COMPARISON BETWEEN WEISSKOPF AND THOMAS–FERMI MODEL FOR PARTICLE EMISSION WIDTHS FROM HOT DEFORMED NUCLEI* **

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The emission widths Γ_n and Γ_p for emission of neutrons and protons are calculated within the Thomas–Fermi model, which we have recently developped, and are compared with those obtained in the usual Weisskopf approach for the case of zero angular momentum. Both methods yield quite similar results at small deformations, but rather important differences are observed for very deformed shapes, in particular for charged particles. A possible generalization of the model for emission of α-particles is also discussed.

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1. Introduction

The observation of light particles in particular neutrons, protons and α-particles emitted in the course of the fission process from an excited and rotating nucleus between the compact initial shape to the scission point can provide interesting information on the fission process. We shell, in particular,

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study the light-particle emission width Γ_ν for a particle of type ν from hot and deformed but nonrotating ($L=0$) nuclei. The emission widths are calculated within the Thomas–Fermi model and the standard Weisskopf approach [1].

2. Weisskopf theory

According to the Weisskopf evaporation theory [1] the partial decay width $\Gamma_\nu^{\mu\kappa}(E^*,L)$ for emission of a light particle of type ν with energy ε_μ and orbital angular momentum ℓ_κ from a generally deformed compound nucleus with excitation energy E^* and rotational angular momentum L is given by

$$\Gamma_\nu^{\mu\kappa}(E^*,L) = \frac{2S_\nu+1}{2\pi\hbar\rho(E^*,L)} \sum_{L_{\rm R}=|L-\ell_\kappa|}^{L+\ell_\kappa} \int\limits_{\varepsilon_\mu-\Delta\varepsilon_\mu/2}^{\varepsilon_\mu+\Delta\varepsilon_\mu/2} w_\nu(\varepsilon,\ell_\kappa)\,\rho_{\rm R}(E^*_{\rm R},L_{\rm R})\,d\varepsilon\ , \tag{1}$$

where the level densities are given by [2]

$$\rho(E^*,L) = (2L+1)\left(\frac{\hbar^2}{2\mathcal{J}}\right)^{3/2}\sqrt{a}\,\frac{{\rm e}^{2\sqrt{aE^*}}}{12E^{*2}}\,. \tag{2}$$

Here $\mathcal{J}$ represents the moment of inertia and a the level density parameter of the compound nucleus at given deformation. $E^*_{\rm R}$, $L_{\rm R}$ and $\rho_{\rm R}$ are the excitation energy, angular momentum and level density of the residual nucleus, and S_ν is the intrinsic spin of the emitted particle. w_ν is the transmission coefficient evaluated in the semiclassical approach for emission of a particle of type ν, with energy ε and angular momentum ℓ_κ. This quantity also depends on mass and charge number, nuclear deformation and on the surface point of the deformed compound nucleus from which and the direction into which the particle ν is emitted.

3. Thomas–Fermi approximation

The Thomas–Fermi Approach (TFA) [4] calculates the transition rates $\Gamma_\nu^{\mu\kappa}$ through the probability that a light particle which inpinges on the nuclear surface at the surface point $\vec{r}_0^{\,\prime}$ with a velocity $\vec{v}^{\,\prime}$ is actually transmitted.

The number n_ν of particles of type ν which are emitted per time unit through the surface S of the fissioning nucleus is given by

$$n_\nu = \int\limits_{\rm S} d\sigma \int d^3p'\, f_\nu(\vec{r}_0^{\,\prime},\vec{p}^{\,\prime})\, v'_\perp(\vec{r}_0^{\,\prime})\, w_\nu(v'_\perp(\vec{r}_0^{\,\prime}))\,. \tag{3}$$

Primed quantities like the momentum $\vec{p}\,'$ and the velocity $\vec{v}\,'$ always refer to the body-fixed frame. $v'_\perp$ is the velocity component perpendicular to the surface at the surface point $\vec{r}_0'$ and $w_\nu(v'_\perp(\vec{r}_0'))$ is the classical transmission coefficient for emission of a particle of type ν.

The momentum of the particle of mass m in the laboratory frame is given by $\vec{p} = m\vec{v}\,' + m\vec{\omega}\times\vec{r}\,'$, with $\vec{\omega}$ the angular velocity of the nucleus in this frame.

The quantity $f_\nu(\vec{r}\,',\vec{p}\,')$ appearing in Eq. (3) is the classical phase-space distribution function (so called Wigner function)

$$f_\nu(\vec{r}\,',\vec{p}\,') = \frac{2}{h^3}\frac{1}{1+\exp\left[\frac{1}{T}\left(\frac{p'^2}{2m}+U_\nu-\omega\ell'-\mu_\nu\right)\right]}\,, \qquad (4)$$

where μ_ν is the chemical potential and ℓ' the body-fixed angular momentum in the direction of the rotation axis.

