QCD TESTS THROUGH HADRONIC FINAL-STATE MEASUREMENTS*

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Modern-day 'testing' of (perturbative) QCD is as much about pushing the boundaries of its applicability as about the verification that QCD is the correct theory of hadronic physics. This talk gives a brief discussion of a small selection of topics: factorisation and jets in diffraction, power corrections and event shapes, the apparent excess of b-production in a variety of experiments, and the matching of event generators and NLO calculations.

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1. Introduction

The testing of QCD is a subject that many would consider to be well into maturity. The simplest test is perhaps that α_s values measured in different processes and at different scales should all be consistent. It suffices to take a look at compilations by the PDG [2] or Bethke [3] to see that this condition is satisfied for a range of observables, to within the current theoretical and experimental precision, namely a few percent. There exist many other potentially more discriminatory tests, examples explicit measurements of the QCD colour factors [4] or the running of the b-quark mass [5] — and there too one finds a systematic and excellent agreement with the QCD predictions. A significant amount of the data comes from HERA experiments, and to illustrate this, figure 1 shows a compilation of a subset of the results on α_s, as compiled by ZEUS [1].

In the space available however, it would be impossible to give a critical and detailed discussion of the range of different observables that are used to verify that QCD is 'correct'. Rather let us start from the premise that, in light of the large body of data supporting it, QCD *is* the right theory of hadronic physics, and consider what then is meant by 'testing QCD'.

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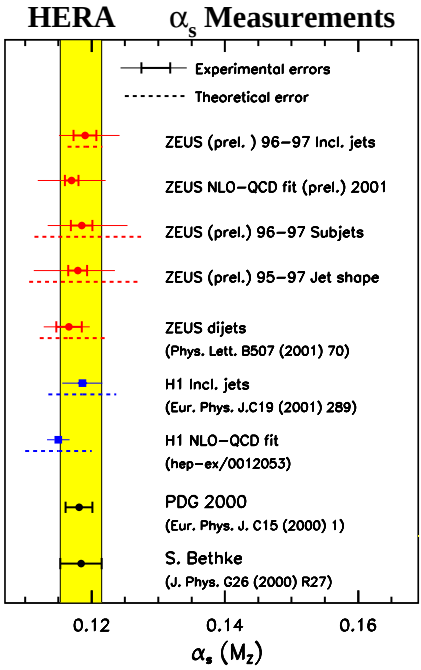


Fig. 1. A compilation of HERA α_s measurements, taken from [1].

One large body of activity is centred around constraining QCD. This includes such diverse activities as measuring fundamental (for the time being) unknowns such as the strong coupling and the quark masses; measuring quantities such as structure functions and fragmentation functions, which though formally predictable by the theory are beyond the scope of the tools currently at our disposal (perturbation theory, lattice methods); and the understanding, improvement and verification of the accuracy of QCD predictions, through NNLO calculations, resummations and projects such as the matching of fixed-order calculations with event-generators. One of the major purposes of such work is to provide a reliable 'reference' for the inputs and backgrounds in searches for new physics.

A complementary approach to testing QCD is more about exploring the less well understood aspects of the theory, for example trying to develop an understanding of non-perturbative phenomena such as hadronisation and diffraction, or the separation of perturbative and non-perturbative aspects of problems such as heavy-quark decays; pushing the theory to new limits as is done at small-x and in studies of saturation; or even the search for and study of qualitatively new phenomena and phases of QCD, be they within immediate reach of experiments (the quark–gluon plasma, instantons) or not (colour superconductors)!

Of course, these two branches of activity are far from being completely separated: it would in many cases be impossible to study the less well understood aspects of QCD without the solid knowledge that we have of its more 'traditional' aspects — and it is the exploration of novel aspects of QCD that will provide the 'references' of the future.

The scope of this talk is restricted to tests involving final states. Final states tend to be highly discriminatory as well as complementary to more inclusive measurements. We shall consider two examples where our understanding of QCD has seen vast progress over the past years, taking us from a purely 'exploratory' stage almost to the 'reference' stage: the question of jets and factorisation in diffraction (Section 2); and that of hadronisation corrections in event shapes (Section 3). We will then consider two questions that are more directly related to the 'reference' stage: the topical issue of the excess of b-quark production seen in a range of experiments (Section 4); and then the problem of providing Monte Carlo event generators that are correct to NLO accuracy, which while currently only in its infancy is a subject whose practical importance warrants an awareness of progress and pitfalls.

For reasons of lack of space, many active and interesting areas will not be covered in this talk, among them small-x physics, progress in next-to-next-to-leading order calculations, questions related to prompt photons, the topic of generalised parton distributions and deeply-virtual Compton scattering, hints (or not) of instantons, a range of measurements involving polarisation and so on. Many of these subjects are widely discussed in other contributions to both the plenary and parallel sessions of this conference, to which the reader is referred for more details.

2. Jets in diffraction and factorisation

Factorisation, for problems explicitly involving initial or final state hadrons, is the statement that to leading twist, predictions for observables can be written as a convolution of one or more non-perturbative but universal functions (typically structure or fragmentation functions) with some perturbatively calculable coefficient function.

