HIGH ENERGY QCD AT e^+e^-, pp AND ep COLLIDERS*

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We review recent high energy QCD data and phenomenological developments on small-x and BFKL like effects. Data of ep, e^+e^- and pp colliders are discussed, and prospects for future colliders are given.

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1. High energy QCD

In this contribution we will study strong interactions at large partonic center of mass (CMS) energy s and momentum transfer t such that $s \gg |t|$. This is the domain of small x and the region where we could expect BFKL theory to be applicable.

In this paper we report on progress of measurements which study the parton evolution in the 'parton ladder' as depicted for an ep collision in Fig. 1. This is often also called the study of the hard or the perturbative or BFKL pomeron. We will examine footprints for such BFKL signals. The signals studied are:

- Structure functions.
- Forward jet measurements in ep.
- Forward high p_{T} particle measurements in ep.
- Dijets in pp.
- Hard color singlet exchange in pp and ep.
- Double tagged events in ee.

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- Vector Meson production in *ep* and *ee*.
- New ideas on small x measurements.
- A few common challenges in *ee*, *ep* and *pp* data.
- Instantons searches.

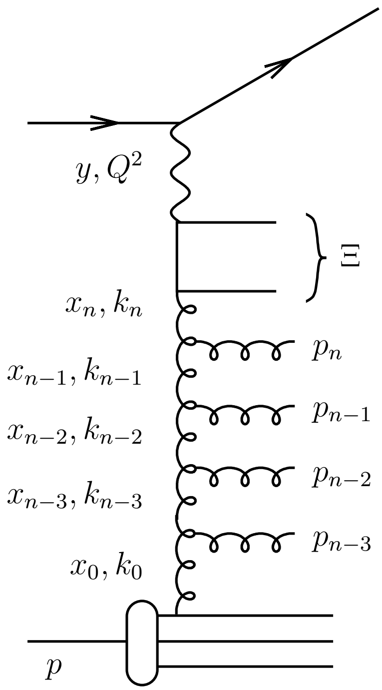


Fig. 1. Ladder diagram for multi-gluon emission in low-x *ep* collisions.

We will report both on present and possible future low-x data for these different interactions, shown in Fig. 2. The present data consists of results from HERA ($ep, \sqrt{s} = 320$ GeV), Tevatron ($p\overline{p}, \sqrt{s} = 1800$ GeV) and LEP ($e^+e^-, \sqrt{s} = 90$–210 GeV). The first two colliders have terminated run I a few years ago and now start run II in order to collect 10 times and 100 times more data respectively. LEP has concluded its data taking in the year 2000.

Future data will include data from the LHC ($pp, \sqrt{s} = 14$ TeV, startup expected in 2007), and possible that of a linear collider LC ($ep, \sqrt{s} = 500$–1000 GeV and $\gamma\gamma, \sqrt{s} = 400$–$800$ GeV) and a collider such as THERA ($ep, \sqrt{s} = 1$ TeV).

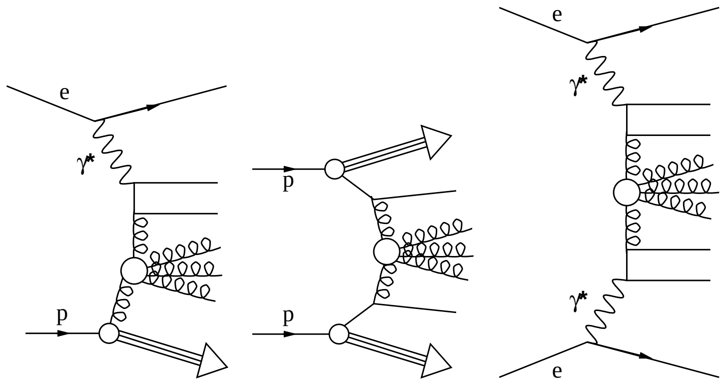


Fig. 2. Sketch of processes for small-x signatures in high energy collisions in ep, pp (or $p\overline{p}$) and e^+e^- collisions.

2. Parton evolution

Basically, there are three different parton evolution equations at our disposal: the DGLAP (Dokshitzer, Gribov, Lipatov, Altarelli, Parisi) [1], BFKL (Balitski, Fadin, Kuraev, Lipatov) [2] and CCFM (Catani, Ciafaloni, Fiorani, Marchesini) [3] equations.

The DGLAP equations resum $\alpha_{\rm s} \ln Q^2$ terms. This implies a strong ordering of the parton $k_{\rm T}$ in the ladder, and predicts the Q^2 evolution of parton distributions, *i.e.* once a parton distribution is given at any scale Q_0^2 it can be predicted at any other Q^2 value.

The BFKL equation resums $\alpha_{\rm s} \ln 1/x$ terms. It implies strong ordering in x but no ordering of the parton $k_{\rm T}$ in the ladder which follows in fact a diffusion pattern. These equations are expected to ultimately describe the low-x behaviour of processes.

The CCFM equation interpolates between DGLAP and the BFKL limits, and is based on angular ordering and colour coherence. In the appropriate limits it will produce the DGLAP and BFKL approximation.

In the case of DGLAP collinear factorisation and for BFKL/CCFM $k_{\rm T}$ factorisation can be used to calculate cross sections of processes. The parton distributions to be used in the latter case are the so called unintegrated ones: $f(x, k_{\rm T}^2, Q^2)$.

The DGLAP equation is well know and studied since quite some time. The study of the BFKL equation began in earnest just before and during the HERA data taking. In LO it predicts a power increase of the cross section $\sigma_{\rm BFKL} \sim s^{\lambda} \sim (1/x)^{\lambda}$ with $\lambda = 0.5$ for $\alpha_{\rm s} = 0.2$. In 1998, after an heroic effort, the NLO corrections to the kernel were finalised [4]. These corrections turned out to be very large and may turn λ zero or even negative.

Since then the effects of higher orders have been studied for measurable processes. A phenomenological determination of the effective λ via analyses of subleading terms or studying the structure of the divergences [5] (*e.g.* consistency constraints, effective $\alpha_{\rm s}$, collinear resumation, order-by-order consistency of the $\ln Q^2$ terms, rescaling Y, dipole cascades *etc.*) lead to the observation that the 'NLO' value of λ seems to converge to a range of 0.15–0.3. Complete NLO calculations for measurable processes are also in preparation, *e.g.* [6].

Several tools exist to confront predictions with experimental low-x data. For BFKL studies we have:

- BFKL Monte Carlo program [7, 8].
- Monte Carlo programs without $k_{\rm T}$ ordering (CDM ... not truly BFKL).
- HERWIG and PYTHIA versions with BFKL for specific processes.
- BFKL analytical and numerical calculations (many ...).

For CCFM predictions we have:

- The CASCADE Monte Carlo program [9]. This includes a CCFM backward evolution, has been fitted to the F_2 data to fix the parameters, and final state results can be predicted without further tuning.
- The LDC Monte Carlo [11].
- The SMALLx program [10].
- CCFM analytical and numerical calculations (many ...).