The potential U_ν felt by the particle of type ν is taken as

$$U_\nu(\vec{r}\,') = V_{\mathrm{SW}}^{(\nu)}(\vec{r}\,') + V_{\mathrm{Cb}}(\vec{r}\,')\,\delta_{\nu,p}\quad, \qquad (5)$$

where $V_{\mathrm{SW}}^{(\nu)}$ is a Saxon–Woods mean-field potential of standard depth, radius and diffuseness and V_{Cb} is the Coulomb potential experienced only by protons.

Let us just mention that one can easily show the link of this microscopic classical expression with the Weisskopf's formulation. Then the expression

$$\frac{d^2 n_\nu}{d\varepsilon_\mu d\ell_\kappa}\Delta\varepsilon\,\Delta\ell$$

represents the emission probability per unit time of a particle ν with given final energy ε_μ and given final angular momentum ℓ_κ [5] corresponding to the Weisskopf formula given above:

$$\Gamma_\nu^{\mu\kappa} = \frac{d^2 n_\nu}{d\varepsilon_\mu d\ell_\kappa}\Delta\varepsilon\,\Delta\ell\,. \qquad (6)$$

Emission rates can be then easily determined for neutrons and protons where the distribution function $f_\nu(\vec{r}\,',\vec{p}\,')$ is known.

We have shown analytically [5] that for spherical, non rotating nuclei both models lead to practically the same partial decay rates.

4. Phase-space distribution function for α-particle

The determination of the phase-space distribution function is much more intricate in the case of α-particles, which are composite particles. We are at present working on a model in which one assumes that the α-particle is described by a distribution function f_α built from two correlated proton and neutron pairs. Following a suggestion by Dietrich we write [6]:

$$f_\alpha(\vec{R},\vec{P}) = \frac{1}{h^3}\int I_{n_1} I_{n_2} I_{p_1} I_{p_2}\, d\sigma_{n_1}\, d\sigma_{n_2}\, d\sigma_{p_1}\, d\sigma_{p_2}\, \delta\left(\sigma_\alpha\right). \tag{7}$$

where σ_{ν_i} denotes spin coordinate of particle ν_i and $\delta\left(\sigma_\alpha\right)$ is the projection operator on the spin of the α-particle.

Such 4 correlated nucleons (2 protons, 2 neutrons) which create an α-particle should be close to each other and have almost parallel momenta. This is the basic Ansatz for the temperature dependent probability $f_\alpha(\vec{R},\vec{P})$ to find an α cluster with momentum $\vec{P}$ at the position $\vec{R}$ in a nucleus of temperature T:

$$f_\alpha(\vec{R},\vec{P}) = \frac{1}{h^3} I_{n_1} I_{n_2} I_{p_1} I_{p_2}\,, \tag{8}$$

where *e.g.* for the first neutron function I_ν is given by the interval:

$$I_{n_1} = \mathcal{N}_{\beta_1\beta_2}\int \tilde{f}_{n_1}\left(\vec{r_1},\vec{p_1}\right) \mathrm{e}^{-\frac{\left(\vec{r_1}-\vec{R}\right)^2}{2\beta_1^2}}\, \mathrm{e}^{-\frac{\left(\vec{p_1}-\frac{\vec{P}}{4}\right)^2}{2\beta_2^2\hbar^2}}\, d^3r_1\, d^3p_1\,, \tag{9}$$

with:

$$\tilde{f}_n(\vec{r},\vec{p}) = \frac{1}{1+\exp\left[\frac{1}{T}\left(\frac{p^2}{2m} - U_n(\vec{r}) - \omega l_x - \mu_n\right)\right]}\,, \tag{10}$$

where β_1 is the width of the Gaussian coordinate distribution and β_2 the corresponding width in momentum space with the overall normalization factor

$$\mathcal{N}_{\beta_1\beta_2} = \frac{1}{(2\pi\beta_1\beta_2\hbar)^3}\,.$$

The emission width $\mathit{\Gamma}_\alpha$ for α-particles is then obtained in our Thomas–Fermi model through Eq. (8).

5. Results

Figure 1 gives a comparison of the neutron and proton emission rates obtained in the two approaches for the nucleus ^{188}Pt for different values of the collective (deformation) coordinate $q = R_{12}/R_0$ (see Ref. [3]).

