an even number of fields from the standard model (SM) by SUSY particles. If one replaces and odd number of SM fields by SUSY particles, one cannot conserve the total angular momentum. Thus this minimal extension of the SM to the Minimal Supersymmetric Standard Model (MSSM) one has a new quantum number which one calls R parity. It is $R = +1$ for an even number of SUSY particles and $R = -1$ for an odd number of SUSY particles.

The mass of the three neutrinos in supersymmetry is zero originally and their acquire mass by mixing with the four neutralinos: The photino $\widetilde{\gamma}$, the zino $\widetilde{Z}^0$, the Higgsino $\widetilde{H}_1^0$ and a second Higgsino $\widetilde{H}_2^0$. But in the MSSM one cannot mix standard particles like the three neutrinos ν_e, ν_μ and ν_τ with SUSY particles. This violates R parity.

One now argues that R parity is until now not a quantity which is conserved by a symmetry principle. Thus one expects to have also terms in the Lagrangian which violate R parity conservation.

$$\begin{aligned} W_{\not R} = {} & \lambda_{ijk} L_i L_j E_k^{\mathrm{c}} + \lambda'_{ijk} L_i Q_j D_k^{\mathrm{c}} \\ & + \mu_j L_j H_z + \lambda''_{ijk} U^{\mathrm{c}} D_j^{\mathrm{c}} \\ & + \{\text{soft } \not R \text{ terms}\}. \end{aligned} \tag{17}$$

Here L_i are the left-handed lepton superfields of the three families defined in Eq. (13) and Q_j are the left-handed super quark fields, where j runs over

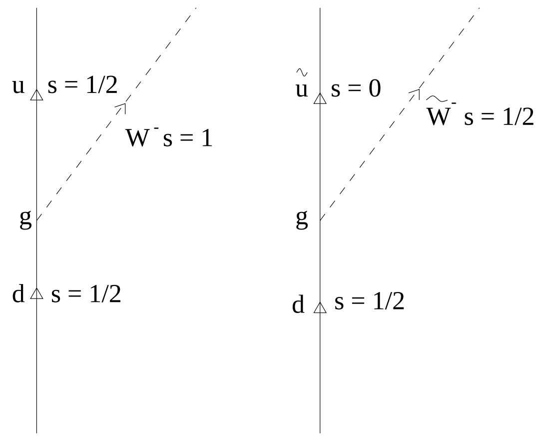

Fig. 3. The left diagram is showing in the standard model (SM) the microscopic process of the weak beta decay changing a "down" into an "up"-quark and the emission of a W^- vector boson with the coupling constant g. The right side shows a diagram in the minimal supersymmetric model (MMSM) which changes a "down"-quark into a SUSY-"up"-quark and into a SUSY-vector boson $\widetilde{W}^-$ with the same coupling constant g.

the three families (13). The superfields E_k and D_k correspond to the right-handed lepton and quark super fields.

$$E_{k=1} = \begin{pmatrix} e^- \\ \tilde{e}^- \end{pmatrix}_{\text{right}} \; ; \; D_{k=1} = \begin{pmatrix} d \\ \tilde{d} \end{pmatrix}_{\text{right}} . \tag{18}$$

The index k can run over the three families $k = 1, 2, 3$. The upper index "c" in Eq. (17) indicates the charge conjugate state. The soft SUSY breaking terms contain only superfields. The coupling constant λ''_{ijk} and the corresponding term in the soft R-parity breaking terms are put to zero to prevent a fast decay of the protons.

The R-parity breaking terms allow for a vacuum expectation value of the SUSY neutrinos. These and the μ_j terms yield tree diagrams for the mixing of the neutrinos with the neutralinos (photinos, zinos, higgsinos) by eliminating the four neutralinos (one has two neutral higgsinos) one obtains a separable mass matrix.

$$\begin{aligned} \sum_{\beta=1}^{3} m_{\alpha\beta}\; c^i_\beta \;&=\; m_i\; c^i_\alpha \text{ with: } \; m_{\alpha\beta} = a_\alpha \cdot a_\beta \, , \\ a_\alpha \sum_\beta a_\beta\; c^i_\beta \;&=\; m_i\; c^i_\alpha \text{ with: } \; a_{\nu_e}, a_{\nu_\mu}, a_{\nu_\tau} . \end{aligned} \tag{19}$$

For such a separable mass matrix of rank 1, one has two mass eigenvalues, which are zero. This can be easily seen from the lines three and four from Eq. (16). The vector in flavour space $(a_{\nu e}, a_{\nu\mu}, a_{\nu\tau})$ allows in this three dimensional space two vectors $\{c^i\}$ which are orthogonal to $\{a\}$ and thus one has two mass eigenvalues which are zero. The hierarchical neutrino mass spectrum must have the form as shown in figure 2.