DGLAP predictions are provided by various Monte Carlo programs.

3. Structure functions

The first DIS workshop in 1993 in Durham also marked the release of — and debate on — the first low-x data. During that workshop the first F_2 measurements from HERA were shown, reaching x values down to a few times 10^{-4}, which showed a strong rise of F_2 with decreasing x. This observation was at first sight not incompatible with the LO BFKL power given above. The rise of F_2 at small x has been confirmed ever since with much more data and an improvement in precision of a factor 10.

However, it was soon realised that leading twist NLO DGLAP equations are perfectly able to describe the data down to x values of a few times 10^{-5}. Where are the expected large $1/x$ logarithms hiding? (Some recent discussion can be found in [12].) The DGLAP evolution was shown to be rather robust and able to describe the data, but perhaps at the expense of having a rather unnatural behaviour of the resulting gluon distribution at small Q^2 values, where it can become negative. Such behaviour could result from higher twists contributions to the data, or from a genuine breakdown of the theory, but it remains difficult to proof the latter. Any inconsistency with a measurement and analysis of $F_{\rm L}$ in the HERA region could shed light on this matter, but it is doubtful that a precise enough measurement can be made at HERA within its anticipated lifetime, as presented in [13].

Despite the success of the DGLAP equations for the inclusive observable F_2, attempts to test unified BFKL+DGLAP descriptions of F_2 also do a good job (see *e.g.* [14]), if not better, but obviously makes the description more complicated. Recently R. Thorne showed that when including low-x resumation a substantially better fit of the data can be obtained than with a pure DGLAP fit alone. The results of this analysis are shown in Fig. 3. Does the F_2 data start to reach a precision that will allow to detect the effect of small-x terms? The present measurement errors are around 1–2%

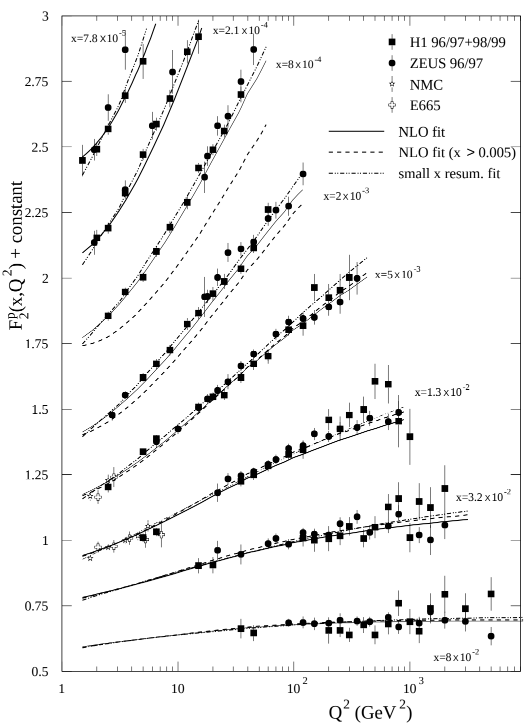


Fig. 3. Proton structure function data as a function of Q^2 at fixed x. Several fits are shown, following the MRST prescription, one of which includes small-x resumation.

now in a large region of the phase space. Future HERA-II data will be able to extend the region of precise data further and perhaps even improve the overall precision. Hence there is some hope that future F_2 measurements may become one of the referees in the small x discussion.

4. Final states

4.1. The first ideas

By 1995 it was already realized that F_2 may be a too inclusive observable to reveal the BFKL dynamics. At the time the interest started to turn to final state variables. The underlying idea was to study the behaviour of the partons in the 'ladder' as shown in Fig. 1.

Proposed variables at the time where

- Forward jets in *ep*.
- Forward energy flow in *ep*.
- Mueller–Navelet jets in *pp*.
- Jet angular de-correlation in *ep*.

The global energy flows, although encouraging at first [16], finally were concluded to be too dependent on the non-perturbative fragmentation phase, and have no longer been pursued. The initial BFKL effects on jet angular de-correlations were found to be below the expected resolution, but this topic has been recently revisited [17] in order to extract the unintegrated gluon distribution in the proton.

Since then a number of new ideas have been put forward such as the measurement of forward particles, vector meson production at large t, $\gamma^*\gamma^*$ scattering, *etc.*

4.2. Forward jets in ep

The idea [18] of this measurement is to choose low-x events as shown in Fig. 4 with a jet with $x_{\rm jet}(= x_1) \sim E_{\rm jet}/E_{\rm proton}$ to be large, *i.e.* around 0.1, and to choose $E^2_{\rm Tjet}(= k^2_{\rm T1}) \sim Q^2$. The latter condition suppresses DGLAP evolution while the former selects event with a large partonic ladder. The measurement exploits the strong $k_{\rm T}$ ordering expected in the DGLAP evolution to suppress this contribution. These jets are kinematically close to the forward proton direction in the HERA lab frame, and hence labelled forward jets. Typical experimental cuts select jets with 7 to 10 degrees of angle w.r.t. the proton and $E_{\rm T}$ values of 4–5 GeV [19, 20]. Fig. 4 and Fig. 5 show the data compared to calculations. Pure DGLAP based models generally

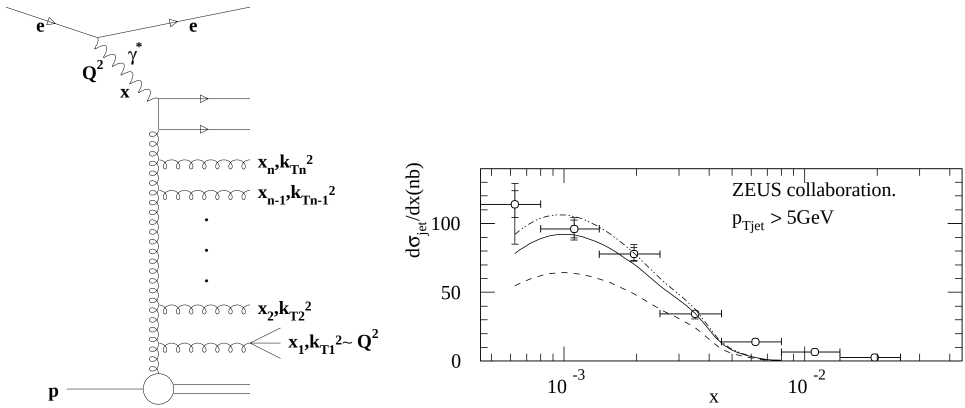


Fig. 4. (left) Parton evolution in the ladder approximation. The kinematics of forward jets is indicated. (right) Forward jet DIS cross section as function of x. BFKL calculations including the consistency constraint are overlaid.

