# THE GROUND STATE PROPERTIES OF SPHERICAL NUCLEI CALCULATED BY HARTREE–FOCK–BOGOLUBOV PROCEDURE WITH THE GOGNY D1S FORCE* **

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The selfconsistent Hartree–Fock–Bogolubov (HFB) calculation with the Gogny force D1S was performed for the spherical nuclei: isotopes of Ca, Sr, Sn, Sm, Pb, Th, isotones with magic neutron numbers 50, 82, 126 and $\beta$-stable nuclei. The shell effects were extracted by Strutinsky procedure from Hartree–Fock energy and obtained in this way macroscopic part of potential energy was approximated by the liquid drop formula of Myers–Świątecki type. The nuclear radii were calculated by HFB method with the Gogny D1S force for the same group of nuclei. They were approximated by the isospin dependent 3 parameter formula, which reproduced the experimental data very well, what is shown.

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## 1. Introduction

The selfconsistent Hartree–Fock–Bogolubov [1] method with the two body effective nucleon–nucleon forces is one of the best tool to investigate the nuclear properties. The binding energies, single particle levels, sepa-

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ration energies, radii and fission lifetimes should be reproduced in such a calculation when phenomenological parameters are well established.

We have performed HFB calculation with the Gogny effective interaction D1S [2]. This interaction is very successful in description of spherical and deformed nuclei in the ground states, however, we have restricted our calculation only to spherical nuclei: isotopes of Ca, Sr, Sn, Sm, Pb, Th, isotones with magic neutron number 50, 82, 126 and $\beta$-stable spherical nuclei.

## 2. Basic formulae and results

### *2.1. Energy*

We have chosen the representative group of spherical nuclei to get the systematics of their binding energies and radii. First we have performed the Hartree–Fock (HF) calculation without pairing with the Gogny D1S [2] force. Then, we have extracted the shell effects from the HF energies by Strutinsky [3] averaging method. We have taken the selfconsistent single particle levels $e_\nu$ and smoothed them to $\tilde{e}_\nu$ by modified gaussian function looking carefully for the proper gaussian width $\gamma$ giving the plateau for Strutinsky shell corrections

$$E_{\mathrm{shell}} = \sum_\nu e_\nu - \sum_\nu \tilde{e}_\nu \,. \tag{1}$$

In Fig. 1 we can see the dependence of neutron, proton and total shell corrections on proton number $Z$ for nuclei with magic neutron numbers. The magic numbers 50 and 82 are well reproduced by the minimal proton shell corrections.

In the next step we have subtracted the total shell correction from the HF energy to obtain the pure macroscopic one

$$E_{\mathrm{macr}} = E_{\mathrm{HF}} - E^p_{\mathrm{shell}} - E^n_{\mathrm{shell}} \,. \tag{2}$$

This energy served us to get the parameters of the liquid drop model of Myers–Świątecki type [4]

$$E_{\mathrm{fit}} = -a_v(1-k_vI^2)A + a_{\mathrm{surf}}(1-k_{\mathrm{surf}}I^2)A^{2/3} + a_{\mathrm{c}1}\frac{Z^2}{A^{1/3}} - a_{\mathrm{c}2}\frac{Z^2}{A}\,, \tag{3}$$

which correspond to the selfconsistent Gogny theory. After performing the least square fit we have obtained the following set of parameters:

$$\begin{aligned} a_v &= 15.652\ \mathrm{MeV}, & k_v &= 1.916\ \mathrm{MeV},\\ a_{\mathrm{surf}} &= 18.919\ \mathrm{MeV}, & k_{\mathrm{surf}} &= 2.104\ \mathrm{MeV},\\ a_{\mathrm{c}1} &= 0.727\ \mathrm{MeV}, & a_{\mathrm{c}2} &= 1.987\ \mathrm{MeV}, \end{aligned} \tag{4}$$

