# SHAPE COEXISTENCE IN $^{98}$Mo*

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Quadrupole deformation parameters of the $^{98}$Mo nucleus in two first $0^+$ (ground and excited) states are determined using Coulomb Excitation method. Matrix elements were determined using the GOSIA code and then analysed using the Quadrupole Sum Rules formalism. Shape coexistence in $^{98}$Mo manifests in the very different triaxiality of the two $0^+$ states. The results are compared with previously known data on $^{72,74,76}$Ge isotopes where the similar trend of low-lying $0^+$ states is observed.

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## 1. Introduction

Only four even-even stable nuclei with $Z > 20$, namely $^{72}$Ge, $^{90}$Zr, $^{96}$Zr and $^{98}$Mo, have a $0^+$ first excited state. An easy interpretation within the framework of the collective model would be that these levels are the bandheads of the $\beta$-vibrational bands. However, Coulomb excitation studies of $^{72}$Ge [1] and $^{96}$Zr [2] proved that the ground state is deformed, while the first excited state has a spherical shape and can be interpreted as an intruder state. Due to the high excitation energies and non-collective structure no

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information is available on the nature of the first excited $0^+$ in $^{90}$Zr. $^{98}$Mo nucleus is the best candidate for further investigation of the nature of the shape coexistence in this mass region.

## 2. Experiments

Experimental data were collected with the GEMINI $\gamma$-ray detection system [3], supported by four position sensitive scintillators used for charged particle detection. $^{84}$Kr and $^{136}$Xe beams were provided by JAERI tandem accelerator at Tokai in Japan.

Preliminary data were obtained in a simple experiment with a $^{20}$Ne beam, performed at the Heavy Ion Laboratory in Warsaw with the use of the CUDAC set-up [4].

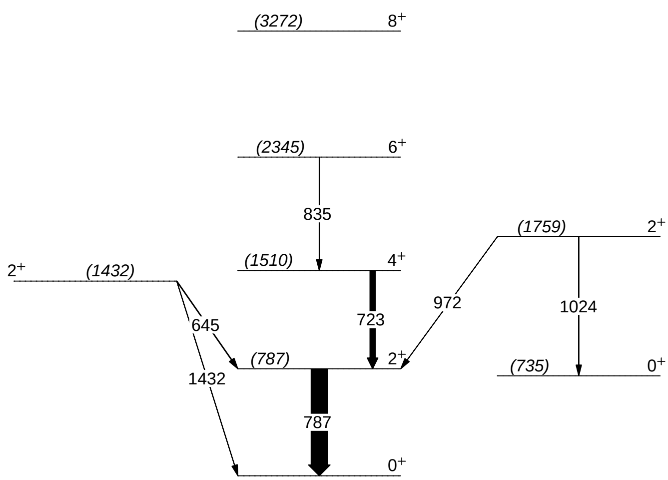


Fig. 1. Low-lying excited states of the $^{98}$Mo nucleus (adopted after [5]). The transitions observed in experiment with the $^{136}$Xe beam are marked with the arrows. All energies are given in keV.

## 3. Data analysis

Excited levels scheme together with available spectroscopic data were adopted after an on-line database of NNDC Brookhaven [5] and combined with measured $\gamma$-ray intensities. Those were used as an input to the Coulomb Excitation Least Squares Fitting code GOSIA [6].

A resulting set of reduced matrix elements was rich and precise enough to perform further analysis of the nucleus shape. For example, the crucial matrix elements connecting $0^+$ states with all observed $2^+$ states were determined with accuracy better than 15%.

### *3.1. Quadrupole sum rules*

The E2 operator expressed in the principal axes frame of the nucleus may be parameterized using two parameters $Q$ and $\delta$:

$$\begin{aligned} E(2,0) &= Q\cos\delta, \\ E(2,1) &= E(2,-1) = 0, \\ E(2,2) &= E(2,-2) = \frac{1}{\sqrt{2}}\,Q\sin\delta. \end{aligned}$$

The $Q$ parameter is a measure of an overall deformation, while the $\delta$ parameter is a measure of triaxiality. $Q$ and $\delta$ are equivalent to commonly used deformation parameters $\beta$ and $\gamma$, except that they are related to the charge distribution rather than mass distribution.

An invariant is needed to evaluate intrinsic deformation parameters using quantities measured in the laboratory frame.