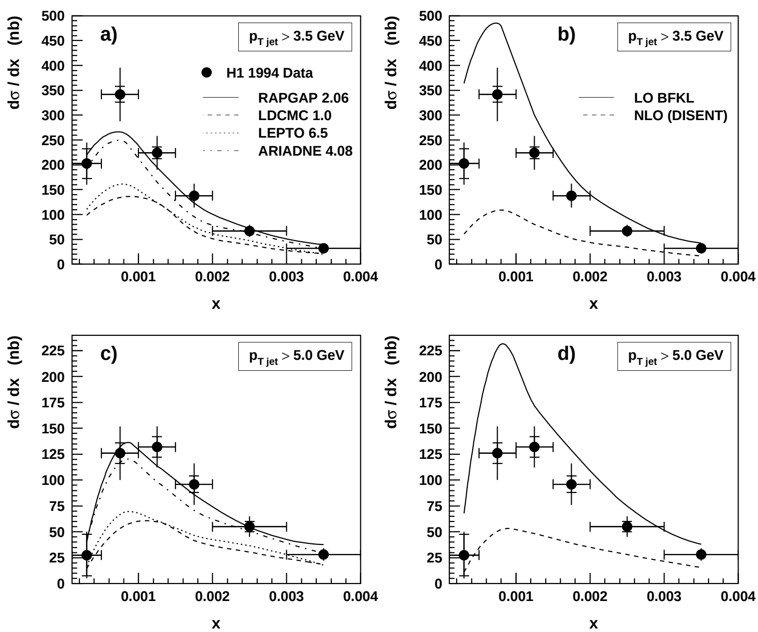


Fig. 5. Forward jet data as function of Bjorken-x, compared to Monte Carlo calculations (left) and numerical calculations (right).

underestimate the data. Pure LO BFKL calculations predict generally too steep a rise. Improved BFKL calculations which include explicit kinematic effects (via Monte Carlo techniques [7]) or so called consistency constraints, which include a large (calculable) part of the subleading effects [21] can describe the data, see Fig. 4. However also DGLAP models with added resolved photon contributions can describe the data equally well [22]. Also the CASCADE Monte Carlo describes well the H1 forward jet data, but slightly worse the ZEUS data, as shown in Fig. 6.

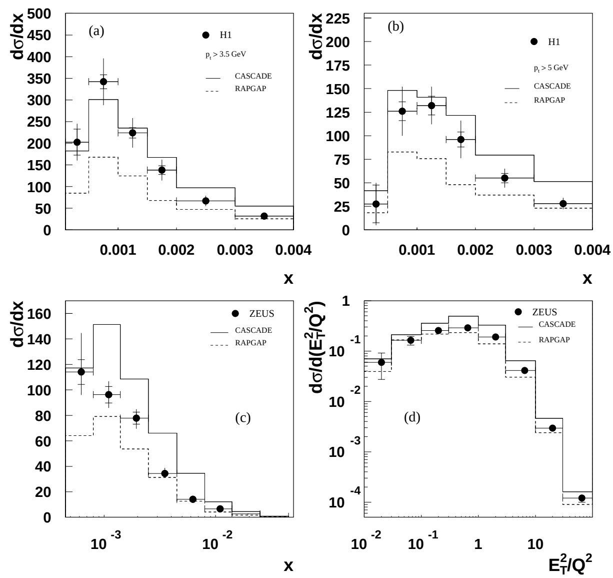


Fig. 6. The cross section for forward-jet production obtained from the Monte Carlo CASCADE at hadron level (solid line); (a)–(c) The cross section for forward-jet production as a function of x, for different cuts in $p_{\rm T}$ compared to H1 data [19] (a)–(b) and compared to ZEUS data [20] (c); (d) The cross section for forward-jet production as a function of $E_{\rm T}^2/Q^2$ compared to [23].

In all the conclusion on the forward jets from HERA is not unambiguous. Possibly the length of the ladder accessible at HERA is still to small. The late B. Andersson conjunctured that the number of gluons emitted in the ladder of Fig. 4 with a $p_{\rm T} > 1.5$ GeV is approximately $\ln(x_{\rm jet}/x)/2$, *i.e.* 3 to 4 gluons for $x \sim 10^{-4}$. This is still far from the asymptotic region and the random walk of the gluon emission is still strongly affected by momentum conservation constraints, reducing the BFKL effect and discriminative power with respect to other explanations. To increase the reach HERA would need to be able to detect forward jets closer to the outgoing proton, which is prohibited with the layout of present HERA experiments and interaction region. Progress could be expected at THERA, where the reach in x could be increased by an order of magnitude. Furthermore, if special attention to the interaction region/instrumentation lay-out is made, one can expect [24] that the different approaches, which are presently successful in describing the data (BFKL, resolved photons, dipole cascades) can be distinguished. Ideally forward jets with an angle down to 1 degree need to be tagged.

4.3. Forward high $p_{\rm T}$ particles

Forward jets have the disadvantage of being fairly extended objects, depend on jet finding algorithms and are sensitive to the calorimetry hadronic energy scales. A complementary approach is to study forward high $p_{\rm T}$ particles. Generally lower angles (*i.e.* longer ladders) can be reached, and such analyses have different systematics. On the other hand fragmentation functions are needed to calculate the rates. H1 has published forward π^0 data [25] which have been compared with various models. The BFKL calculations including the consistency constraint [26] give a good description of those data. The resolved photon approach was found to describe the data less well. On this conference H1 has released new measurements with larger statistics and presented more differentially [27], *e.g.* studying the $E_{\rm T}$ associated with the tagged π^0. Some of the results are shown in Fig. 7. The conclusion is however not yet clear: in some regions CASCADE performs better, in others the resolved photon approach is in better agreement. The failure of CASCADE in this measurement could be due to the quark splitting functions which are presently not yet taken into account in this model.

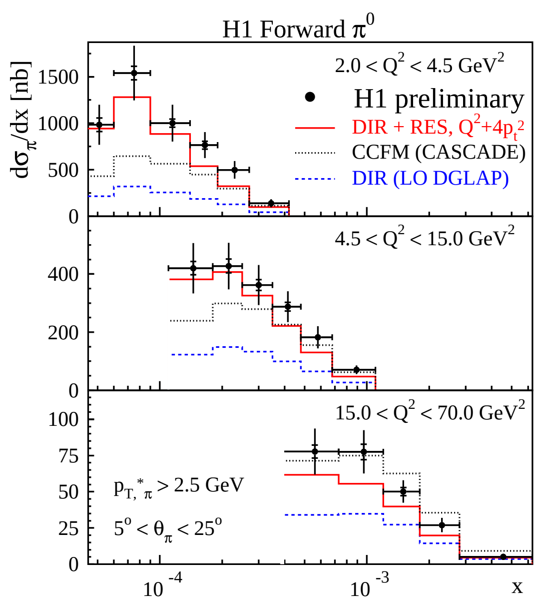


Fig. 7. Inclusive π^0 meson production cross sections as a function of x for $p_{\rm T} > 5$ GeV in three regions of Q^2. Comparisons are made with different models.

