COLLECTIVE QUADRUPOLE EXCITATIONS IN TRANSURANIC NUCLEI* **

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The collective excitations in transuranic nuclei ($94 \leq Z \leq 102$, $146 \leq N \leq 158$) are studied within the model based on the general Bohr Hamiltonian modified by including the coupling with the pairing vibrations. Preliminary results on superdeformed states in ^{254}No are also presented.

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1. Introduction

Fast developing spectroscopy of very heavy nuclei provides us with new data on their excited states [1,2]. On the theoretical side there are several attempts to describe rotational bands of those nuclei (microscopic-macroscopic methods [3], self-consistent mean field calculations with Skyrme [4, 5] or Gogny forces [6]). In [7] we have shown the first results on energies and E2 transition probabilities of fermium and nobelium isotopes within the model based on the generalized Bohr Hamiltonian. The present work covers a wider region of nuclei with $94 \leq Z \leq 102$ and $146 \leq N \leq 158$. We present also some preliminary results on superdeformed states.

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The model that we use to describe quadrupole collective states is an extension of the generalized Bohr Hamiltonian through including, in an approximate way, the effects coming from coupling with the pairing vibrations. The collective variables we consider are: Bohr deformation parameters (β, γ), Euler angles Ω and Δ_n and Δ_p which are the pairing gaps for neutrons and protons respectively.

The Hamiltonian of the model can be written as

$$\hat{\mathcal{H}}_{\rm CQP} = \hat{\mathcal{H}}_{\rm CQ}(\beta, \gamma, \Omega; \Delta_p, \Delta_n) + \hat{\mathcal{H}}_{\rm CP}(\Delta_p, \Delta_n; \beta, \gamma)\,. \tag{1}$$

The first term of (1) is the standard generalized Bohr Hamiltonian and the second term describes collective pairing vibrations of protons and neutrons. The dependence of $\hat{\mathcal{H}}_{\rm CQ}$ on $\Delta_{n,p}$ appears when we calculate inertial functions (and potential energy) from microscopic theories. Similarly, collective pairing mass parameters depend on deformation of nucleus.

Our approach can be sketched as follows [8]:

1. We determine the $\hat{\mathcal{H}}_{\rm CP}$ from the nucleon single-particle potential applying the cranking or generator coordinate method. After solving the appropriate eigenequation we obtain the ground state of $\hat{\mathcal{H}}_{\rm CP}$. The specific feature of this state is that probability density as a function of $\Delta_{(n,p)}$ has its maximum for arguments lower than obtained from the solution of the BCS equations.

2. In the next step we evaluate the inertial functions entering $\hat{\mathcal{H}}_{\rm CQ}$ taking the corresponding most probable $\Delta_{(n,p)}$. Finally we solve eigenproblem for $\hat{\mathcal{H}}_{\rm CQ}$ as a matrix equation in a properly constructed basis.

Results presented below have been obtained using Nilsson potential with the standard set of parameters [9], while strength of the pairing forces was determined in [10] from the mass differences. The collective potential energy is calculated using Strutinsky macroscopic-microscopic method.

Important point is that we do not introduce any additional parameters nor readjust any of existing ones.

2. Energy levels and E2 transition probabilities

The model presented above has been successfully applied in several regions of nuclei with the mass number $100 \leq A \leq 160$ [8]. It seemed to be interesting to test its predictions for much heavier systems, especially in the context of recent experimental successes of in-beam spectroscopy. We discuss in this section the energy levels and E2 transition probabilities of nuclei with $94 \leq Z \leq 102$ and $146 \leq N \leq 158$.

2.1. Energy levels

In Figs. 1–5 we show theoretical and experimental [11] 2_1^+ level (g.s. band) and bandheads of β and γ vibrational bands. They are sufficient for testing theoretical predictions with experiment because most nuclei in this region are good rotors, with similar values of moment of inertia in g.s., β and γ bands (in experiment and in theory, see as *e.g.* ^{250}Cf nucleus — Fig. 6). Note also that in Figs. 1–5 both experimental and theoretical values of the first 2^+ state are multiplied by 5.