Zero-coupled products of E2 operators are rotational invariants. The simplest product may be expressed as:

$$\langle i|\,[\mathrm{E2}\times\mathrm{E2}]_0\,|i\rangle = \frac{Q^2}{\sqrt{5}}.$$

Operator products may be expressed using the intermediate state expansion formula:

$$\langle i\|\,[\mathrm{E2}\times\mathrm{E2}]_0\,\|i\rangle = \frac{1}{\sqrt{(2I_i+1)}}\sum_t \langle i\|\mathrm{E2}\|t\rangle\langle t\|\mathrm{E2}\|i\rangle \left\{ \begin{array}{ccc} 2 & 2 & 0 \\ I_i & I_i & I_t \end{array} \right\}. \tag{1}$$

To get information on triaxiality, the expectation value of the $\delta$ parameter must be determined. Therefore the higher order invariant is needed:

$$\langle i|\{[\mathrm{E2}\times\mathrm{E2}]_2\times\mathrm{E2}\}_0|i\rangle = \sqrt{\frac{2}{35}}Q^3\cos 3\delta.$$

The similar evaluation using the intermediate state expansion formula yields:

$$\langle i|\{[\mathrm{E2}\times\mathrm{E2}]_2\times\mathrm{E2}\}_0|i\rangle = \frac{1}{(2I_i+1)}\sum_{t,u}\langle i\|\mathrm{E2}\|u\rangle\langle u\|\mathrm{E2}\|t\rangle\langle t\|\mathrm{E2}\|i\rangle \left\{ \begin{array}{ccc} 2 & 2 & 2 \\ I_i & I_t & I_u \end{array} \right\}. \tag{2}$$

Sums over intermediate states in equations (1) and (2) extend over all states of the system, which may be reached by a single E2 transition from the state of interest $i$.

Reduced matrix elements in the above equations are determined experimentally.

## 4. Shape parameters of $0_1^+$ and $0_2^+$ states

Figures 2 and 3 show the quadrupole deformation parameters, calculated for both $0^+$ states in $^{98}$Mo nucleus, in comparison with available data from selected Ge isotopes [1, 7, 8] which also have a low-lying second $0^+$ state.

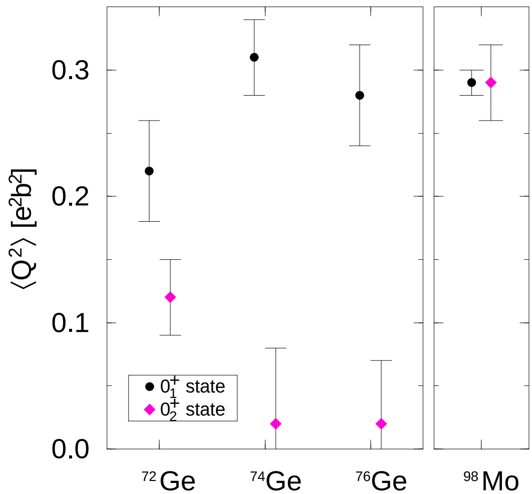


Fig. 2. Mean values of $Q^2$ parameter found for the two first $0^+$ states in $^{98}$Mo and $^{72,74,76}$Ge nuclei.

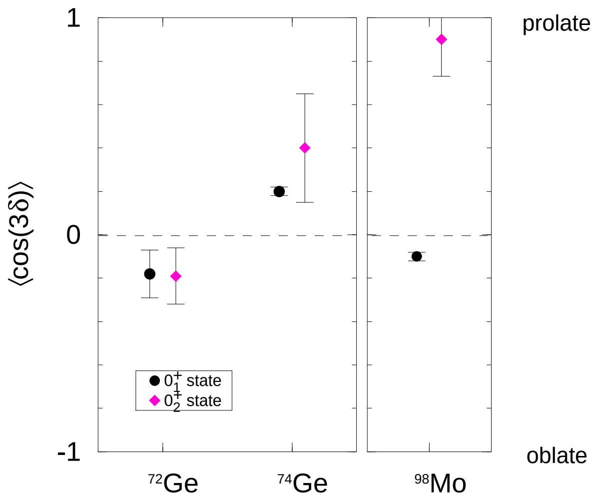


Fig. 3. Mean values of $\cos 3\delta$ parameter found for the two first $0^+$ states in $^{98}$Mo and $^{72,74}$Ge nuclei.

In case of Ge isotopes the ground state is deformed, while the second $0^+$ state tends to be spherical. For $^{98}$Mo nucleus overall deformation is the same for both $0^+$ states.

The triaxiality remains the same for each of Ge isotopes. On the contrary, the ground state of $^{98}$Mo is triaxial, while the first excited state has a prolate shape.

## 5. Summary

The collective properties of the first $0^+$ states in $^{98}$Mo nucleus have been investigated in a model-independent way using multiple Coulomb excitation. The result shows a clear shape coexistence of the ground and the first excited $0^+$ states. While the overall quadrupole deformation (intrinsic $Q$ moment) remains almost constant, the nucleus undergoes the transition from triaxial ($0^+_1$) to prolate ($0^+_2$) shape. The result is in sharp contrast to those obtained for Ge isotopes, although their energy-level structures are similar.

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