4.4. Dijets in pp

The forward jets studied in *ep* collisions have emerged from the original idea of Mueller and Navelet [28], who suggested to study di-jets in *pp* (or in practice at the Tevatron in $p\overline{p}$) with a large rapidity difference, allowing for a gluon ladder to develop between the two jets. The D0 collaboration has analysed such events in terms of low-x phenomenology. The azimuthal angle de-correlations between jets have been studied as a function of the jet distance in rapidity. The result was at first slightly discouraging, showing that both naive and somewhat improved BFKL calculations overestimated the de-correlation effect. A fixed order QCD calculation on the other hand underestimated the effect, but the general purpose Monte Carlo program HERWIG does a good job. HERWIG, which contains angular ordering in its QCD showers, could of course just have that part of the BFKL effect included which is most relevant for this de-correlation.

Meanwhile it has been shown that improved BFKL calculations either by using the consistency constraint [29] or effective rapidity [30] can provide also a reasonably good description of the *pp* dijet de-correlation data.

To avoid the strong dependence on the steeply falling parton densities at small-x, it has been proposed [28, 31] to study the dijet cross section at fixed x_i, the fractional momenta of the partons in the proton, at different CMS energies. In an analysis performed by D0 [32], jets have been selected with $E_{\mathrm{T},i} > 20$ GeV and $|\eta_i| < 3$ and $400 < (Q^2 = E_{\mathrm{T},1}E_{\mathrm{T},2}) < 1000$ GeV^2. The cross section ratio at fixed x_i for two CMS energies has been measured:

$$R = \frac{\sigma(\sqrt{s_a})}{\sigma(\sqrt{s_b})} = \frac{\exp\left(\lambda\left(\Delta\eta_a - \Delta\eta_b\right)\right)}{\sqrt{\Delta\eta_a/\Delta\eta_b}}. \tag{1}$$

The result of the cross section ratio at $\sqrt{s_a} = 1.8$ TeV and $\sqrt{s_b} = 630$ GeV is shown as a function of the average $\langle\Delta\eta\rangle$ for $\sqrt{s_b} = 630$ GeV in Fig. 8. At large $\langle\Delta\eta\rangle$ the dijet cross section increases almost by a factor of 3 between the two CMS energies. The strong increase leads to a large value for λ namely: $\lambda = 0.65 \pm 0.07$. The exact LO pQCD calculation leads to a falling cross section. The LO BFKL calculation (labelled LLA in Fig. 8) predicts $\lambda = 0.45$ for $\alpha_{\mathrm{s}}(20\ \mathrm{GeV}) = 0.17$. A complete NLO BFKL calculation is unfortunately not yet available. Surprisingly, the highest $\lambda = 0.6$ is obtained by HERWIG.

However, it has recently been pointed out [30] that an interpretation of these results is ambiguous because of differences in the definition of the cross sections between the D0 data and the original Mueller–Navelet proposal, related to the momentum fractions x_i which is based on the assumption of 2-body kinematics, and an upper bound on the momentum transfer Q^2. These effects can only be neglected in the asymptotic limit *i.e.* at large s and large

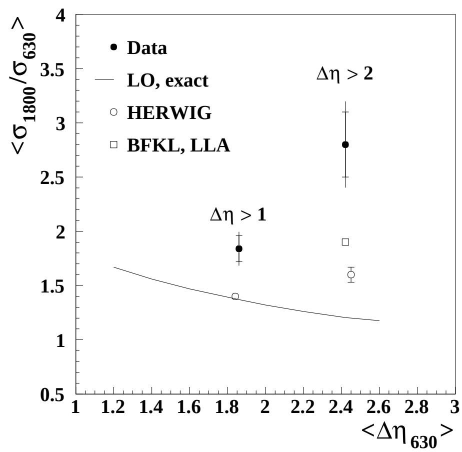


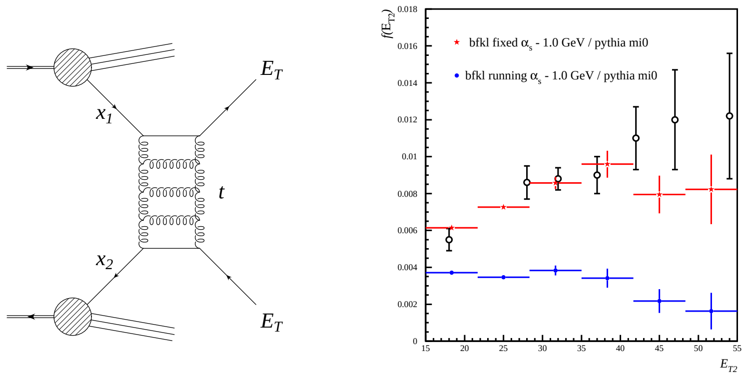
Fig. 8. Ratio of the D0 dijet cross section at a CMS energy $\sqrt{s}$ = 1800 GeV and $\sqrt{s}$ = 630 GeV for $\Delta\eta > 1$ and $\Delta\eta > 2$ as a function of the mean rapidity difference of the jets at $\sqrt{s}$ = 630 GeV.

$\Delta\eta$, which is not yet reached at Tevatron. Furthermore, the requirement of two jets with the same minimum E_{T} is particularly critical [33], since large logarithms, so called Sudakov logs, which not connected with BFKL effects may mask any BFKL effects. Hence for a more clean BFKL test this measurement should be redone with asymmetric E_{T} cuts!

A larger phase space would also be commendable. At the LHC the ATLAS and CMS detectors will allow for such a measurement to be made up to values of $\Delta\eta$ of 9 units and perhaps even beyond. Predictions for the LHC are shown at this conference in [8].

A further measurements to test BFKL in *pp* collisions is the study of hard colour singlet exchange events. These are events as depicted in Fig. 9(a), consisting of two hard jets separated by a rapidity gap. An analysis presented in [34], and shown in Fig. 9(b) reveals that when a full Mueller–Tang calculations is used, the multiple interaction corrections are undone and α_{s} is kept fixed, the model describes the D0 data very well. The predictions are made with HERWIG, including the above calculation. It should be noted that the CDF [35] data do not agree with the D0 [36] data at large rapidity. It would be useful to sort this measurement out with the upcoming Tevatron run-II data. A similar observation is reported in [37], based on BFKL + consistency constraints and including non-asymptotic effects in the rapidity variable definition, all included in PYTHIA. Agreement with the D0 but with not the CDF data is reported.

Recently such measurements have also been made at HERA [38] but were found to be inconclusive for BFKL claims at this stage.



(a) (b)

Fig. 9. (a) The jet-gap-jet topology in *pp* scattering resulting from colour singlet exchange. (b) Gap fraction as function of E_{T} compared with the D0 data.