While factorisation has long been established in inclusive processes [7] it has been realised in the past few years [8] that it should also hold in more exclusive cases — in particular for diffraction, in terms of diffractive parton distributions $f_{a/p}^{\mathrm{diff}}(x, x_{\mathbb{P}}, \mu^2, t)$, which can be interpreted loosely as being related to the probability of finding a parton a at scale μ^2 with longitudinal momentum fraction x, inside a diffractively scattered proton p, which in the scattering exchanges a squared momentum t and loses a longitudinal momentum fraction $x_{\mathbb{P}}$. These kinematic variables are illustrated in figure 2.

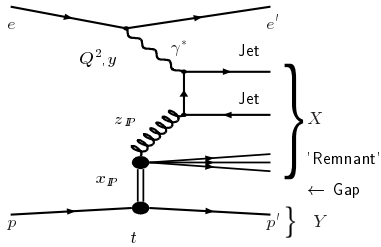


Fig. 2. Illustration of diffractive kinematics. Figure taken from [6].

The dependence of the diffractive parton distributions on so many variables means that without a large kinematical range (separately in x, $x_{I\!P}$ and Q^2, while perhaps integrating over t) it is a priori difficult to thoroughly test diffractive factorisation. An interesting simplifying assumption is that of Regge factorisation, where one writes [9]

$$f_{a/p}^{\mathrm{diff}}(x, x_{I\!P}, \mu^2, t) = |\beta_p(t)|^2 x_{I\!P}^{-2\alpha(t)} f_{a/I\!P}(x/x_{I\!P}, \mu^2, t) \qquad (1)$$

the interpretation of diffraction being due to (uncut) pomeron exchange (first two factors), with the virtual photon probing the parton distribution of the pomeron (last factor).

As yet no formal justification exists for this extra Regge factorisation. Furthermore given that diffraction is arguably related to saturation and high parton densities (assuming the AGK cutting rules [10]) one could even question the validity of arguments for general diffractive factorisation, which rely on parton densities being low (as does normal inclusive factorisation).

The experimental study of factorisation in diffraction relied until recently exclusively on inclusive F_2^d measurements. This was somewhat unsatisfactory because of the wide range of alternative models able to reproduce the data and even the existence of significantly different forms for the $f_{a/I\!P}(x/x_{I\!P}, \mu^2, t)$ which gave a satisfactory description of the data within the Regge factorisation picture. However diffractive factorisation allows one to predict not only inclusive cross sections but also jet cross sections. Results in the Regge factorisation framework are compared to data in figure 3 (taken from [11]), showing remarkable agreement between the data and the predictions (based on one of the pomeron PDF fits obtained from F_2^d). On the other hand, when one considers certain other models that work well for F_2^d the disagreement is dramatic, as for example is shown with the soft colour neutralisation models [12, 13] in figure 4. Despite this apparently strong confirmation of diffractive factorisation, a word of warning is perhaps needed. Firstly there exist other models which have not been ruled out (for

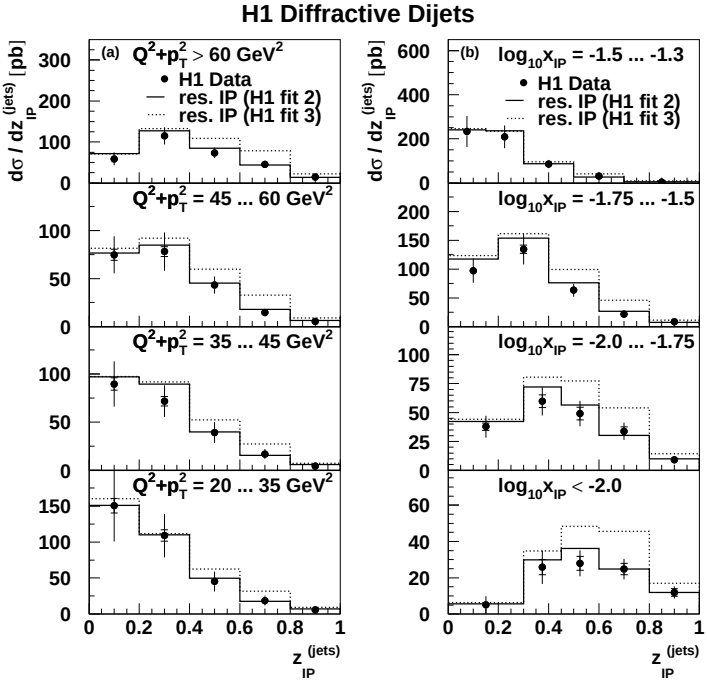


Fig. 3. Comparisons of H1 diffractive dijet cross sections with predictions obtained using the assumption of Regge factorisation [11].

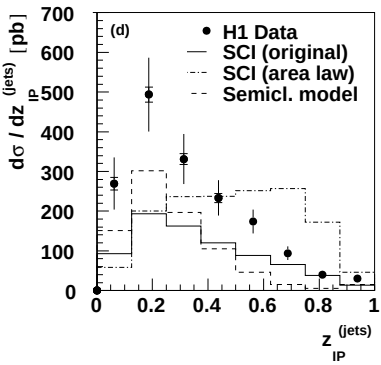


Fig. 4. A comparison between diffractive dijet data and results from the soft-colour interaction (SCI) [12] and semiclassical [13] models. Figure adapted from [11].

example the dipole model [14]). In these cases it would be of interest to establish whether these models can be expressed in a way which satisfies some effective kind of factorisation.