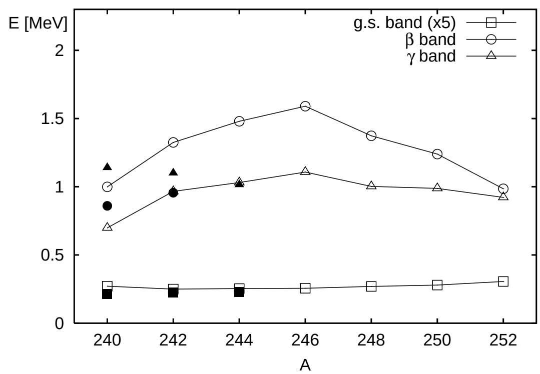

Fig. 1. Energy levels of Pu isotopes. In Fig. 1–6 open (solid) symbols denote theoretical (experimental) values

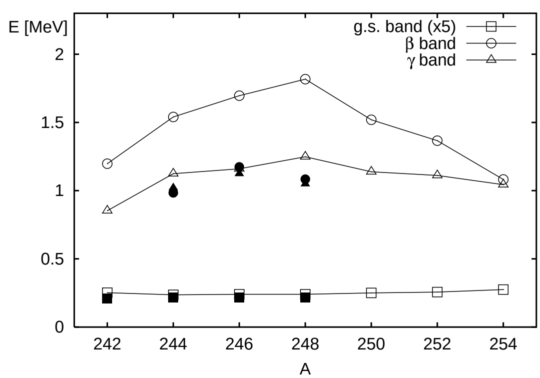

Fig. 2. Energy levels of Cm isotopes

The ground state bands are reproduced quite well, theoretical 2_1^+ level lies typically 4-5 keV above experimental one. The situation with γ bands is also not bad, but the failure in describing β bands indicates that our approach needs some improvement. One possible cause of this failure could be the fact that most of considered nuclei have nonzero values of higher defor-

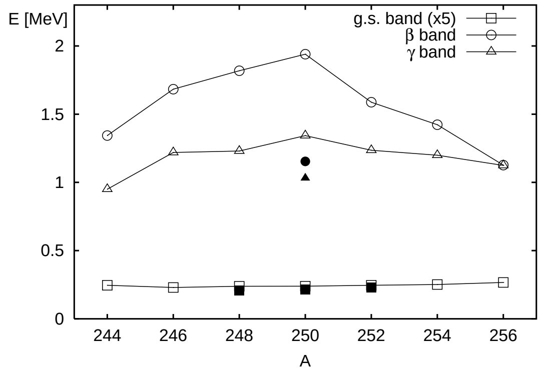

Fig. 3. Energy levels of Cf isotopes, see also caption to Fig. 1

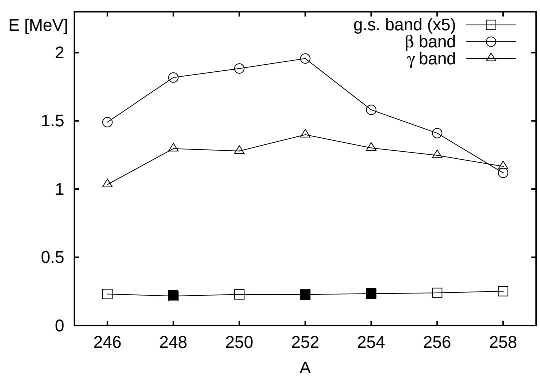

Fig. 4. Energy levels of Fm isotopes, see also caption to Fig. 1

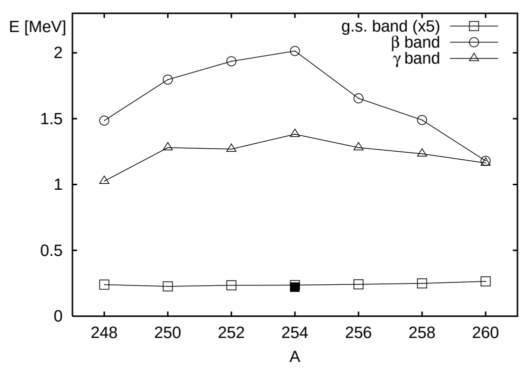

Fig. 5. No isotopes, see also caption to Fig. 1

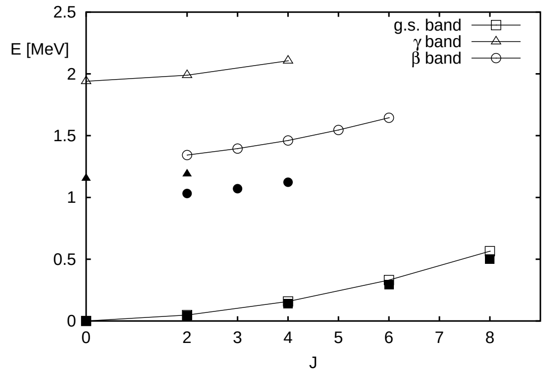

Fig. 6. More detailed picture of ^{250}Cf levels

mations ($\lambda = 4, 6$). Such deformations could be partially included into our model if we would take single particle spectrum from selfconsistent calculations. In such a case (β, γ) will correspond to components of a quadrupole mass tensor and not to a shape of equipotential surfaces.