4.5. Inclusive hadronic cross section in $\gamma^*\gamma^*$ collisions

When studying the total hadronic cross section $\gamma^*\gamma^* \to$ *hadrons* in e^+e^- collisions, the difficulties connected with the hadronic and thus extended nature of incoming particles can be avoided. The virtuality Q^2 of the γ^* controls the transverse size $\propto 1/\sqrt{Q^2}$ of the hard processes. For $Q^2 \gg \Lambda^2_{\mathrm{QCD}}$ a complete perturbative calculation is possible. For small virtualities Q_i^2 of one of the virtual photons and for large CMS energies W of the $\gamma^*\gamma^*$ system the cross section contains large logarithms of the form:

$$\sigma_{\gamma^*\gamma^*} \propto \exp\left(\lambda \ln \frac{W^2}{\sqrt{Q_1^2 Q_2^2}}\right) = \exp\left(\lambda\, Y\right).$$

The cross section $\sigma_{\gamma^*\gamma^*} \to$ hadrons is often considered as a golden BFKL signature. In LO a strong increase of $\sigma_{\gamma^*\gamma^*}$ at high Y is expected [39]. NLO corrections [40] however predict a much more suppressed cross section at high Y.

In a recent analysis [41], L3 has presented cross sections corrected for radiative effects using e^+e^- data at a CMS energy of $\sqrt{s_{ee}} = 189$–209 GeV. The cross section is measured in the kinematic region: $4 < Q_i^2 < 44$ GeV^2, $W > 5$ GeV, electron energy $E_i > 40$ GeV and the polar electron angle $30 < \theta_i < 66$ mrad and shown in Fig. 10. A fixed order calculation in LO as well as in NLO [42] is able to describe the data at low Y ($Y < 5$). However, at the largest Y ($5 < Y < 7$), the calculations are below the data by 3–4 standard deviations.

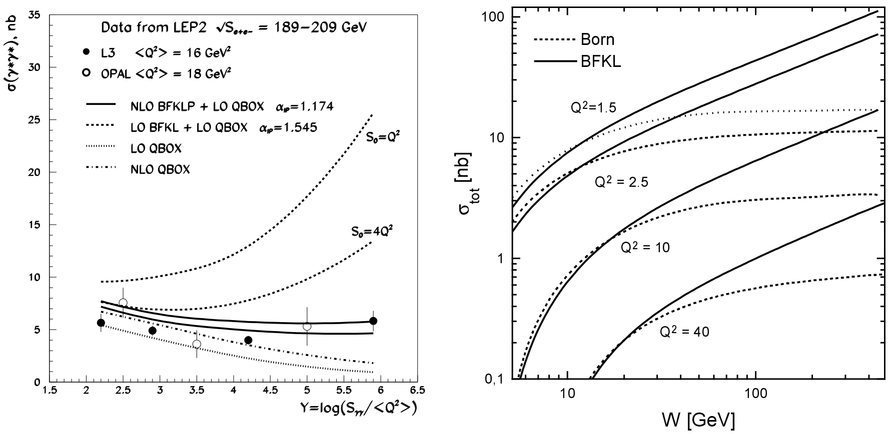


Fig. 10. (left) The energy dependence of the total cross section for highly virtual photon collisions from the L3 and OPAL collaborations, compared with LO and BLM fixed scale NLO BFKL calculations. (right) Predictions for the cross section for e^+e^- interactions as function of CMS energy.

OPAL [43] has analysed e^+e^- collisions at CMS energies of $\sqrt{s_{ee}} =$ 189–209 GeV. The measured cross section $\sigma_{\gamma^*\gamma^*}$ (for $E_i > 0.4\,E_b$, where E_b is the beam energy, $34 < \theta_i < 55$ mrad and $W > 5$ GeV) for an average $Q^2 = 17.9$ GeV^2 is shown in Fig. 10 as a function of Y. While being in general in agreement with the L3 data the evidence for BFKL effects is smaller in those data.

To establish small-x effects in $\gamma^*\gamma^*$ collisions unambiguously, an *ee* collider such as TESLA will be needed. In [44] it was shown that BFKL calculations which include the consistency constraint — and are in agreement with the LEP data — predict in the TESLA regime 1000 events/year for the Born process and 2600 events if BFKL is added to the signal. Predictions are shown in Fig. 10. Hence despite the suppression of the cross section due to subleading terms this effect will be still a large and perfectly measurable. It needs however electron taggers which go down to about 30 mrad to observe and measure the scattered electrons.

4.6. Vector meson production

Following the same idea as the for virtual photon scattering in e^+e^- other processes with point-like sources can be obtained via vector meson scattering in $\gamma\gamma$ or γp interactions by involving either heavy vector mesons or scattering at large $|t|$ values. H1 has studied the diffractive reaction $\gamma p \to J/\psi + Y$

in the context of BFKL [45]. Predictions have been made [46] and can be compared to data. The calculations are found to agree well with the data, as shown in Fig. 11.

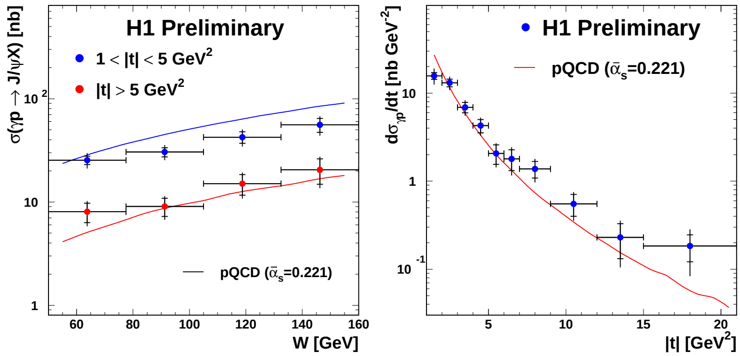


Fig. 11. The photon–proton differential cross section $\sigma(\gamma p \to J/\psi X)$ for J/ψ production with proton dissociation in the kinematic range $Q^2 < 1.0$ GeV^2 for two different intervals of t, compared with BFKL calculations (with $\overline{\alpha_s} = 0.221$). (right) The photon–proton differential cross-section $d\sigma/d|t|$ for $50 < W_{\gamma p} < 160$ GeV compared with the BFKL calculation.

In [47] a comparison is made of ZEUS vector meson data $\gamma p \to VM + X$ at large $|t|$. The $|t|$ dependence is compared with pQCD calculations (few parameter fits) namely BFKL predictions and a two-gluon calculation. It clearly shows that the BFKL calculation describes the data very well, while the two-gluon calculation fails, see Fig. 12.

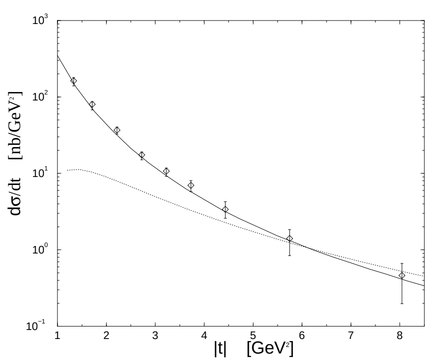


Fig. 12. Comparison of BFKL (solid line) and two-gluon (dotted line) calculations with ZEUS data for ρ meson production.