Other important provisos are that a diffractive PDF fit based on more recent F_2^d data has a lower gluon distribution and so leads to diffractive dijet predictions which are a bit lower than the data, though still compatible to within experimental and theoretical uncertainties [15]. And secondly that the predictions themselves are based on the Rapgap event generator [16] which incorporates only leading order dijet production. It would be of interest (and assuming that the results depend little on the treatment of the 'pomeron remnant,' technically not at all difficult) to calculate diffractive dijet production to NLO with programs such as Disent [17] or Disaster++ [18], using event generators only for the modelling of hadronisation correction, as is done in inclusive jet studies.

3. Hadronisation

Another subject that has seen considerable experimental and theoretical progress recent years is that of hadronisation. Even at the relatively high scattering energies involved at LEP and the Tevatron, for many final state observables non-perturbative contributions associated with hadronisation are of the same order of magnitude as next-to-leading order perturbative contributions and cannot be neglected. With the advent of NNLO calculations in the foreseeable future the need for a good understanding of hadronisation becomes ever more important.

Until a few years ago, the only way of estimating hadronisation corrections in final-state measurements was by comparing the parton and hadron levels of Monte Carlo event generators. Such a procedure suffers from a number of drawbacks. In particular the separation between perturbative and non-perturbative contributions is ill-defined: for example event generators adopt a prescription for the parton level based on a cutoff; on the other hand, in fixed-order perturbative calculations no cutoff is present, and the perturbative integrals are naively extended into the non-perturbative region — furthermore the 'illegally-perturbative' contribution associated with this region differs order by order (and depends also on the renormalisation scale).

Additionally, hadronisation corrections obtained from event generators suffer from a lack of transparency: the hadronisation models are generally quite sophisticated, involving many parameters, and the relation between these parameters and the hadronisation corrections is rarely straightforward.

In the mid 1990's a number of groups started examining approaches for estimating hadronisation corrections based on the perturbative estimates of observables' sensitivity to the infrared. This leads to predictions of non-perturbative corrections which are suppressed by powers of $1/Q$ relative to the perturbative contribution (for a review see [19]). One of the most successful applications of these ideas has been to event shapes, for which (in

the formalism of Dokshitzer and Webber [20])

$$\langle \mathcal{V}_{\rm NP} \rangle = \langle \mathcal{V}_{\rm PT} \rangle + c_{\mathcal{V}} \mathcal{P} \,, \qquad \mathcal{P} \equiv \frac{2C_F}{\pi} \frac{\mu_I}{Q} \left\{ \alpha_0(\mu_I) - \alpha_{\rm s}(Q) - \mathcal{O}\left(\alpha_{\rm s}^2\right) \right\} \,, \quad (2)$$

where $c_{\mathcal{V}}$ is a perturbatively calculable observable-dependent coefficient and $\mathcal{P}$ governs the size of the power correction. The quantity $\alpha_0(\mu_I)$, which can be interpreted as the mean value of an infrared finite effective coupling in the infrared (up to an infrared matching scale μ_I, conventionally chosen to be 2 GeV), is hypothesised to be *universal*. The terms in powers of $\alpha_{\rm s}$ are subtractions of pieces already included in the perturbative prediction for the observable.

It is interesting to see the progress that has been made in our understanding of these effects. The first predictions for the $c_{\mathcal{V}}$ coefficients were based on calculations involving the Born configuration plus a single 'massive' (virtual) gluon. Fitting α_0 and $\alpha_{\rm s}$ to data for mean values of e^+e^- event-shapes, using the original predictions for the $c_{\mathcal{V}}$, leads to the results shown in figure 5.

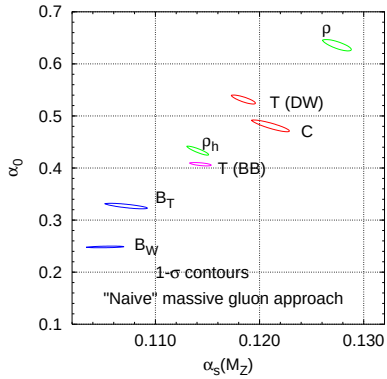


Fig. 5. Results of fits to e^+e^- mean event shapes using original, 'naive' calculations for $c_{\mathcal{V}}$.

At the time of the original predictions, however, much of the data used to generate figure 5 was not yet in existence (which is perhaps fortunate — had figure 5 been around in 1995, the field of $1/Q$ hadronisation corrections might not have made it past early childhood). Rather, various theoretical objections (*e.g.* [21]) and the gradual appearance of new data, especially for the broadenings, forced people to refine their ideas.

Among the developments was the realisation that to control the normalisation of the $c_{\mathcal{V}}$ it is necessary to take into account the decay of the massive, virtual, gluon (the reason for the two thrust results in figure 5 was the existence of two different conventions for dealing with the undecayed

massive gluon) [22]. It was also realised that it is insufficient to consider a lone 'non-perturbative' gluon, but rather that such a gluon must be taken in the context of the full structure of soft and collinear perturbative gluon radiation [23]. Another discovery was that hadron-masses can be associated with universality breaking $1/Q$ power corrections in certain definitions of observables [24] and when testing the universality picture all observables should be measured in an appropriate common 'hadron-mass' scheme.