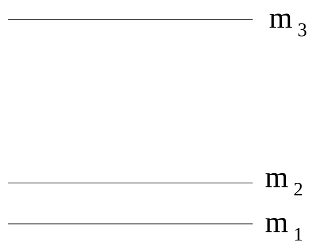

Fig. 4. On the tree level two eigenvalues of the neutrino mass matrix are zero and one is only different from zero. The inclusion of loop diagrams (see Fig. 3) yield for all masses values different from zero. But one expects, that the hierarchical structure of the neutrino masses remain. Two masses very small and the third neutrino mass larger.

On the tree level we have therefore $m_1 = m_2 = 0$ and $m_3 \neq 0$. From the atmospheric neutrino measurement of SuperKamiokande, one therefore obtains immediately for the third neutrino mass:

$$2 \times 10^{-2} \text{ eV} \leq m_3 \leq 10^{-1} \text{ [eV]}. \tag{20}$$

This naturally cannot be the final truth. So we have to calculate also loop diagrams.

The lepton–slepton and the quark–squark loops yield again a separable contribution to the neutrino mass matrix in the three dimensional neutrino flavour space. One has therefore now a rank tree spearable matrix of which the mass eigenvalues are all different from zero.

$$m_{\alpha\beta} = \sum_{k=1}^{3} a_\alpha^k \; a_\beta^k. \tag{21}$$

The upper index k runs over the tree diagrams, the lepton loop and the quark loop diagrams since we are assuming CP conservation the matrix $m_{\alpha,\beta}$ is symmetric and the separable rank three mass matrix (18) has altogether nine parameters. On the other side we have only five experimental quantities: the three mixing angles $\theta_{12}, \theta_{13}, \theta_{23}$ and the differences of the masses squared $\Delta m_{21}^2, \Delta m_{32}^2$. We therefore must reduce the nine parameters in the 3×3 neutrino mass matrix (18) from 9 to a maximum of 5. This

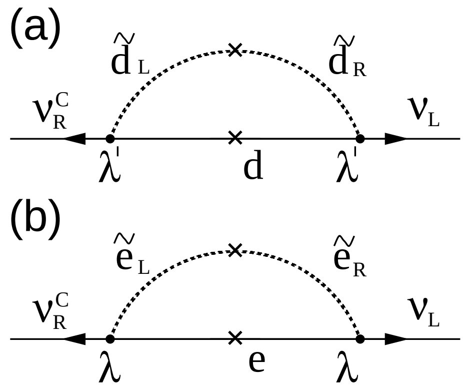

Fig. 5. The quark–squark (a) and the lepton–slepton (b) 1-loop contributions to the neutrino masses. The crosses on the lines denote the left-right mixing by a mass term.

is possible with a horizontal flavour symmetry. Such a horizontal flavour symmetry [12–16] of U(1) type has been used to explain the quark masses. The U(1) field, which we want to call θ has a new type of charge $C_\theta = -1$, which is not conserved in R-parity violating processes. Thus this horizontal U(1) field forbids R-parity violation. The anomaly of the U(1) field is canceled by the Green–Schwarz mechanism.

The U(1) charge conservation is broken by a vacuum expectation value of this field $\langle\theta\rangle \neq 0$. The R-parity violating terms in the Lagrangian (14) have in form of an effective Lagrangian for example for the term involving one lepton and two quark superfields the following form:

$$\begin{aligned} \mathcal{L}(\not{R}) &= \ldots g L_i Q_j D_k^c \vartheta^{n_{ijk}} + \ldots \\ &\quad \Longrightarrow \ldots \lambda'_{ijk} L_i Q_j D_k^c + \ldots \\ &\quad \text{with}:\ \lambda'_{ijk} \propto g\langle\vartheta\rangle^{n_{ijk}}. \end{aligned} \tag{22}$$

The power n_{ijk} of the ϑ field in the effective R-parity violating Lagrangian conserves the charge C_θ. This determines the exponent n_{ijk}. Here automatically a sum over all three families over the indices i, j and k is included. As long as the vacuum expectation value of the U(1) field ϑ is zero R-parity is conserved. But if the vacuum expectation value of θ is different from zero $\langle\theta\rangle \neq 0$ one obtains terms with R-parity violation. It is now possible to reduce the six coupling constants which are needed for the loop diagrams to only one.

$$\lambda'_{i33} = \lambda'_{333} \begin{pmatrix} \varepsilon^4 \\ \varepsilon \\ 1 \end{pmatrix},$$

$$\lambda_{i33} \;=\; \lambda'_{333} \left(\begin{array}{c} (\varepsilon^4 - 1)\varepsilon^4 \\ (\varepsilon - 1)\varepsilon \\ 0 \end{array} \right) . \tag{23}$$

Since the loop diagrams in figure 3 are proportional to the mass of the intermediate particles squared, one needs only to include in the loops the particles of the third family.