Vector meson production is also a good tool to study low-x in two-photon collisions. The diagram is presented in Fig. 13(left). At TESLA, with a CMS energy of 500 GeV, one expects a few hundred events of the type $\gamma\gamma \to J/\psi J/\psi$ per year. In [49] the cross sections in the presence of BFKL are predicted, see Fig. 13(right). Also $\rho\rho$ production at large t and Q^2 would be a good channel to study but there are no predictions yet.

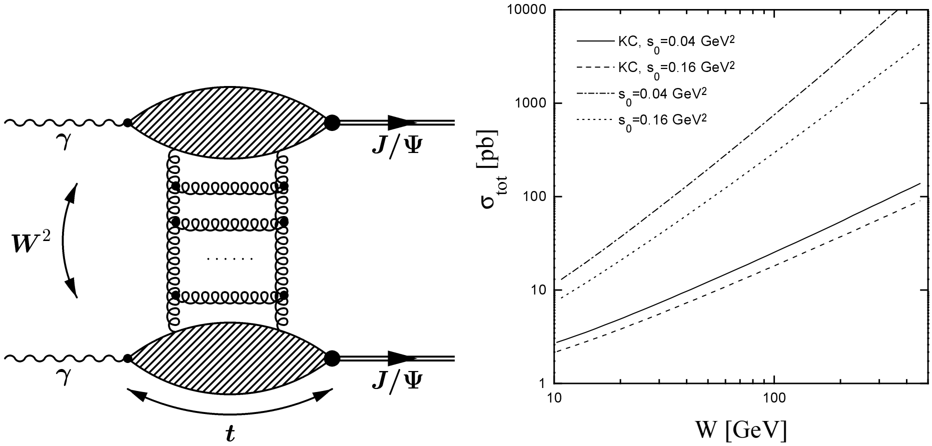


Fig. 13. (left) The pomeron exchange mechanism of the process $\gamma\gamma \to J/\psi J/\psi$. (right) Energy dependence of the cross section for the process $\gamma\gamma \to J/\psi J/\psi$. Upper curves are LO BFKL predictions while the two lower curves include the consistency constraint, both for two values of the cut-off.

Recently DELPHI [48] has reported the observation of the process $\gamma\gamma \to J/\psi + X$ and deduced a cross section $\sigma(J/\psi \to \mu\mu) = 25.2 \pm 10.2$ pb. There is however no chance for a 'BFKL' measurement with the LEP J/ψ data.

4.7. Review of the hadronic measurements

We arrive at the following table for BFKL/CCFM measurements at the various colliders (Table I).

Clearly in most cases the BFKL or CCFM calculations can describe the data (and at the same time pure DGLAP calculations fail), but often with different approximations or corrections to LO BFKL. Do we have a consistent picture and predictive power? Especially the 'consistency constraint' approach seems to be successful for most of the data shown above. However complete NLO BFKL calculations for these variables are eagerly awaited for.

TABLE I

Measurements confronted with BFKL/CCFM descriptions, and alternative successful explanations.

Process	BFKL/CCFM	Other/comments
Forward jets in (ep)	yes	Resolved photons?
Forward particles (ep)	maybe	Resolved photons?
Azimuth jet de-correlation (pp)	yes	HERWIG also ok
R(di-jets,$\sqrt{s}$) (pp)	?	?
Hard color singlet (pp)	yes (D0), ? (CDF)	CDF & D0 data?
Hard color singlet (ep)	yes	enhanced γ exch.
$\gamma^*\gamma^*$ scattering	yes	effect small at LEP
Vector mesons at large t	yes	

4.8. Future of low-x measurements

It is imperative that a consistent and systematic study of the available measurements of low-x and related phenomena is performed. To that end the good news is the start-up of the low-x Collaboration, which resulted out of a workshop organised at Lund around this topic. The collaboration is open and everybody interested in this subject should join it. Its first manifesto can be found on [50].

Not all options and new ideas to detect BFKL effects in the already collected or future data have been exhausted yet. The different ordering in $k_{\rm T}$ in the gluon ladder between DGLAP and BFKL will remain the important signal to hunt for. To this end it will most certainly be required to look not just at one but at the same time different objects (particles, jets) in the ladder, and to their correlations. Some of the possible new ideas, which should be explored further, include

- Particle or jet correlations.
- Number of jets with an $E_{\rm T}$ above a given threshold.
- Azimuthal correlations in the ladder.
- Long range particle correlations in the ladder.
- $R(E_{\rm T1}/E_{\rm T2})$ of jets for different CMS energy.
- $p_{\rm T}$ compensation effects in the ladder.

- Forward b-quark production in pp *e.g.* at the LHC.
- ... and hopefully more which we have not thought of yet ....

Furthermore it was observed in [51, 52] that some small x effects are enhanced in the scattering of polarised beams. A polarised ep collider would be the ideal tool here

An example of a new process is the forward production of b quarks at the LHC. The signal is given by the diagram $gg \to bbbb$ while the background is $gg \to bb$. Fig. 14 shows the preliminary calculation [8], where 2 b's are detected as function of a minimum rapidity value away from zero. The signal is calculated without including possible BFKL effects. The CMS detector will detect b-jets for a rapidity up to 2–2.5, the region where the $bbbb$ signal starts to dominate over the bb background. Hence BFKL studies in this channel can be performed at the LHC.

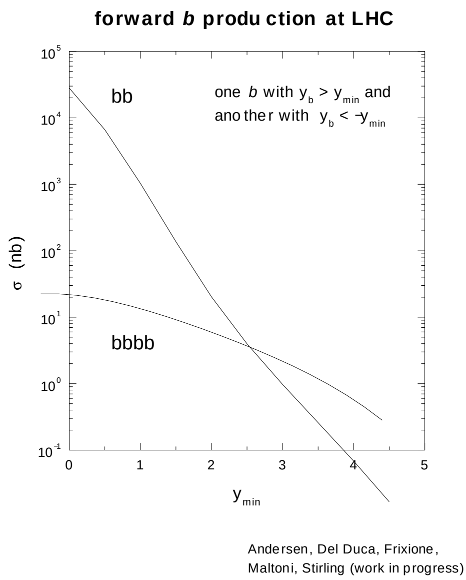


Fig. 14. The production of two b quarks as function of their distance from the CMS center, for the $bbbb$ signal and bb background, without including BFKL effects (which would enlarge the $bbbb$ signal).

The production of b quarks turns out to be actually quite a challenge for pQCD. While the production of charm quarks is generally well predicted by pQCD, b-quark production gives a headache! In all cases namely $pp \to bX, ep \to bX$ and $\gamma\gamma \to bX$ the data is well above the predictions, typically by a factor 2 to 3. Have these discrepancies a common origin? Are these relevant to the understanding of low-x? Indeed CASCADE is able to describe the Tevatron data and also reduces the discrepancy with the HERA data. It would be interesting to see how well it can predict the 2-photon data. Hence the 'b-problem' can certainly be relevant for low-x, but also other explanations are proposed to solve the issue, as discussed in [53].