Results incorporating these theoretical developments are shown in figure 6. As well as e^+e^- mean event shapes we also include recent results using resummed DIS event shapes [25], fitted to H1 distributions [26]. The agreement between observables, even in different processes, is remarkable, especially compared to figure 5, and a strong confirmation of the universality hypothesis[1].

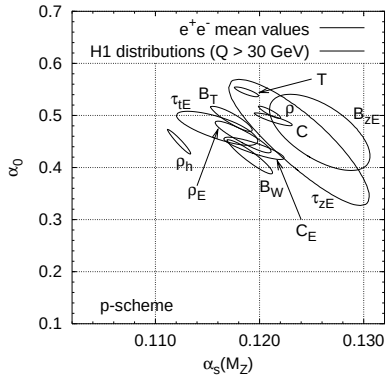


Fig. 6. Results of fits to e^+e^- mean event shapes and DIS distributions with state-of-the-art account of NP contributions.

This is not to say that the field has reached maturity. In the above fits the approximation has been made that non-perturbative corrections just shift the perturbative distribution [29], however there exists a considerable amount of recent work which examines the problem with the more sophisticated 'shape-functions' approach [30] in particular in the context of the Dressed Gluon Exponentiation approximation [31]. An important point also is that all the detailed experimental tests so far are for 2-jet event shapes, where there exists a solid theoretical justification based on the Feynman tube model [32], *i.e.* longitudinal boost invariance. It will be of interest to see what happens in multi-jet tests of $1/Q$ hadronisation corrections where one introduces both non-trivial geometry and the presence of gluons in the

[1] It should be noted that results for certain e^+e^- distributions [27] and DIS means [26, 28] are not quite as consistent. Though this remains to be understood, it may in part be associated with the particular fit ranges that are used.

Born configuration [33]. Finally we note the provocative analysis by the Delphi collaboration [34] where they show that a renormalisation-group based fit prefers an absence of hadronisation corrections, at least for mean values of event shapes, as well as leading to highly consistent values for α_s across a range of event-shapes.

4. Heavy quark (b) production

For light quarks (and gluon) it is impossible to make purely perturbative predictions of their multiplicity or of their fragmentation functions because of soft and collinear divergences. For heavy quarks however, these divergences are cut off by the quark mass itself, opening the way to a range of perturbative predictions and corresponding tests of QCD.

It is therefore particularly embarrassing that there should be a significant discrepancy in most experiments (but not all, *e.g.* [37]) where the QCD bottom production cross section has been measured. The situation is shown in figure 7 for Tevatron, HERA and LEP results, illustrating the systematic excess of a factor of three between measurements and NLO calculations. To add to the puzzle, the agreement for charm production (which if anything should be worse described because of the smaller mass) is considerably better across a range of experiments (see *e.g.* the lower-right plot of figure 7).

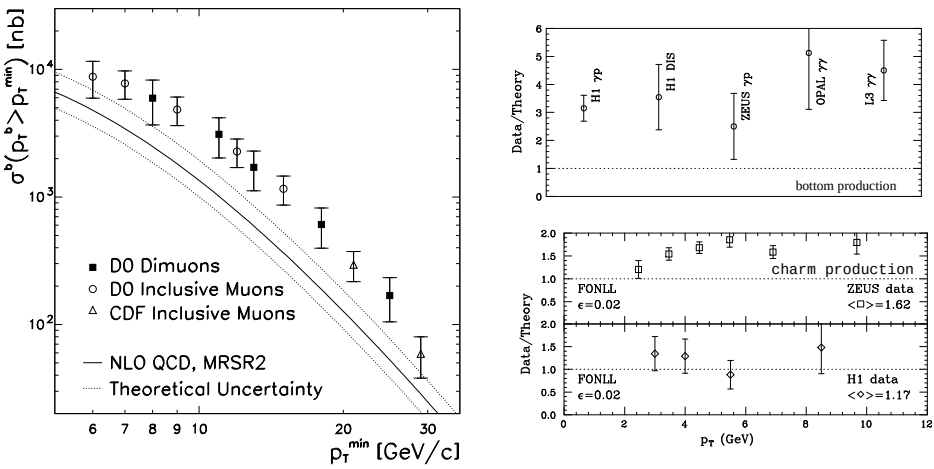


Fig. 7. Left: b-quark p_t distribution at the Tevatron [35]; upper right: summary of open b cross sections in γp, DIS and $\gamma\gamma$ collisions, normalised to theoretical expectations (figure taken from [36]); lower right: ratio of experiment to theory for the charm p_t distribution at HERA (taken from [36]).

Aside from the intrinsic interest of having a good understanding of b-production in QCD, one should keep in mind that b-quarks are widely relied upon as signals of Higgs production and in searches for physics beyond the standard model, so one needs to have confidence in predictions of the QCD background.

We shall discuss a couple of explanations that have been proposed for the excess at the Tevatron (the excesses in other experiments are more recent and have yet to be addressed in the same detail). Indeed, one hypothesis is precisely that we are seeing a signal of light(ish) gluino production. Another is that bottom fragmentation effects have been incorrectly accounted for. A third explanation, discussed in detail in another of the opening plenary talks [38] is associated with unintegrated k_t distributions and small-x resummations.

4.1. The SUSY hypothesis

In [39] it has been argued that a possible explanation of the Tevatron b-quark excess is the production of a pair of light gluinos with a mass of order 14 GeV which then decay to sbottoms (~ 3.5 GeV) and bottoms, as in figure 8. The mixing angles are chosen such that the sbottom decouples from the Z at LEP, accounting for its non-observation there.

