ON THE EXISTENCE OF NEUTRINO-LESS DOUBLE ELECTRON CAPTURE TRANSITIONS*

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The nuclear method to discover Majorana neutrinos is the neutrinoless $\beta\beta$ decay. We propose to study the inverse process accompanied by a photon emission, the neutrinoless radiative double electron capture. Chances for such a decay are estimated. These favour studies of 0^+0^+ nuclear transitions of small energy release in high Z atoms.

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1. Introduction

The question of possible existence of massive neutrinos is one of the most intriguing topics of the present day physics. This general question is followed by a more specific one. What is the nature of the neutrinos, are these Dirac or Majorana fermions? Nuclear physics, with the studies of double β, β decays allows, in principle, to resolve this question for the electron neutrino ν_e. If it is a Majorana particle by definition it is equal to its charge conjugate. Thus the neutrino produced in one act of neutron decay may be absorbed in another act of neutron decay. This corresponds to the reaction

$$(A, Z-2) \to (A, Z) + e + e\,. \tag{1}$$

Such a process, visualised in Fig. 1 and denoted by $0\nu\beta\beta$, would be followed by the emission of two correlated electrons. The unique signature of this process is that the sum of the energies of the two β electrons is constant and equal to the total double β decay energy. Thanks to this the $0\nu\beta\beta$ electrons could be seen on the background of much more numerous final products $\bar{\nu}, \bar{\nu}, e, e$ from the normal $\beta\beta$ decays. While typical times of the double β

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decays are of the order of 10^{20}y, the processes $0\nu\beta\beta$ are expected to last 10^{24}y. For the description of the physics involved we refer to books [1,2] and recent experimental reviews [3,4].

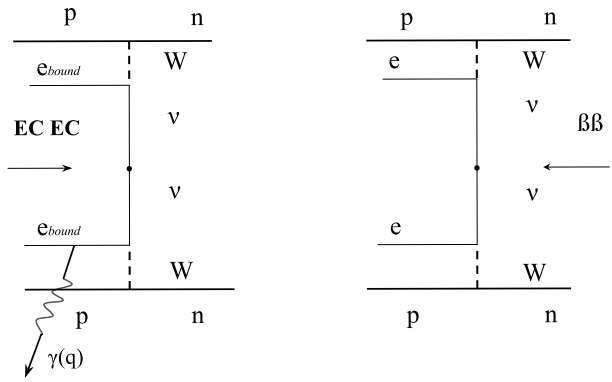

Fig. 1. Diagrams for the 0ν double beta decay (right) and radiative double electron capture (left) processes.

In this note we suggest to study the inverse of neutrinoless double β decay, *i.e.* the neutrinoless double electron capture (see also [5,6]). This process is forbidden in the first order because of the energy conservation requirement. A third body is needed to carry away the excess energy. The most obvious candidate for this energy carrier is a photon. In a usual electron capture decay such a radiative process is rare (branching ratio $10^{-3} \div 10^{-4}$) and the energy spectrum of the so-called internal bremsstrahlung photons is continuous (*cf. e.g.* [7]). On the other hand, the rate of the neutrinoless double electron capture, 0νEC EC, is presumably determined solely by this radiative effect, at least for the case of neutrino being a Majorana particle. There are experimental advantages in studying such processes: (1) The photon energy is well defined, equal to the total decay energy diminished by the binding energies of the two electrons captured. This provides a convenient observable easy to distinguish from the continuous background, random as well as physical one. (2) Contrary to the β electrons, the photons can easily escape from quite thick layers of the source material without energy degradation. Large quantities of the decaying isotope can thus be used and the detectors can be separate from the sources. The price to pay is a sizable reduction of the transition rates. These are discussed in Section 2. The experimental questions and the optimal choice of the isotopes are discussed in Section 3, some calculations are presented in the Appendix.

2. Comparison of the $0\nu\beta\beta$ and 0ν EC EC γ transition rates

The rate $\Gamma(0\nu\beta\beta)$ for the double neutrinoless beta decay of Eq. 1 may be factorised into nuclear and leptonic parts (see *e.g.* [3])

$$\Gamma(0\nu\beta\beta) = G^{0\nu} \mid M^{0\nu}(A, Z-2 \to Z) \mid^2 (m_\nu/m_e)^2 \,. \tag{2}$$