5. Future colliders

In the previous section we already showed where future colliders can contribute to the specific BFKL measurements.

LHC will be the next new collider and will allow to study in more detail the low-x region in general [54–56]. Fig. 15 shows the kinematic plane in x and Q^2 which can be probed by the LHC. The plot safely stops at a $Q^2_{\rm min}$ of 100 GeV^2 which could be reached with jets and photons in the central detector. However new ideas are forming to have a better coverage in rapidity, perhaps as far down as $|\eta| = 7$–8 and to reach lower mass scales, such that x values down to 10^{-6}–10^{-7} can be probed [56]. This would be an ideal environment to study *e.g.* parton saturation effects.

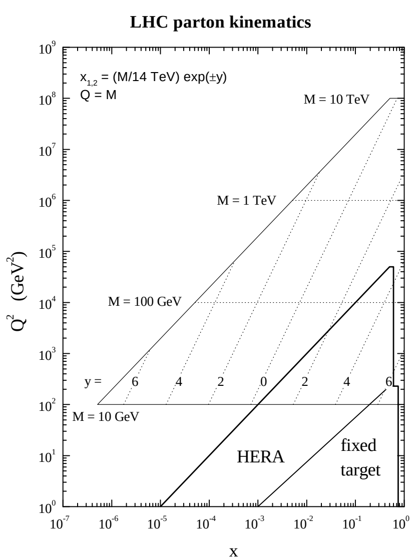


Fig. 15. The kinematic plane (x, Q^2) and the reach of the LHC, together with that of the existing data (HERA, fixed target). Lines of constant pseudo-rapidity are shown to indicate the kinematics of the produced objects in the LHC centre of mass frame.

Another intriguing option is to measure the total two-photon production cross section at higher energies. Presently the measurement made by L3 at LEP shows that this cross section rises much faster with $\gamma\gamma$ CMS energy than the hadron–hadron cross sections in this regime. A Donnachie–Landshoff type of fit of the soft pomeron slope gives a value of $\epsilon = 0.215 \pm 0.019$ compared to a value of $\epsilon = 0.09$–0.11 in measurements of hadron–hadron collisions. What is the origin of this large slope? If confirmed by data from other LEP experiments, this would be a very exciting observation. While we have data now only up to $\sqrt{s}_{\gamma\gamma} = 175$ GeV extending this measurement towards higher energies would be very important. Such an opportunity can be offered by the LHC [56, 57], if the low angle scattered protons can be detected, or better by a linear collider and its photon-collider option [58]. For the latter, possible measurements of the total cross section up to $\sqrt{s}_{\gamma\gamma} =$ 400–800 GeV could be made.

Finally I would like to mention instantons. The discovery and understanding of instantons will be of prime importance for the study of the non-perturbative QCD. So far these have been studied theoretically and experimentally only for ep collisions, but there is no reason not to consider e^+e^- and pp. While still no positive signal is reported, the H1 analysis starts to exclude regions of parameter phase space [59, 60]. I would encourage our theory friends to make predictions for signals to hunt for instantons at e^+e^- and pp colliders, especially for the LHC.

6. Summary

The small-x data tells us that:

- Wherever we look at purposely selected regions or processes we see deviations from the DGLAP type of predictions. Clearly something more is needed at small-x.

- BFKL calculations can often accommodate for these effects *e.g.* forward jets and particles, vector mesons, Tevatron data, and in $\gamma^*\gamma^*$ scattering.

- However often agreement with data is only reached when using special choices using phenomenological approaches to the subleading terms, fixed α_s, *etc.* A complete consistent picture is still somewhat lacking. Do we really have predictive power with these calculations for different and new processes?

- What is clearly needed at the end are complete NLO calculations for experimental variables, some of these calculations are in progress now.

- The CASCADE/CCFM Monte Carlo works well. Is this the way to go to understand low-x? There is presently a debate between the LDC and CASCADE groups on how to include non-singular terms, which affects the predictions.

- The LHC will significantly — and hopefully decisively — open the phase space. Also at linear colliders a number of key processes can be studied.

In other words "To BFKL or not to BFKL": I would say the jury is still out.

There are many other issues which clearly deserve attention in the grand study of *ee*, *ep* and *pp* collisions:

- What is wrong with the *b*-quark predictions in all hadronic collisions?

- Does the $\gamma\gamma$ cross section rises faster than hadron–hadron cross section?

- Instantons: there is still an important discovery opportunity for this phenomenon, and it should be fully exploited also at *ee* and *pp* colliders.

I would like to thank the organisers for inviting and supporting me to this successful and inspiring conference. I would also like to thank my colleagues for the discussion and information to this report, in particular: J. Andersen, J. Bartels, T. Carli, V. Del Ducca, J. Forshaw, H. Jung, J. Kwiecinski, A.D. Martin, G. Poludniowski, J. Stirling and R. Thorne.

REFERENCES

[1] V.N. Gribov, L.N. Lipatov, *Sov. J. Nucl. Phys.* **15**, 438 (1972); G. Altarelli, G. Parisi, *Nucl. Phys.* **B126**, 298 (1977); Yu.L. Dokshitzer, *Sov. Phys. JETP* **46**, 641 (1977).

[2] E.A. Kuraev, L.N. Lipatov, V.S. Fadin, *Sov. Phys. JETP* **44**, 443 (1976); *Sov. Phys. JETP* **45**, 199 (1977); I.I. Balitsky, L.N. Lipatov, *Sov. J. Nucl. Phys.* **28**, 822 (1978).

[3] M. Ciafaloni, *Nucl. Phys.* **B 296**, 49 (1988); S. Catani, F. Fiorani, G. Marchesini, *Phys. Lett.* **B234**, 339 (1990); *Nucl. Phys.* **B336**, 18 (1990); G. Marchesini, *Nucl. Phys.* **B445**, 49 (1995).

[4] V.S. Fadin, L.N. Lipatov, *Phys. Lett.* **B429**, 127 (1998); see also: G. Camici, M. Ciafaloni, *Phys. Lett.* **B412**, 396 (1997) (Erratum, **B417**, 390 (1997); G. Camici, M. Ciafaloni, *Phys. Lett.* **B430**, 349 (1998).

[5] S.J. Brodsky *et al.*, *JETP Lett.* **70**, (1999); J. Blümlein, A. Vogt, *Phys. Rev.* **D57**, 1 (1998); **D58**, 014020 (1998); C. Schmidt, *Phys. Rev.* **D60**, 074003 (1999); J.R. Forshaw, D.A. Ross, A. Sabio Vera, *Phys. Lett.* **B455**, 273 (1999); J. Kwiecinski, A.D. Martin, P. Sutton, *Z. Phys.* **C71**, 585 (1996).

[6] J. Bartels *et al.*, `hep-ph/028130`; J. Bartels, D. Colferai, G.P. Vacca, `hep-ph/0206290`; *Eur. Phys. J.* **C24** , 83 (2002).

[7] L. Orr, J. Stirling, `hep-ph/0102226`.