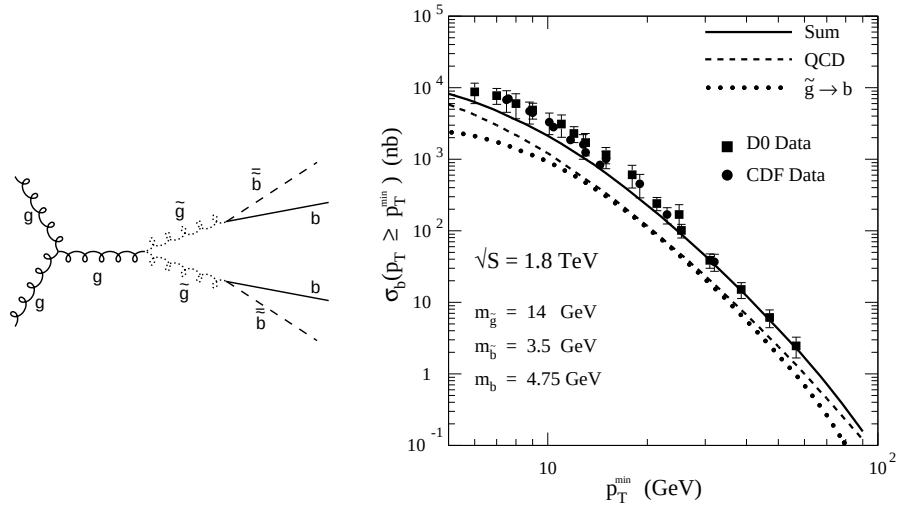


Fig. 8. Above: a potential SUSY contribution to the b-quark excess. Below: the resulting modification of the bottom p_t distribution at the Tevatron [39].

At moderate and larger p_t, the contribution from this process is about as large as that from NLO QCD and so it brings the overall production rate into agreement with the data.

There are a number of other consequences of such a scenario: one is the production of like-sign b quarks (as in the Feynman graph of figure 8), which could in principle be observed at the Tevatron, although it would need to be disentangled from B_0-$\bar{B}_0$ mixing. Another is that the running of α_s would be modified significantly above the gluino mass, leading to an increase of about 0.007 in the running to M_Z of low Q measurements of α_s. This seems to be neither favoured nor totally excluded by current α_s measurements.

Though they have not provided a detailed analysis, the authors of [39] also consider the implications for HERA. There it seems that the enhancement of the b-production rates is too small to explain the data (because of the suppression due to the gluino mass).

4.2. The fragmentation explanation

In any situation where one sees a significant discrepancy from QCD expectations it is worth reexamining the elements that have gone into the theoretical calculation. Various groups have considered issues related to b fragmentation and found significant effects, which could be of relevance to the Tevatron results (see for example [40]). However, a recent article by Cacciari and Nason [41] is particularly interesting in that it makes use of the full range of available theoretical tools to carry out a unified analysis all the way from the e^+e^- data, used to constrain the b-quark fragmentation function, through to expectations for the Tevatron. It raises a number of important points along the way[2].

To be able to follow their analysis it is worth recalling how one calculates expectations for processes involving heavy quarks. The cross section for producing a b-quark with a given p_t (or even integrated over all p_t) is finite, unlike that for a light quark. This is because the quark mass regulates (cuts-off) the infrared collinear and soft divergences which lead to infinities for massless quark production. But infrared finiteness does not mean infrared insensitivity and to obtain a B-meson p_t distribution from a b-quark $\hat{p}_t$ distribution, one needs to convolute with a fragmentation function,

$$\underbrace{\frac{d\sigma}{dp_t}}_{\text{measured, } e.g.\ \ B_0} = \int d\hat{p}_t dz \quad \underbrace{\frac{d\sigma}{d\hat{p}_t}}_{\text{PT QCD, } b \text{ quark}} \quad \underbrace{D(z)}_{\text{fragmentation}} \quad \delta(p_t - z\hat{p}_t)\,. \qquad (3)$$

The details of the infrared finiteness of the b-quark production are such that $\langle zD(z)\rangle$ is $1-\mathcal{O}(\Lambda/m_b)$, where the origin of the Λ/m_b piece is closely related to that of the Λ/Q power corrections discussed in the previous section [42].

[2] The reader is referred to their article for full references to the 'ingredients' used at different stages of the analysis.

There are various well-known points to bear in mind about fragmentation functions. Firstly, in close analogy to the hadronisation corrections discussed earlier (and of course structure functions), the exact form for the fragmentation function will depend on the perturbative order at which we define Eq. (3). Secondly, while for $p_t \sim m_b$ we are free to use fixed order (FO) perturbative predictions, for $p_t \gg m_b$ there are large logarithmically enhanced terms, which need to be resummed. The technology for doing this currently exists to next-to-leading logarithmic (NLL) order. In the intermediate region $p_t \gtrsim m_b$ the two approaches can be combined to give FONLL predictions [43, 44] (strictly this can be used even for $p_t \gg m_b$).