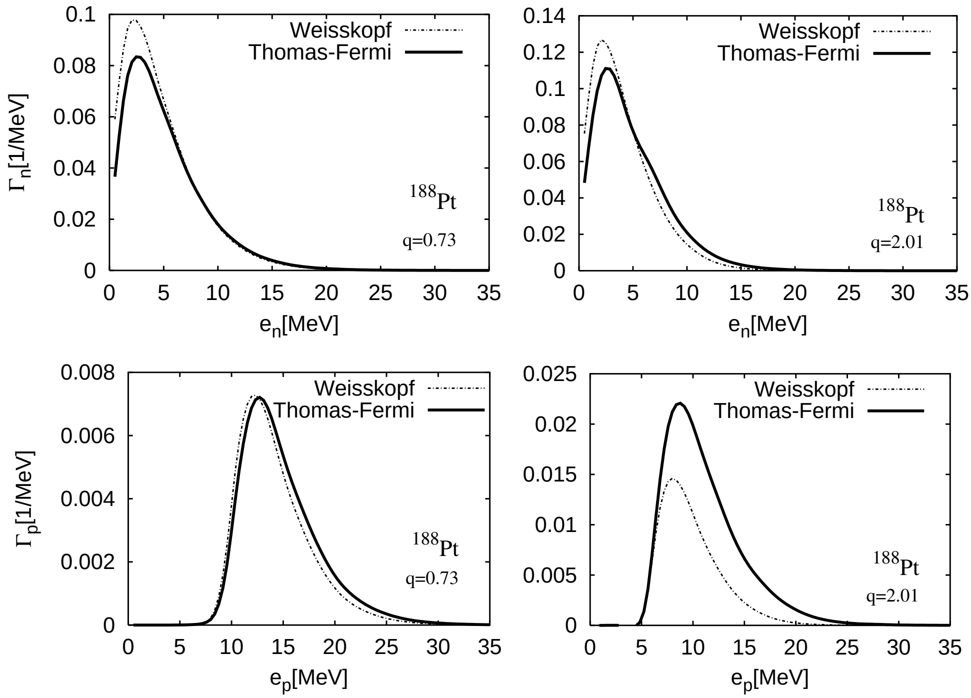

Fig. 1. Comparison between the emission rates Γ_n and Γ_p for neutrons and protons obtained in the Weisskopf and the distribution-function approach.

As one can notice both approaches yield emission rates that are almost identical for small deformations. For large nuclear deformations (close to the scission configuration) this is still the case for the neutron width Γ_n, whereas the proton emission width Γ_p is much larger in the Thomas–Fermi approach as compared to the Weisskopf model.

Figure 2 shows the behaviour of the phase-space distribution function for α-particles for different Gaussian width β_2 in momentum space. One notices that this function is very sensitive to its momentum distribution. The comparison between the α emission width obtained in the Weisskopf theory and the Thomas–Fermi approach shown in figure 3 was obtained for a value of the Gaussian momentum width parameter of $\beta_2 = 0.0685\,\mathrm{fm}^{-1}$.

In this figure one can see that the α-particle can be only emitted with very large energy. Here we have made the assumption that the potential experienced by α-particle is the sum of the four single particle potential of neutrons and protons.

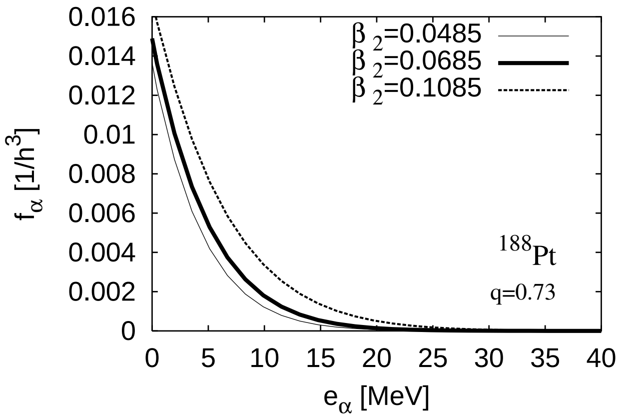

Fig. 2. Distribution function for α-particle for different Gaussian width β_2 in momentum space.

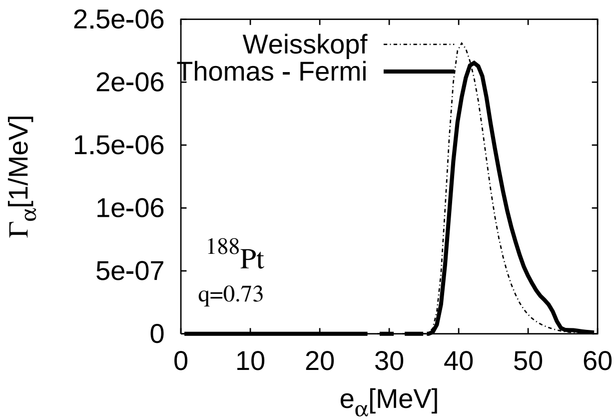

Fig. 3. Comparison between the emission rate Γ_α for α-particle obtained in the Weisskopf and the distribution-function approach.

6. Conclusion

Summing up the results obtained in our research one may say that the phase-space distribution function approach gives results not much different from the Weisskopf evaporation theory. But the new model describes also the emission rates for light particles in a given direction of space. We can say that for neutrons both approches yield very similar results.

The behaviour of the deformation dependence of the proton emission width Γ_p can be understood if one keeps in mind that the Coulomb barrier which the charged particles have to overcome depends (for deformed nuclei) on the direction of the emitted particle. It leads to a large enhancement of the emission width of protons from very deformed nuclei.

Investigating the phase-space distribution function approach for α-particles one can conclude that it depends, in a very sensitive way, on its momentum distribution and on the parameters of the single particle potential. These parameters will be worked out in the future. We did not attempt to do this here also because of the lack of the corresponding experimental data.

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