ATOMIC STATES OF Σ HYPERONS AND ΣN INTERACTION*

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(Received December 11, 2001)

Model F of the Nijmegen baryon–baryon interaction is used to determine the strong complex s.p. potential of Σ^-, and to calculate the strong-interaction shifts and widths of the lowest observed levels of Σ^- atoms. The results obtained are in satisfying agreement with the experimental data.

PACS numbers: 13.75.Ev, 36.10.Gv

1. Introduction

Observed properties of Σ^- atoms, *i.e.*, strong-interaction shifts ε and widths Γ of the lowest observed levels, provide us with valuable information on the strong interaction between Σ^- and the nucleons, as well as on the nucleon density distribution in the nucleus of the Σ^- atom. In a recent comprehensive phenomenological analysis of the existing Σ^- data Batty, Friedman, and Gal [1] found the following striking property of the single particle (s.p.) strong-interaction potential of Σ^-: it is repulsive inside the nucleus and attractive outside. The need for the repulsion arose when new data were included into the analysis, namely the results of Powers *at al.* [2], especially their precise data on the Σ^-Pb atom.

This behavior of Σ^- s.p. potential found in the analysis of Σ^- atoms is consistent with the analysis of the pion spectra measured in (K^-, π) reactions, which suggests a Σ s.p. potential repulsive inside nuclei [3, 4] (with a substantial positive Lane potential V_τ [5]). This repulsion follows directly from the observed shift of the pion spectra toward higher Σ energies compared to the quasi-free spectrum.

In the paper reported here [6], we consider the Nijmegen models of the baryon-baryon interaction: models D [7], F [8], Soft-Core (SC) model [9],

* Presented at the XXVII Mazurian Lakes School of Physics, Krzyże, Poland, September 2–9, 2001.

and the New Soft-Core (NSC) model [10], and want to find out whether any of them is at the same time consistent with the pion spectra measured in (K^-,π) reactions and leads to the observed properties of Σ^- atoms. In our analysis, we apply the effective $\Sigma^- N$ interaction in nuclear matter, $\mathcal{K}$, obtained within the Low Order Brueckner (LOB) theory with the above interaction models by Yamamoto, Motoba, Himeno, Ikeda, and Nagata [11], and by Rijken, Stoks, and Yamamoto [10] (the so called YNG interactions).

The single-particle (s.p.) potential V of the Σ^- moving with momentum $\hbar k_\Sigma$ in nuclear matter with nucleon density ρ and neutron excess $\alpha = (N-Z)/A$ has the form [5]:

$$V_{\rm NM}(k_\Sigma,\rho,\alpha) = V_0(k_\Sigma,\rho) + \tfrac{1}{2}\alpha V_\tau(k_\Sigma,\rho)\,. \tag{1}$$

Here, we ignore terms connected with spin excess, considered in [12], which are usually negligibly small.

Expressions for the isoscalar potential V_0 and for the Lane potential V_τ in terms of the effective ΣN interaction $\mathcal{K}$ are given in [5]. When we apply the expression for V_0 to the YNG effective ΣN interactions, we see[1] that only model F of the Nijmegen baryon-baryon interaction leads to repulsive V_0 at nucleon densities $\rho \gtrsim 0.05$ fm^{-3} encountered inside nuclei, and to attractive V_0 at lower densities encountered in the nuclear surface. All the remaining models lead to attractive V_0 at all densities. This means that only model F leads to the Σ s.p. potential which is in qualitative agreement with the phenomenological analysis [1] of Σ^- atoms and also with the pion spectra measured in the (K^-,π) reactions.

The important question is whether model F can explain quantitatively the measured properties of Σ^- atoms. It is our purpose to show that this is indeed the case. We do it by calculating with the help of model F the energy shifts ε and widths Γ of the Σ^- atomic levels, and showing that they are reasonably close to experimental data.