Eq. (2) involves nuclear matrix elements $M^{0\nu}(A, Z-2 \to Z)$. (We consider only the $0^+ \to 0^+$ transitions to avoid parallel single β decays.) The leptonic contributions including the final state electron wave functions and final phase space elements are contained in $G^{0\nu}$. Factor m_ν/m_e reflects the simplest assumption of Majorana neutrino which is adopted here. The way the neutrino mass arises in Eq. (2) is indicated in appendix. The reverse process — the double electron capture —

$$(A, Z) + e + e \to (A, Z-2) \tag{3}$$

is not allowed, due to the energy-momentum conservation. Emission of an additional particle: *e.g.* a photon, an electron or a majoron is necessary. The first possibility is studied here, *i.e.*

$$(A, Z) + e + e \to (A, Z-2) + \gamma \,. \tag{4}$$

Again, the capture rate may be factorised into nuclear, leptonic and photon parts

$$\Gamma(0\nu\text{EC EC}\gamma) = G^{0\nu\gamma} \mid M^{0\nu}(A, Z \to Z-2) \mid^2 (m_\nu/m_e)^2 \mid M^\gamma \mid^2 \,. \tag{5}$$

The factors involved in Eq. (5) differ from those of Eq. (2) by the final phase space, the transition energy and the electron wave functions. The nuclear transition element is of similar nature though it connects different nuclei. An additional factor M^γ introduced into Eq. (5) gives the probability of photon emission. This is due to the electron–photon coupling $eA^\mu J^e_\mu$ and the intermediate electron propagator:

$$M^\gamma = [\frac{e}{\sqrt{2q}m_e}]\sqrt{f_{\text{E,M}}}\,, \tag{6}$$

where e is the electron charge, q and $\sqrt{2q}$ are the photon momentum and photon w.f. normalisation. The term in the square brackets gives an order of magnitude estimate. Finer calculations, outlined in appendix, give corrective factors $f_{\text{E,M}}$ for electric and magnetic transitions. Those depend on electron states and the photon energy. In this section we give an estimate (order of magnitude) for the rate of the radiative capture process. This is presented as a retardation factor $R(\gamma)$ defined by the ratio

$$R(\gamma) = \frac{\Gamma(0\nu\text{EC EC}\gamma)}{\Gamma(0\nu\beta\beta)} \,. \tag{7}$$

The $0\nu\beta\beta$ processes involve two final electrons described by the Coulomb waves Ψ_f. At the nucleus the latter are

$$\mid \Psi_f(0) \mid^2 = \frac{2\pi\eta}{1-\exp(2\pi\eta)}, \tag{8}$$

where η is the Coulomb parameter for electrons. The final state interactions and the phase space element are given by integral (see [1])

$$\varrho(0\nu\beta\beta) = \int \frac{d\boldsymbol{p}d\boldsymbol{p}'}{(2\pi)^6}\delta(\Delta E - e(p) - e(p')) \mid \Psi_f(0) \mid^4 \approx (\frac{\Delta E)^5}{(120\pi^4)}, \tag{9}$$

where ΔE is the energy release for the $\beta\beta$ decay. Eq. (9) is an analogue of the Fermi function used in the ordinary β. The last relation is known as Primakoff–Rosen approximation. An equivalent expression for double electron captures involves the initial electron interactions and the final phase space element. For an estimate let us use $1S$ atomic levels and the corresponding wave functions

$$\mid \Psi_{1S}(0) \mid^2 = \frac{(m_e Z\alpha)^3}{\pi}, \tag{10}$$

where $\alpha = e^2/4\pi$. The final state phase space is determined by the energy release ΔE^{c}

$$\varrho(0\nu\mathrm{EC\ EC}\gamma) = \int \frac{d\boldsymbol{q}}{(2\pi)^3}\delta(\Delta E^{\mathrm{c}} - q). \tag{11}$$

The retardation factor becomes

$$R(\gamma) = \frac{\varrho(0\nu\mathrm{EC\ EC}\gamma) \mid \Psi_{1S}(0) \mid^4 \mid M^\gamma \mid^2}{\varrho(0\nu\beta\beta)} = \Big(\frac{m_e}{\Delta E}\Big)^5 \ \frac{30 \times 16\pi q\alpha^7 Z^6 f_{\mathrm{E,M}}}{m_e}. \tag{12}$$

The photon momentum q and the factors $f_{\mathrm{E}}, f_{\mathrm{M}}$ depend strongly on the energy release ΔE^{c}. These are given in Eqs. (A.8),(A.6) of the Appendix; f_{M} corresponds to M1 transitions from $2S, 1S$ and f_{E} corresponds to E1 transitions from $2P, 1S$ atomic states. We note that the transitions involving two $1S$ electrons are forbidden by the angular momentum conservation.

3. Optimal targets

The factors determining the optimal choice of the nucleide to be studied are first of all those affecting the decay rates: the decay energy and the atomic number, Z. The third factor, the nuclear matrix element, though of course important, varies much more slowly from a nucleus to nucleus; a crude estimate of its values can suffice for the first order-of-magnitude

consideration. The cost affecting factors, such as the isotopic abundance and the availability of the source material, should be included on a different basis.