Having established these points we can consider what has been done by Cacciari and Nason [41]. Firstly they discuss moments of the fragmentation function $\langle z^{N-1} D(z) \rangle$. This is because for a steeply falling perturbative $\hat{p}_t$ distribution in Eq. (3), $\frac{d\sigma}{d\hat{p}_t} \sim 1/\hat{p}_t^N$, after integrating out the δ-function to give $\hat{p}_t = p_t/z$, one obtains the result

$$\left.\frac{d\sigma}{dp_t}\right|_{B-\text{meson}} = \langle z^{N-1} D(z) \rangle \left.\frac{d\sigma}{d\hat{p}_t}\right|_{b-\text{quark}}, \tag{4}$$

where for the Tevatron $N \simeq 5$.

The cleanest place to constrain b fragmentation is in e^+e^- collisions. Figure 9 shows moments of the momentum fraction (with respect to $Q/2$) carried by B-mesons as measured by Aleph [45]. The (magenta) dot-dashed curve shows the purely perturbative NLL prediction, which is clearly above the data. The dashed curve shows what happens when one includes the convolution with an $\varepsilon = 0.006$ Peterson fragmentation function [46]. Why this particular function? Simply because it is the one included in certain Monte Carlo event generators and used widely by experimental collaborations that have compared measured and theoretical p_t distributions. The data point for the $N = 5$ moment is 50% higher than the theoretical expectation with this fragmentation function.

Of course we don't expect agreement: the $\varepsilon = 0.006$ Peterson is widely used in Monte Carlos where one has only leading-logs. But we are interested in NLL calculations and the fragmentation function needs to be refitted. The authors of [41] take the functional form of [47], fitted to the $N = 2$ moment, to give the solid curve.

The next step in the Cacciari and Nason analysis should simply have been to take the FONLL calculation of bottom production at the Tevatron [43], convolute with their new fragmentation function and then compare to data. This however turns out to be impossible for most of the data, because it has already been deconvoluted to 'parton-level' (in some cases with the $\varepsilon = 0.006$ Peterson fragmentation function). So they are only able to compare with the

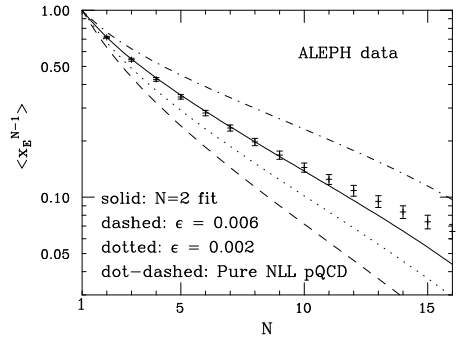


Fig. 9. Moments of the momentum fraction carried by B-mesons in e^+e^-, compared to NLL predictions with and without fragmentation functions [41].

recent CDF data [48] for B-mesons, shown in the left-hand plot of figure 10. The dashed curve is the central result, while the solid ones are those obtained when varying the factorisation and renormalisation scales by a factor of two[3]. The dotted curve shows the results that would have been obtained with the Peterson fragmentation function. Predictions with FO (generally used in previous comparisons) rather than FONLL would have been 20% lower still.

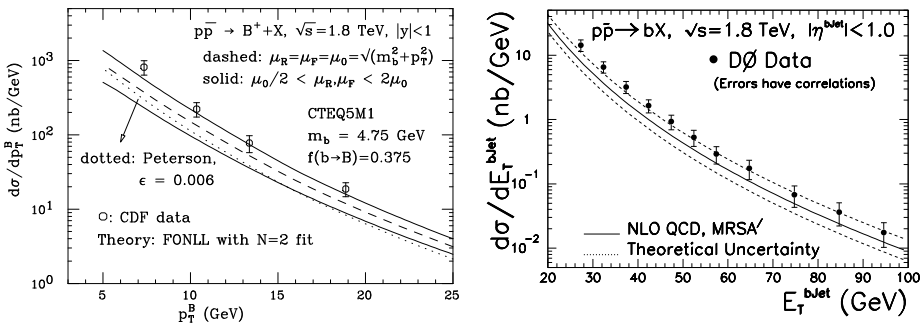


Fig. 10. Left: result for B-meson production [48] compared to the FONLL prediction with the '$N = 2$' fragmentation function [41]. Right: results for b-jet production [50] compared to the NLO predictions [51].

Another interesting approach to the problem is to eliminate the fragmentation aspects altogether, which can be achieved by looking at the $E_{\rm t}$ distribution of b-jets, without specifically looking at the b momentum [51].

[3] A point worth keeping in mind [49] is that the central scale choice $\mu = \sqrt{p_{\rm t}^2 + m_b^2}$ is not universally accepted as being optimal — indeed for $p_{\rm t} \gtrsim m_b$, a scale choice of $\mu = p_{\rm t}$ is equally justifiable, and would have a non-negligible effect on the predictions.

This has been examined by the D0 collaboration [50] and the comparison to NLO predictions is shown in the right-hand plot of figure 10. Though in a slightly different E_t range, the relation between theory and data is similar to that in the Cacciari–Nason approach for B-mesons: there is a slight excess in the data but not significant compared to the uncertainties. A minor point to note in the study of b-jets is that there are contributions $\alpha_s^n \ln^{2n-1} E_t/m_b$ from soft and collinear logs in the multiplicity of gluons which can then branch collinearly to $b\bar{b}$ pairs [52]. At very large E_t these terms would need to be resummed.