2. The theoretical scheme

To determine ε and Γ, we solve the Schrödinger equation, which describes the motion of Σ^- in the Σ^- atom:

$$\left[-\frac{\hbar^2}{2\mu_{\Sigma A}}\Delta + V_{\rm C}(r) + \mathcal{V}(r)\right]\Psi = \mathcal{E}\,\Psi\,, \tag{2}$$

where $\mu_{\Sigma A} = M_\Sigma M_A/(M_\Sigma + M_A)$ is the Σ^--nucleus (of mass M_A) reduced mass (M_Σ is the mass of Σ^-), and $V_{\rm C}$ is the Coulomb interaction between Σ^- and the nucleus.

[1] Compare Fig. 1 in Ref. [13].

Because of the $\Sigma\Lambda$ conversion process $\Sigma^- p \to \Lambda n$, the strong interaction potential $\mathcal{V}$ is complex, $\mathcal{V} = V + iW$, and consequently the eigenvalue $\mathcal{E}$ is also complex, with its imaginary part connected with the width of the level, $\mathcal{E} = E - i\Gamma/2$. For the strong interaction energy shift ε, we have $\varepsilon = E_{\mathrm{C}} - E$, where E_{C} is the pure Coulomb energy, *i.e.*, the eigenvalue of equation (2) without the strong interaction potential $\mathcal{V}$. Notice that ε is positive for downward shift of the level. The measured energy of γ transition to the level is then increased by ε.

To calculate the real and absorptive strong interaction potentials V and W, we apply the local density approximation: the Σ^- atom is treated at each point as Σ^- moving in nuclear matter with the local nuclear density of the Σ^- atom.

2.1. Expression for V

Let us consider a Σ^- atom with proton and neutron density distributions $\rho_p(r)$ and $\rho_n(r)$ respectively. At any distance r, we treat the system as nuclear matter with total nucleon density $\rho(r) = \rho_p(r) + \rho_n(r)$ and with neutron excess $\alpha(r) = [\rho_n(r) - \rho_p(r)]/\rho(r)$, and with a Σ^- hyperon with momentum $k_\Sigma \approx 0$. [The last approximation is justified by the very weak dependence of the Σ s.p. potential in nuclear matter on k_Σ found in [5], and by the relatively small magnitude of Σ momenta in Σ^- atoms.] To get the value of the Σ^- s.p. potential in Σ^- atom at a distance r, we calculate $V_{0,\tau}(k_\Sigma, \rho(r))$ at $k_\Sigma = 0$ by applying the expressions given in [5] with the YNG effective interactions of [11] (and [10]). In this way we obtain the isoscalar and the Lane potentials in Σ^- atom at a distance r,

$$V_0(r) = V_0(k_\Sigma\!=\!0, \rho(r)), \quad V_\tau(r) = V_\tau(k_\Sigma\!=\!0, \rho(r))\,, \tag{3}$$

and the total nuclear s.p. Σ^- potential,

$$V(r) = V_0(r) + \tfrac{1}{2}\alpha(r) V_\tau(r)\,. \tag{4}$$

2.2. Expression for W

Here we follow the procedure applied in [14] in explaining the early data on Σ atomic widths. A slightly simplified form of our expression (5) for W_{NM} in terms of the $\Sigma\Lambda$ conversion cross section was used before in [15].

First, let us consider a Σ^- hyperon moving with momentum $\hbar k_\Sigma$ in nuclear matter with total and proton densities ρ, ρ_p. The width Γ_{NM} of this state is connected with the absorptive potential $W_{\mathrm{NM}} = -\frac{1}{2}\Gamma_{\mathrm{NM}}$. By applying the optical theorem to the Brueckner reaction matrix $\mathcal{K}$ — as was shown in [15] and [14] — one obtains for $W_{\rm NM}$:

$$W_{\rm NM}(k_\Sigma,\rho,\rho_p) = -\frac{1}{2}\nu\rho_p\frac{\hbar^2}{\mu_{\Sigma N}}\langle k_{\Sigma N} Q\sigma\rangle\,, \tag{5}$$

where $\langle\ \rangle$ denotes the average value in the Fermi sea, $\hbar k_{\Sigma N}$ is the $\Sigma^- N$ relative momentum, $\mu_{\Sigma N}$ is the $\Sigma^- N$ reduced mass, Q is the exclusion principle operator, ν is the ratio of the effective to the real nucleon mass, and σ is the total cross section for the $\Sigma\Lambda$ conversion process.