The decay rate arguments for $\beta\beta$ decays favour strongly large values of ΔE (*cf.* the $(\Delta E)^5$ factor in Eq. (9)). On the other hand, high rates for the radiative captures are obtained with *small* $\Delta E^{\rm c}$. This requirement arises in the electromagnetic factors $f_{\rm M,E}$. The latter, for not too small transition energies, indicate $1/\Delta E^4$ dependence. This effect is only partly off-set by the phase space and photon wave function normalisation factors entering as q in Eq. (12). As a result, we expect the $(\Delta E)^{-3}$ dependence of the radiative neutrinoless double electron capture rate.

Strong enhancement of the radiative captures is obtained at large Z due to increase of electron densities in the nuclear region. The requirement of high electron density in the nuclear region for the radiative electron capture results in the extremely strong Z^6 dependence of the 0ν double electron capture rate (*cf.* Eq. (12)).

The retardation factor $R(\gamma)$ is typically 10^{-3}–10^{-5} and the radiative neutrinoless capture is much slower than the neutrinoless double beta decay. However, there are exceptional cases of large Z and small $\Delta E^{\rm C}$. Thus for $\Delta E^{\rm C} < m_e$ and $Z \approx 80$ the retardation $R(\gamma)$ is close to unity.

We can introduce a relative retardation factor

$$R_{\rm rel} = \frac{Z^6}{(Q - B_K - B_L)^3} \tag{13}$$

(where the Bs are the electron binding energies) as a rough rate indicator to compare various nuclei undergoing double electron capture transitions. Table I lists a few nucleides as possible candidates for experimental studies. It is seen that the $R_{\rm rel}$ factor favours strongly the ^{180}W nucleide. The nuclear matrix elements for the transitions listed in Table I have yet to be calculated. The estimates given below are based on the QRPA calculations

TABLE I

The relative enhancement factors and expected lifetimes.

Atom	abundance %	Q(EC, EC), keV	$R_{\rm rel}$, keV^{-3}	$T_{1/2}$(y), $m_\nu = 1$ eV
$^{92}_{42}$Mo	15.84	1648.6	1.3	$10^{31\pm1}$
$^{108}_{48}$Cd	0.875	262	1.0×10^3	$10^{28\pm1}$
$^{180}_{74}$W	0.12	145	3.9×10^5	$2.5 \times 10^{25\pm1}$
$^{196}_{80}$Hg	0.146	820	6.5×10^2	$2 \times 10^{28\pm1}$

for ECEC transition matrices $M^{0\nu} \approx 4$ of Ref. [8] which include both the GT and F transitions. Making crude estimates and taking into account the approximations made in deriving Eq. (12) we arrive at the decay rates of Table I with order of magnitude uncertainties. The corresponding feasibility estimates suffer thus from similar inaccuracies. The improved estimates will be the subject of a forthcoming publication.

Leaving the cost arguments aside and assuming 1 ton of the source material and the correspondingly larger amount of the high resolution detector (be it high purity Ge or a large bolometer) it seems to be feasible to design experiments for the 0ν double electron capture process in ^{180}W with the count rates of the order of $100 \times 10^{\pm 2}$ counts per year. The counting efficiency of about 10% has been assumed with the γ–K X-ray coincidence requirement to reduce the random background to a tolerable level. Prior to proper calculations of the nuclear matrix elements and the reduction of the present uncertainty factor, we consider this estimate as encouraging. The most sensitive double β experiment of the Heidelberg–Moscow team with 11 kg of 76Ge gives the limit $m_\nu < 0.3$ eV [4]. The plan (GENIUS, [4]) is to increase the source (detector) material to 10 t, which should improve the sensitivity for m_ν down to about 2×10^{-2} eV. We feel that the objective of finding out the nature of the electron neutrino (Dirac or Majorana) and of determining its mass calls for trying different methods and that the radiative double electron capture may be an attractive alternative to the double β technique.

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Appendix A

The photon emission amplitude

Few formulas for the photon emission amplitudes are collected in this appendix. The presentation is reduced to the basics and concerns three topics:

(i) the construction of a conserved current,

(*ii*) the calculation of E1 transition from $2P, 1S$ atomic electrons,

(*iii*) the calculation of M1 transition from $2S, 1S$ atomic electrons.