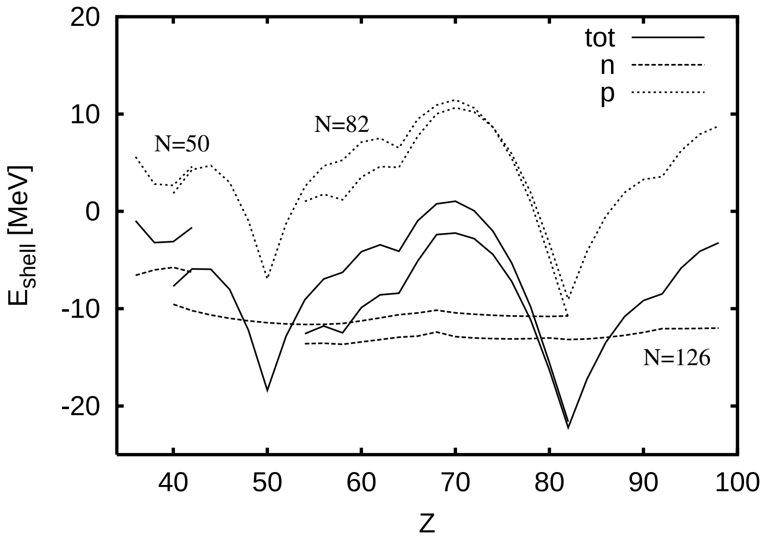


Fig. 1. Gogny shell corrections of protons, neutrons (dashed lines) and total (solid lines) for nuclei with magic neutron numbers $N = 50, 82, 126$ in dependence on $Z$.

In Fig. 2 we can see the difference between macroscopic energy (2) and this evaluated by formula (3) with the set (4) of parameters for nuclei with magic neutron numbers. The maximum discrepancy about 2.5 MeV reaches the 0.5 % error. We have obtained similar results for the other groups of nuclei.

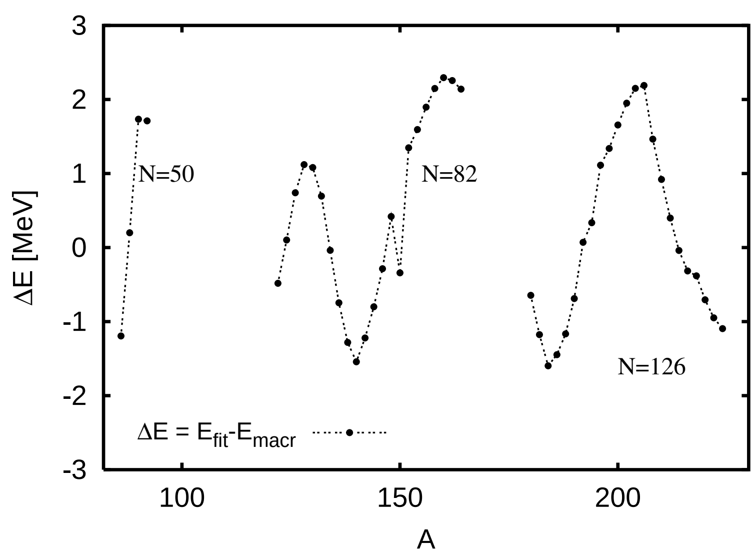


Fig. 2. The difference between macroscopic and approximated by formula (3) with the set (4) of parameters energy for nuclei with magic neutron numbers 50, 82, 126.

### *2.2. Radii*

We have also gathered the root mean square radii of proton, neutron, charge and mass distribution of the spherical nuclei in order to find their isospin dependence like in Ref. [5].

The theoretical Root Mean Square Radius (RMSR) was approximated by the formula obtained with uniform density distribution

$$\sqrt{\langle r^2 \rangle} = \sqrt{\frac{3}{5}}\, r_0\, A^{1/3}\,, \tag{5}$$

with the radius constants $r_0$ for neutrons given by the equation

$$r_0^n = 1.17 \left(1 + 0.12 \frac{N-Z}{A} + \frac{3.29}{A}\right) \mathrm{fm}\,, \tag{6}$$

for protons

$$r_0^p = 1.21 \left(1 - 0.14 \frac{N-Z}{A} + \frac{1.83}{A}\right) \mathrm{fm}\,, \tag{7}$$

for charge

$$r_0^{ch} = 1.22 \left(1 - 0.15 \frac{N-Z}{A} + \frac{2.32}{A}\right) \mathrm{fm}\,, \tag{8}$$

and for mass distribution

$$r_0^{\mathrm{tot}} = 1.19 \left(1 + 0.03 \frac{N-Z}{A} + \frac{2.70}{A}\right) \mathrm{fm}\,. \tag{9}$$

It was shown in Ref. [5] that the ratio of neutron to proton radius is almost deformation independent. So the corresponding result obtained within the HFB calculation with the Gogny force for the spherical nuclei

$$\sqrt{\frac{\langle r^2 \rangle^p}{\langle r^2 \rangle^n}} = \frac{r_p}{r_n} = 1.04 \left(1 - 0.27 \frac{N-Z}{A} - \frac{1.12}{A}\right) \mathrm{fm} \tag{10}$$

could be also used for the deformed nuclei.