With the absorptive potential W(r) in a Σ^- atom with total and proton densities $\rho(r)$, $\rho_p(r)$, we proceed similarly as with V and write:

$$W(r) = W_{\rm NM}(\bar{k}_\Sigma,\rho(r),\rho_p(r))\,. \tag{6}$$

Here, we insert for k_Σ in (5) the average momentum of Σ^-, $\bar{k}_\Sigma$.

For the total $\Sigma\Lambda$ conversion cross section σ we shall use the parametrization suggested by Gal, Toker, and Alexander [16].

3. Results and discussion

The proton and neutron density distributions, $\rho_p(r)$ and $\rho_n(r)$ used in our calculation have been obtained from the isomorphic shell model [17,18] (see also [19] and references therein).

For the Coulomb interaction $V_{\rm C}$ in Schrödinger equation (2), we use the potential produced by a uniform charge distribution with radius R, which leads to the same r.m.s. radius $\langle r^2\rangle^{1/2}$ of the charge distribution, $R = \sqrt{3/5}\langle r^2\rangle^{1/2}$. For the r.m.s. radii, we use the empirical values collected in [20].

Our results for ε and Γ are presented in Table I together with the existing experimental data which, however, are relatively inaccurate. Our results appear reasonably close to the experimental data and indicate the consistency of model F with properties of Σ^- atoms. This leads us to the conclusion that among the Nijmegen baryon-baryon interactions, model F and only model F is capable to represent the ΣN interaction both in Σ hypernuclear states and in Σ^- atoms.

Two other aspects of our results worth mentioning are:

1. the role of the finite size of the nuclear charge distribution turns out to be negligible, and

2. the accuracy of the first order perturbation approximation applied in [13] turns out to be very good.

TABLE I

Energy shifts ε, ε^u and widths Γ, Γ^u calculated with model F of the ΣN interaction, respectively for the lower and upper level of the indicated Σ^- atoms together with the experimental results. All energies are in eV.

Nucl.	$n+1\to n$	ε	$\varepsilon_{\rm exp}$	Γ	$\Gamma_{\rm exp}$	ε^u	Γ^u	$\Gamma^u_{\rm exp}$
^{12}C	$4\to3$	8.19	—	22.2	—	0.007	0.011	$0.031\pm0.012^{\rm a}$
^{16}O	$4\to3$	50.0	$320\pm230^{\rm b}$	194.2	—	0.11	0.20	$1.0\pm0.7^{\rm b}$
^{24}Mg	$5\to4$	32.6	$25\pm40^{\rm b}$	50.4	$<70^{\rm b}$	0.08	0.10	$0.11\pm0.09^{\rm b}$
^{27}Al	$5\to4$	67.3	$68\pm28^{\rm b}$	113.2	$43\pm75^{\rm b}$	0.22	0.28	$0.24\pm0.06^{\rm b}$
^{28}Si	$5\to4$	139.9	$159\pm36^{\rm b}$	242.8	$220\pm110^{\rm b}$	0.55	0.70	$0.41\pm0.10^{\rm b}$
^{32}S	$5\to4$	433.8	$360\pm220^{\rm b}$	873.2	$870\pm700^{\rm b}$	2.49	3.43	$1.5\pm0.8^{\rm b}$
^{40}Ca	$6\to5$	27.0	—	42.0	—	0.12	0.15	$0.41\pm0.22^{\rm a}$
^{48}Ti	$6\to5$	44.9	—	104.0	—	0.30	0.48	$0.65\pm0.42^{\rm a}$
^{138}Ba	$9\to8$	32.6	—	73.9	—	0.92	1.34	$2.9\pm3.5^{\rm a}$
^{184}W	$10\to9$	126.7	$214\pm60^{\rm c}$	180.5	$18\pm149^{\rm c}$	3.75	4.24	$2\pm2^{\rm c}$
^{208}Pb	$10\to9$	457.4	$422\pm56^{\rm c}$	773.4	$430\pm160^{\rm c}$	18.9	23.8	$17\pm3^{\rm c}$

[a] Data taken from Ref. [21].

[b] Data taken from Ref. [22].

[c] Data taken from Ref. [2].

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