2.2. *E*2 *transitions*

Theoretical probabilities of transitions $2_1^+ \to 0_1^+$ are underestimated by 20%–30% with respect to experimental ones. We show them in Fig. 7 for Cm isotopes, but the previous statement is true also for other nuclei. The transitions within the ground state band of ^{248}Cm (Fig. 8) are not reproduced perfectly, but one should remember that we do not fit any parameters and do not use effective charges. There is not much experimental information on interband transitions, however we give one example from ^{250}Cf nucleus:

TABLE I

$B(E2)$ probabilities (W.u.) for interband ($\gamma \to$ g.s.) transitions in ^{250}Cf

		Exp	Th
$2^+(\gamma$ band$) \to$ g.s.	4^+	0.21 ± 0.02	0.24
	2^+	3.7 ± 0.4	3.2
	0^+	2.3 ± 0.3	1.9

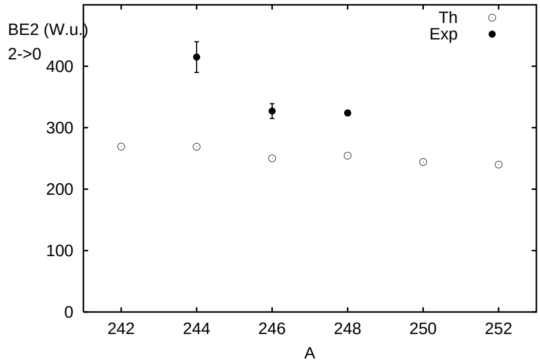

Fig. 7. $E2$ transition probabilities $(2_1^+ \to 0_1^+)$ in Cm isotopes

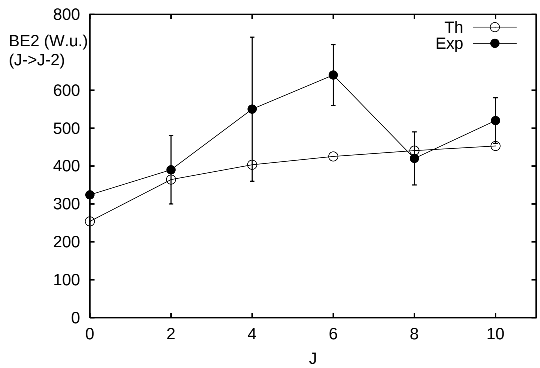

Fig. 8. $E2$ transition probabilities within the ground state band of ^{248}Cm nucleus

3. Superdeformed states

The possibility of a proper description of superdeformed states within our model is an interesting and less standard question. Such states appear as a consequence of an existence of the second minimum of potential energy for large elongation. They have been studied in the framework of general Bohr Hamiltonian approach for the first time (in $A \sim 190$) region by Libert *et al.* in [12].

Some technical problems appear while investigating superdeformed states, *e.g.* the number of basis states should be increased significantly and quite a lot of work is needed for ensuring desired accuracy and stability of results. However after solving these problems a set of superdeformed solutions can be unambiguously identified, *e.g.* by inspecting their average deformations $\bar{\beta}$, $\bar{\gamma}$. These quantities are defined through mean values of invariants β^2 and $\beta^3 \cos 3\gamma$ as: $\bar{\beta} = \sqrt{\langle\beta^2\rangle}$ and $\bar{\gamma} = \arccos(\langle\beta^3 \cos 3\gamma\rangle/\bar{\beta}^3)/3$, where *e.g.* $\langle\beta^2\rangle = \int \Psi^* \beta^2 \Psi \sqrt{g} d\omega$, with g and $d\omega$ denoting the determinant of the metric tensor and the appropriate volume element respectively (for more details see [8]). The wave functions Ψ of nuclear states are obtained by solving the eigenproblem of $\hat{\mathcal{H}}_{\rm CQ}$ as mentioned in Section 1.

We have taken as a test case the nucleus ^{254}No, which attracts much attention recently, but of course the states in the second minimum for this nucleus are not accessible by present experimental techniques. The interesting question if the spontaneous fission halftime is large enough for the superdeformed states to be observed lies beyond the scope of this paper. Note however that the experimental value of $T_{1/2}$ for ^{254}No is quite large (about 50 s, [1, 2]) and the main channel of its decay is the α particle emission.