[8] J. Andersen, *Acta Phys. Pol.* **B33**, 3001 (2002).

[9] H. Jung, `hep-ph/0109102`.

[10] G. Marchesini, B. Webber, *Nucl. Phys.* **B349**, 617 (1991); *Nucl. Phys.* **B386**, 215 (1992).

[11] B. Anderson *et al.*, *Nucl. Phys.* **B467** (1996) 443; *Z. Phys.* **C71**, 613 (1996); *Nucl. Phys.* **B467**, 443 (1996); H. Kharrazhia, L. Lönnblad, *J. High Energy Phys.* **03**, 6 (1998).

[12] S. Forte, R. Ball, `hep-ph/0109235`.

[13] R. Klanner, *Acta Phys. Pol.* **B33**, (2002), next issue.

[14] J. Kwiecinski, A.D. Martin, A.M. Stasto, *Phys. Rev.* **D56**, 3991 (1997).

[15] R. Thorne, *Phys. Lett.* **B474**, 372 (2000); *Phys. Rev.* **D64**, 074005 (2001); R. Thorne, talk given at the Weimar QCD Network Meeting, 2001.

[16] A.J. Askew, D. Graudenz, J. Kwiecinski, A.D. Martin, *Phys. Lett.* **B338**, 92 (1994).

[17] A. Szczurek, N. Nikolaev, W.Schafer, J. Speth, *Phys. Lett.* **B500**, 254 (2001).

[18] A.H. Mueller, *Nucl. Phys. (Proc. Suppl.)* **B18** C125 (1990); *J. Phys. G* **17**, 1443 (1991); J. Bartels, A. de Roeck, M. Loewe, *Z. Phys.* **C54**, 635 (1992); J. Kwiecinski, A.D. Martin, P.J. Sutton, *Phys. Rev.* **D46**, 921 (1992).

[19] H1 Collaboration, C. Adloff *et al.*, *Nucl. Phys.* **B538**, 3 (1999).

[20] ZEUS Collaboration, J. Breitweg *et al.*, *Eur. Phys. J.* **C6**, 239 (1999).

[21] J. Kwiecinski, A. Martin, J. Outhwaite, *Eur. Phys. J.* **C9**, 611 (1999).

[22] H. Jung, L. Jönsson, H. Küster, *Eur. Phys. J.* **C9**, 383 (1999).

[23] ZEUS Collaboration, J. Breitweg, *Phys. Lett.* **B474**, 223 (2000).

[24] H. Jung, L. Lonnblad, `hep-ph/0105122`.

[25] H1 Collaboration, C. Adloff *et al.*, *Phys. Lett.* **B462**, 440 (1999).

[26] J. Kwiecinski, A.D. Martin, J. Outhwaite, *Eur. Phys. J.* **C9**, 611 (1999).

[27] L. Goerlich, *Acta Phys. Pol.* **B33**, 3287 (2002).

[28] A.H. Mueller, H. Navelet, *Nucl. Phys.* **B282**, 727 (1987).

[29] J. Kwiecinski, A.D. Martin, L. Motyka, J. Outhwaite, *Phys. Lett.* **B514**, 355 (2001).

[30] J.R. Andersen *et al.*, *J. High Energy Phys.* **02**, 007 (2001).

[31] L.H. Orr, W.J. Stirling, *Phys. Lett.* **B429**, 127 (1998).

[32] D0 Collaboration, *Phys. Rev. Lett.* **84**, 5722 (2000).

[33] S. Frixione, G. Ridolfi, *Nucl. Phys.* **B507**, 315 (1997); M. Klasen, G. Kramer, *Phys. Lett.* **B366** (1996) 385; H1 Collaboration, *Eur. Phys. J.* **C13**, 415 (2000); T. Carli, `hep-ph/9906541`.

[34] B. Cox, J.R. Forshaw, L. Lonnblad, `hep-ph/9912489`.

[35] F. Abe *et al.*, CDF Collaboration, *Phys. Rev. Lett.* **81**, 5278(1998).

[36] B. Abbot *et al.*, D0 Collaboration, *Phys. Lett.* **B81**, 189 (1998).

[37] R. Enberg, G. Ingelman, L. Motyka, *Phys. Lett.* **B542**, 273 (2002).

[38] H1 Collaboration, C. Adloff *et al.*, *Eur. Phys. J.* **C24**, 517 (2002).

[39] J. Bartels, A. de Roeck, H. Lotter, *Phys. Lett.* **B389**, 742 (1996); J. Bartels, C. Ewerz, R. Staritzbichler, *Phys. Lett.* **B492**, 56 (2000); S.J. Brodsky F. Hautmann, D.E. Soper *Phys. Rev.* **D56**, 6979 (1997).

[40] S. Brodsky, V. Fadin, V. Kim, G. Pivovarov, `hep-ph/0207297.`

[41] L3 Collaboration, P. Achard *et al.*, *Phys. Lett.* **531**, 39 (2002).

[42] M. Cacciari *et al.*, *J. High Energy Phys.* **0102**, 029 (2001).

[43] OPAL Collaboration, G. Abbiendi *et al.*, *Eur. Phys. J.* **C24**, 17 (2002).

[44] J. Kwiecinski, L. Motyka, *Phys. Lett.* **B462**, 203 (1999).

[45] D. Brown, *Acta Phys. Pol.* **B33**, (2002), next issue.

[46] J. Bartels *et al.*, *Phys. Lett.* **B375**, 301 (1996).

[47] J. Forshaw, G. Poludniowski, `hep-ph/0107068` and private communication.

[48] M. Chapkine, talk at Photon2001, Ascona.

[49] J. Kwiecinski, L. Motyka, A. De Roeck, `hep-ph/0001180`.

[50] B. Anderson *et al.*, `hep-ph/0204115.`

[51] J. Bartels, M. Ryskin, `hep-ph/0111306.`

[52] J. Kwiecinski, B. Ziaja, *Phys. Rev.* **D60**, 054004 (1999).

[53] G. Salam, *Acta Phys. Pol.* **B33**, 2791 (2002).

[54] A.D. Matin, R.G. Roberts, W.J. Stirling, R.S. Thorne, *Eur. Phys. J.* **C14**, 133 (2000).

[55] FELIX: a full acceptance detector at the LHC: Letter of Intent: *J. Phys. G* **28**, R117 (2002); http://felix.web.cern.ch/FELIX.

[56] A. De Roeck, *Acta Phys. Pol.* **B33**, (2002), next issue.

[57] K. Piotrzkowski, `hep-ex/0201027.`

[58] G. Pancheri, R. Godbole, A. De Roeck, LC-TH-2001-030, and TESLA TDR.

[59] B. Koblitz, *Acta Phys. Pol.* **B33**, 3263 (2002); H1 collaboration, C. Adloff *et al.*, `hep-ex/0205078`.

[60] A. Utermann, *Acta Phys. Pol.* **B33** (2002), next issue; F. Schrempp, A. Utermann, `hep-ph/0207052.`