So overall, once one has a proper theoretical treatment, including both an appropriate fragmentation function and, where relevant, an FONLL perturbative calculation, it is probably fair to say that the excess of b-production at the Tevatron is not sufficiently significant to be worrisome (or evidence for supersymmetry).

At some of the other experiments where an excess of b-production is observed a number of the same issues arise, in particular relative to the use of the $\varepsilon = 0.006$ Peterson fragmentation function and the presentation of results at parton level rather than hadron level. However fragmentation is less likely to be able to explain the discrepancies, because of the lower p_t range.

5. Event generators at NLO

The problem of matching event generators with fixed order calculations is one of the most theoretically active areas of QCD currently, and considerable progress has been made in the past couple of years. This class of problems is both of intrinsic theoretical interest in that it requires a deep understanding of the structure of divergences in QCD and of phenomenological importance because of the need for accurate and reliable Monte Carlo predictions at current and future colliders.

Two main directions are being followed: one is the matching of event-generators with leading-order calculations of n-jet production (where n may be relatively high), which is of particular importance for correctly estimating backgrounds for new-particle searches involving cascades of decays with many resulting jets. For a discussion of this subject we refer the reader to the contributions to the parallel sessions [53].

The second direction, still in its infancy, is the matching of event generators with next-to-leading order calculations (currently restricted to low numbers of jets), which is necessary for a variety of purposes, among them the inclusion of correct rate estimates together with consistent final states, for processes with large NLO corrections to the Born cross sections (*e.g.* K factors in pp and γp collisions, boson–gluon fusion at small-x in DIS).

While there have been a number of proposals concerning NLO matching, many of them remain at a somewhat abstract level. We shall here concentrate on two approaches that have reached the implementational stage. As a first step, it is useful to recall why it is non-trivial to implement NLO corrections in an event generator. Let us use the toy model introduced by Frixione and Webber [54], involving the emission only of 'photons' (simplified, whose only degree of freedom will be their energy) from (say) a quark whose initial energy is taken to be 1. For a system which has radiated n photons we write a given observable as $O(E_q, E_{\gamma_1}, \ldots, E_{\gamma_n})$. So for example at the Born level, the observable has value $O(1)$. At NLO we have to integrate over the momentum of an emitted photon, giving the following contribution to the mean value of the observable:

$$\alpha \int_0^1 \frac{dx}{x} R(x)\, O(1-x, x)\,, \tag{5}$$

where $R(x)$ is a function associated with the real matrix element for one-photon emission. There will also be NLO virtual corrections and their contribution will be

$$-\alpha\, O(1) \int_0^1 \frac{dx}{x} V(x)\,, \tag{6}$$

where $V(x)$ is related to the matrix element for virtual corrections.

The structure of dx/x divergences is typical of field theory. Finiteness of the overall cross section implies that for $x \to 0$, $R(x) = V(x)$. This means that for an infrared safe observable (*i.e.* one that satisfies $\lim_{x\to 0} O(1-x, x) = O(1)$), the $\mathcal{O}(\alpha)$ contribution to the mean value of the observable is also finite. However any straightforward attempt to implement Eqs. (5) and (6) directly into an event generator will lead to problems because of the poor convergence properties of the cancellation between divergent positively and negatively weighted events corresponding to the real and virtual pieces respectively. So a significant part of the literature on matching NLO calculations with event generators has addressed question of how to recast these divergent integrals in a form which is practical for use in an event generator (which must have good convergence properties, especially if each event is subsequently going to be run through a detector simulation). The second part of the problem is to ensure that the normal Monte Carlo event generation (parton showering, hadronisation, *etc.*) can be interfaced with the NLO event generation in a consistent manner.

One approach that has reached the implementational stage could be called a 'patching together' of NLO and MC. It was originally proposed in [55] and recently further developed in [56] and extended in [57]. There one chooses a cutoff x_{zero} on the virtual corrections such that the sum of Born and virtual corrections gives zero:

$$1 - \alpha \int_{x_{\text{zero}}}^{1} \frac{dx}{x} V(x) \equiv 0\,. \tag{7}$$

It is legitimate to sum these two contribution because they have the same (Born) final state. Then for each event, a real emission of energy x is generated with the distribution $dx/xR(x)$ and with the same cutoff as on the virtuals. The NLO total cross section is guaranteed to be correct by construction:

$$\sigma_{\text{NLO}} \equiv \sigma_0 \alpha \int_{x_{\text{zero}}}^{1} \frac{dx}{x} R(x)\,. \tag{8}$$

The next step in the event generation is to take an arbitrary separation parameter x_{sep}, satisfying $x_{\text{zero}} < x_{\text{sep}} < 1$. For $x > x_{\text{sep}}$ the NLO emission is considered hard and kept (with ideally the generation of normal Monte Carlo showering below scale x, as in the implementation of [57]). For $x < x_{\text{sep}}$ the NLO emission is thrown away and normal parton showing is allowed below scale x[4].

Among the advantages are that the events all have positive and uniform weights. And while the computation of x_{zero} is non-trivial, the method requires relatively little understanding of the internals of the event generator (which are often poorly documented and rather complicated). However the presence of the separation parameter x_{sep} is in principle problematic: there can be discontinuities in distributions at x_{sep}, certain quantities (for example the probability for a quark to have radiated an amount of energy less than some x_{r} which is below x_{sep}) will not quite be correct to NLO and above x_{sep} potentially large logarithms of x_{sep} are being neglected. These last two points mean that for each new observable that one studies with the Monte Carlo program, one should carry out an analysis of the x_{sep} dependence (varying it over a considerable range, not just a factor of two as is sometimes currently done).