To remind the problem, let us consider the transition amplitude for $0\nu, \beta, \beta$ process and specify the factors along the leptonic line in Fig. 1. One has, [1,2],

$$A(0\nu\beta\beta) = (\bar{u}_e \gamma^\lambda P_{\mathrm{L}} S_\nu P_{\mathrm{L}} \gamma^\rho u_e^{\mathrm{c}}) W_{\lambda\rho}\,, \tag{A.1}$$

where: $u, u^{\rm c}$ are the usual and charge conjugated electron spinors, $P_{\rm L}$ projects on the left chirality states, S_ν is the neutrino propagator, $\gamma^\lambda, \gamma^\rho$ are the Dirac matrices and $W_{\lambda\rho}$ is the nuclear matrix element. The latter is left unspecified as it does not affect the radiative transition in a significant way. The time component W_{00} describes the Fermi and W_{ij} the Gamow–Teller transition modes. Eq. (A.1) already assumes the existence of Majorana neutrino which is reflected in the neutrino propagator S_ν. Now, the combination $P_{\rm L} S_\nu P_{\rm L} = m_\nu h_\nu$ is proportional to the neutrino mass m_ν and the spin independent neutrino propagator h_ν, which is called the neutrino potential. In this way m_ν measures the strength of the neutrinoless transitions and the lepton quantum number violation.

The amplitude for the reverse process of double electron capture is of closely related structure

$$A(0\nu) = (\bar{u}_e^{\rm c} P_{\rm L} \gamma^\lambda S_\nu \gamma^\rho P_{\rm L} u_e) W^*_{\lambda\rho}\,. \tag{A.2}$$

For the corresponding radiative capture, the dominant amplitude which describes one radiating electron is

$$A(0\nu\gamma) = e(\bar{u}_e^{\rm c} P_{\rm L} \gamma^\lambda S_\nu \gamma^\rho P_{\rm L} S_e A\gamma u_e) W^*_{\lambda\rho}\,, \tag{A.3}$$

where: S_e is the electron propagator, e is the electron charge, A is the photon field and $A\gamma = A^\mu \gamma_\mu$ describes the photon emission.

The standard procedure for radiative processes is to relate the electron propagator to the initial electron spinor u_e. For a free electron of four-momentum p^μ and a photon of momentum q^μ indicated in Fig. 1 one has

$$S_e A\gamma u_e = \frac{A\gamma}{p\gamma - q\gamma - m_e} u_e = \frac{2Ap - A\gamma q\gamma}{(p-q)^2 - m_e^2} u_e\,, \tag{A.4}$$

where $Ap, A\gamma$ denote four dimensional scalar products. To arrive at this result the free electron Dirac equation has been used. We use this also for the case of a bound electron. The first term Ap in Eq. (A.4) reduces to $Ap = \boldsymbol{Ap}$ in the $A_0 = 0$ gauge. This is equivalent to a nonrelativistic expression for the electric radiation. Strictly speaking this term is not gauge invariant and requires an analogous term due to the proton involved in capture (in the limit of infinite W mass). However, the latter does not contribute to $0^+, 0^+$ transitions and the radiation is emitted by the electron. It comes predominantly from a 2P atomic electron that radiates E1 photon and forms an S wave packet in the continuum. From such a packet the electron undergoes the nuclear capture. The electron wave function at the nucleus $\Psi_{\rm E}(0)$ is given by Eq. (A.4). In the coordinate representation one obtains

$$\Psi_{\rm E}(r=0)_m = \int d\boldsymbol{r} \frac{m_e \exp(-Qr)}{2\pi r} \nabla_m \Psi_{2P}(r)_m \exp(-i\boldsymbol{qr}), \tag{A.5}$$

where the index m denotes the third component of $l = 1$ angular momentum and $Q = \sqrt{-(E_B - q)^2 + m_e^2 + q^2}$ is the effective momentum carried by the electron. Here, E_B is the Dirac energy of the atomic electron and the energy of the photon is $q = \Delta E^{\rm c} - E_B^1 - E_B^2$. The γ emission amplitude factorises out as shown by Eq. (5). The electric factor $f_{\rm E}$ in Eq. (6) is determined by the electron wave function $\Psi_{\rm E}(0)$ that differs from the atomic function $\Psi_{1S}(0)$. The account of this difference and a summation over the initial electron and final photon polarisations produces

$$f_{\rm E} = \frac{1}{2}\left[\frac{2m_e}{B[(Q+1/2B)^2+q^2]}\right]^2 , \tag{A.6}$$

where $B = 1/(m_e \alpha Z)$ is the atomic Bohr radius.

The second term in Eq. (A.4), $A\gamma q\gamma$, corresponds to magnetic M1 transitions from $2S$ electron states. Here, the photon angular momentum is generated by the parallel spins of $2S$ and $1S$ electrons. The wave function of the electron that has radiated a magnetic photon is now

$$\Psi_{\rm M}(r=0) = \int d\boldsymbol{r} \frac{m_e q \exp(-Qr)}{2\pi r} \Psi_{2S}(r)_i \exp(-i\boldsymbol{qr}) . \tag{A.7}$$

The summation over initial the electron spins and final photon polarisations produces

$$f_{\rm M} = \frac{1}{4}\left[\frac{2m_e q}{(Q+1/2B)^2+q^2}\right]^2 . \tag{A.8}$$

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