The quantity ε

$$\varepsilon = \frac{\langle \vartheta \rangle}{M_{U(1)}} \approx 0.23 \tag{24}$$

has been determined in [12–16] by explaining with the same horizontal symmetry the masses of the quarks and the masses of the leptons e^-, μ^- and τ^-.

The zero in Eq. (20) comes from the Pauli principle since one has two identical lepton fields in the third family. The powers of $\varepsilon = 0.23$ can be derived from the charges C_θ of the particles in the first family $C_\theta = -3$, the second family $C_\theta = 2$ and the third family $C_\theta = 1$.

Since we replace in this way six different coupling constants λ'_{i33} and λ_{i33} (with $i = 1, 2, 3$) by one λ'_{333}, one reduces the nine free parameters to four. Now one is able with the five experimental quantities of the three mixing angles and the two squares of the differences of the neutrino masses to determine completely neutrino mass matrix and diagonalize it. Including the uncertainties which show up in the different fits and including also the uncertainties in the CP phases (6) one can now diagonalize the flavour neutrino mass matrix (9) and obtains:

$$\begin{aligned} |m_{\nu 1}| &= 0.000 \div 0.02 \text{ [eV]}, \\ |m_{\nu 2}| &= 0.002 \div 0.04 \text{ [eV]}, \\ |m_{\nu 3}| &= 0.03 \div 1.05 \text{ [eV]}, \\ |\langle m_{\nu_e} \rangle| &= 0.009 \div 0.045 \text{ [eV]} . \end{aligned} \tag{25}$$

Since supersymmetry yields only a Majorana mass term, the three masses are given here as absolute values. They can be positive or negative and even complex if CP violation is allowed. With the inclusion of LSND one is more at the upper end of the interval given in the first three masses and without LNSD one would more prefer the lower values of the intervals. The last value in Eq. (22) is the averaged electron neutrino mass for the neutrinoless double beta decay.

One can now answer the question how large the coupling parameter λ'_{333} can be, so that the masses get not larger than the upper limits given in (22). This is shown in Table III.

TABLE III

Upper limits for the coupling constants λ'_{i33} and λ_{i33} of the R parity violating terms in the Lagrangian (14). The previous values are taken from Rakshit *et al.* [17].

	This work	Previous*	Improvement
λ_{133}	1.7×10^{-3}	3×10^{-3}	1.8
λ_{233}	1.9×10^{-3}	6×10^{-2}	32
λ'_{133}	3.8×10^{-4}	7×10^{-4}	1.8
λ'_{233}	4.3×10^{-4}	0.36	950
λ'_{333}	5.3×10^{-4}	0.48	900

5. Conclusion

From the pre-SNO solar, atmospheric and LSND neutrino oscillation data we took the three mixing angles and the two differences of the masses squared to determine the neutrino masses. Hereby we included and excluded the LSND result to see how it is influencing the final neutrino masses. We found that the Majorana neutrino masses have without inclusion of LSND an average value which is smaller than 0.06 eV and with the inclusion of LSND an average value of around 0.2 eV [18, 19].

On the theoretical side we used the minimal supersymmetric model (MSSM) with R-parity violation. The mass of the neutrino originates in this model by mixing the three neutrinos which are originally massless with the four neutralinos: üphotino $\widetilde{\gamma}$, zino $\widetilde{Z}^0$, higgsino 1 $\tilde{h}^0$ and higgsino 2 $\tilde{h}^0_2$. This yields, if one requests CP conservation, a real neutrino mass matrix which is separable of rank 3. Thus we have nine parameters, but only five experimental quantities, the three mixing angles and the two squares of the differences of the masses, to which we could fit these nine parameters.

By using a horizontal U(1) flavour symmetry, one can reduce six R-parity violating coupling constants to one value. This reduces the nine free parameters to four and allows now to calculate the masses of the three neutrinos which varies roughly from 0 to 1.0 eV. The hierarchical structure is so that the first two masses are quite small, while the third mass can be up to 1 eV. If one omits LSND, one finds an averaged neutrino mass (the parameter $\langle m_{\nu e} \rangle$, which can be determined in the neutrinoless double beta decay) to be smaller than 0.06 eV, while the inclusion of LNSD gives an averaged neutrino mass of the order of 0.2 eV. The supersymmetric model with R-parity violation yields a Majorana neutrino.

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