The potential energy has been calculated using the recent liquid drop model parametrization with a curvature term included proposed by Pomorski and Dudek [13]. For small deformations it gives almost the same results as standard LDM, but for larger β the effects of the curvature term cannot be neglected.

It is probably worthwhile to add a remark on the definition of deformation parameters β, γ used in the paper. We apply the same one as in [8, 14] and which can be expressed by giving formulas for the length of semiaxes of an ellipsoid describing the shape of nucleus:

$$R_k = R_0(\beta,\gamma)\left(1+\sqrt{\frac{5}{4\pi}}\beta\cos\left(\gamma-k\frac{2\pi}{3}\right)\right)\ ,\ k=1,2,3 \qquad (2)$$

with $R_0(\beta,\gamma)$ being determined from the volume conservation. The book [15] (esp. Chapter 6) contains a very useful compilation of different definitions used for nuclear shapes and their mutual relationships.

In Fig. 9 we present all levels with $J = 0, 2, 3, 4$ and $E < 3.8$ MeV, indicating also their average deformations $\bar{\beta}$ and $\bar{\gamma}$. One can easily distinguish the sets of normal and superdeformed solutions. In addition we have plotted in Fig. 10 the probability densities of the ground state and the lowest SD state.

Below we give the energies of the lowest SD states in ^{254}No.

first band	
0^+	$E_{\rm SD,1}$=3.074 MeV (above the ground state)
2^+	$E_{\rm SD,1}$ + 15 keV
4^+	$E_{\rm SD,1}$ + 45 keV
β band	
0^+	$E_{{\rm SD},\beta}$=$E_{\rm SD,1}$ + 580 keV
2^+	$E_{{\rm SD},\beta}$ + 15 keV

It is also interesting to compare our results with cranked HFB calculations with Gogny forces of Egido and Robledo [6]. They estimated the energy of the first rotational SD 2^+ state (relative to SD 0^+_1) as 17.3 keV.

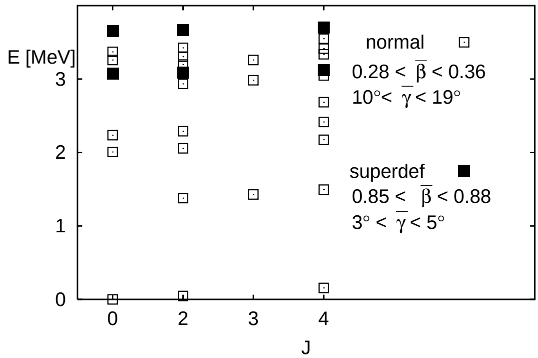

Fig. 9. Energy levels below 3.8 MeV in ^{254}No. Open squares denote "normal" states *i.e.* with $0.28<\bar{\beta}<0.36$ and $10°<\bar{\gamma}<19°$ while full squares correspond to superdeformed states with $0.85<\bar{\beta}<0.88$ and $3°<\bar{\gamma}<5°$

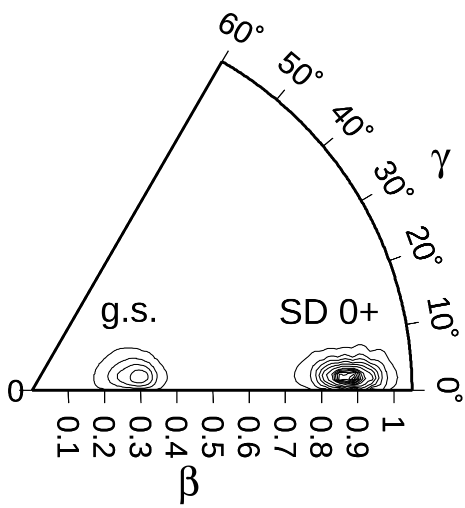

Fig. 10. Contour map of the probability density $|\Psi|^2\sqrt{g}$ as a function of (β,γ) for the ground and the first superdeformed state (^{254}No). The lines are plotted for the values of density differing by 2, starting from 0.2 (in units [rad^{-1}])

4. Conclusions

To conclude briefly: the model developed in [8] gives reasonable description of low lying states of the very heavy nuclei however some improvements are still needed to get better treatment of β vibrations. It gives also the promising perspective of the study of superdeformed states, especially in the context of recent experimental works [16].

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