In Fig. 3 we compare values of charge root mean square radii calculated with radius constant (8) and the experimental ones [6] for $\beta$-stable nuclei. The agreement is good.

In Fig. 4 there are marked theoretical (formula (6)) and experimental values of neutron RMSR [7]. As we can see almost all theoretical results are shifted down in comparison with experimental data, but they are contained within error-bars.

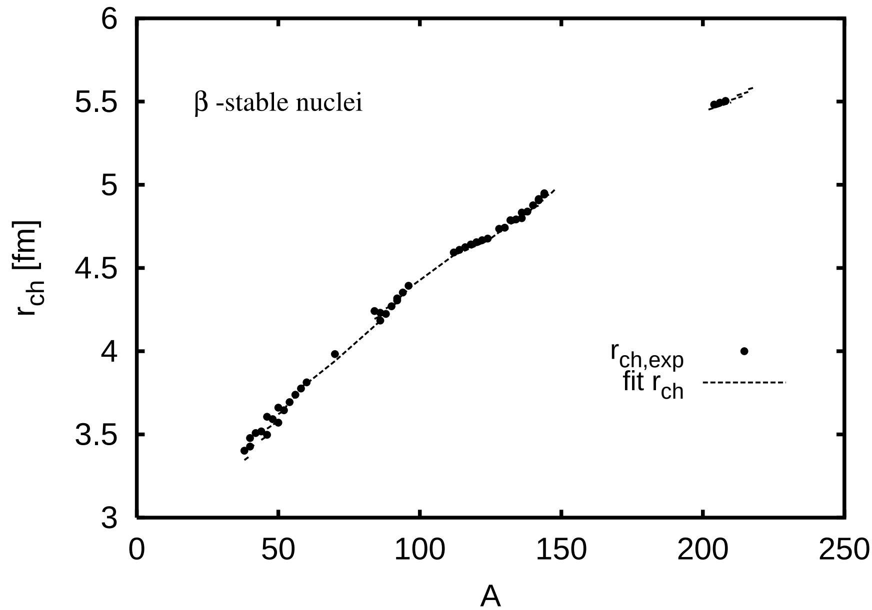


Fig. 3. The charge root mean square radii calculated by formula (8) (lines) and the experimental ones (dots) in dependence on mass number $A$ for $\beta$-stable nuclei.

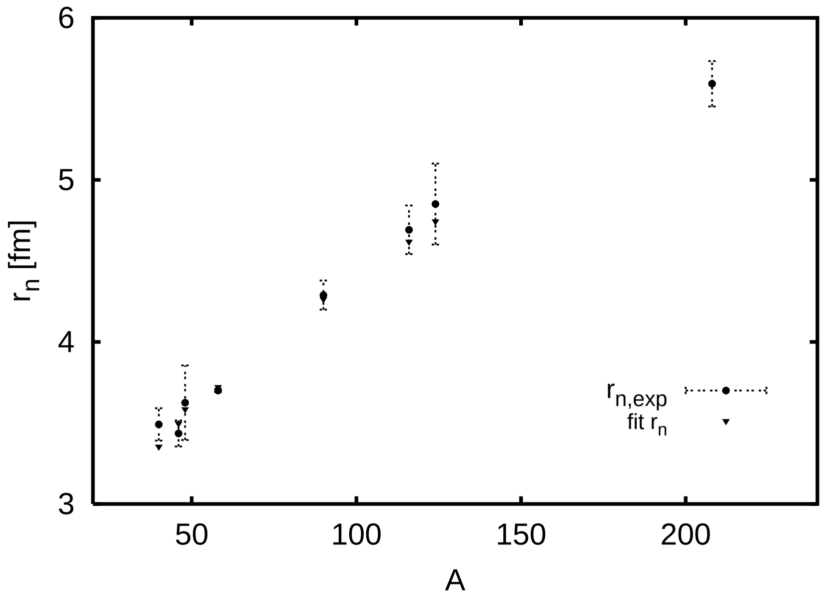


Fig. 4. The neutron RMSR calculated with radius constant (6) (triangles) and the experimental data (dots) in dependence on mass number $A$.

## 3. Conclusions

The following conclusions can be drawn from our calculations:

1. The D1S Gogny force gives the average binding energy of nuclei close to that obtained by the liquid drop model of Myers and Świątecki.

2. The radii of neutron and charge distribution agree well with experimental data.

3. The neutron radii can be estimated from the ratio of Gogny proton to neutron radii.

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