[4] For simplicity, many important but sometimes tricky technical details have been left out. This will also be the case for the merging procedure discussed lower down.

A rather different approach (which we refer to as 'merging') has been developed by Frixione and Webber in [54][5]. They specify a number of conditions that must be satisfied by a Monte Carlo at NLO (MC@NLO): *(i)* all observables should be correct to NLO; *(ii)* soft emissions should be treated as in a normal event generator and hard emissions as in an NLO calculation; *(iii)* the matching between the hard and soft regions should be smooth. Their approach exploits the fact that Monte Carlo programs already contain effective real and virtual NLO corrections,

$$\pm\alpha\frac{dx}{x}M(x) \quad \text{for} \quad {}^{\text{real}}_{\text{virtual}}\,. \tag{9}$$

Because Monte Carlo programs are designed to correctly reproduce the structure of soft and collinear divergences, $M(x)$ has[6] the property that for $x \to 0$, $M(x) = R(x) = V(x)$, *i.e.* the divergent part of the NLO corrections is already included in the event generator. This can be exploited when adjusting the Monte Carlo to be correct to NLO, because the regions that need adjusting are the hard regions, but not the (soft) divergent regions. Specifically the method introduced in [54] can be summarised by the formula

$$\begin{aligned} &I_{\text{MC,Born}} - \alpha\, I_{\text{MC,Born}} \int \frac{dx}{x}\,(V(x) - M(x)) \\ &+\alpha \int \frac{dx}{x}\,(R(x) - M(x))\, I_{\text{MC,Born}+x}\,. \end{aligned} \tag{10}$$

$I_{\text{MC,Born}}$ is to be read 'interface to Monte Carlo.' It means that one should generate a Monte Carlo event starting from the Born configuration (or from the Born configuration plus a photon in the case of $I_{\text{MC,Born}+x}$). Since at the Born level, $I_{\text{MC,Born}}$ already contains effective real and virtual corrections which go as $\pm\alpha M(x)/x$, when evaluating the NLO corrections to the MC, these pieces should be subtracted from the full NLO matrix elements. Because $M(x)$ and $R(x)$ (or $V(x)$) have the same $x \to 0$ limit, the real and virtual integrals are now individually finite and well-behaved, which means that the Monte Carlo only needs only a small, $\mathcal{O}(\alpha)$, correction in order for it to be correct to NLO. Illustrative results from this approach are shown in figure 11 for the transverse momentum distribution of a W^+W^- pair in hadron-hadron collisions. In the low transverse momentum region (which requires resummation — the pure NLO calculation breaks down) MC@NLO

[5] A number of aspects of the work of Collins and collaborations [58] may actually be equivalent, though presented in a rather different framework. Related issues are discussed also in [59].

[6] Or rather, 'should have.' In practice the divergence structure of large-angle soft-gluon emission is not always properly treated in event generators, which leads to some extra complications in the MC@NLO approach.

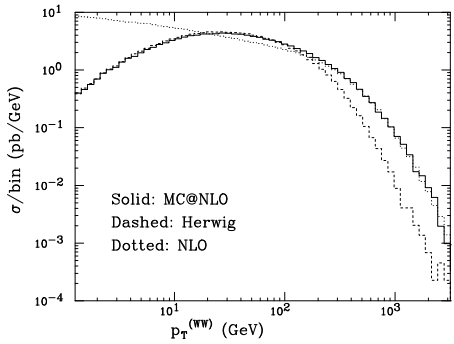


Fig. 11. Transverse momentum distribution of W^+W^- pairs in pp collisions calculated at NLO, with HERWIG (multiplied a K-factor) and with MC@NLO [54].

clearly coincides with the HERWIG results, while at high transverse momentum it agrees perfectly with the NLO calculation (default HERWIG is far too low).

So this procedure has several advantages: it is a smooth procedure without cutoffs; the predictions are guaranteed to be correct at NLO and it does not break the resummation of large logarithms. From a practical point of view it has the (minor) drawback of some events with negative weights, however the fraction of negative weight events is low (about 10% in the example shown above) and they are uniform negative weights, so they should have little effect on the convergence of the results. Another limitation is that to implement this method it is necessary that one understand the Monte Carlo event generator sufficiently well as to be able to derive the function $M(x)$, *i.e.* the effective NLO correction already embodied in the event generator. This however is almost certainly inevitable: there is no way of ensuring a truly NLO result without taking into account what is already included in the event generator.

6. Conclusions: testing QCD?

An apology is perhaps due at this stage to those readers who would have preferred a detailed discussion of the evidence from final-state measurements in favour of (or against) QCD as *the* theory of hadronic physics. I rather took the liberty of reinterpreting the title as 'Tests and perspectives of our *understanding* of QCD through final-state measurements.' Such tests are vital if we are to extend the domain of confidence of our predictions, as has been discussed in the cases of diffraction and power corrections.

The tests of course should be well thought through: some considerations that come out of the still to be fully understood b-excess story are (a) the

importance (as ever) of quoting results at hadron level, not some ill-defined parton level; and (b) that if carrying out a test at a given level of precision (*e.g.* NLO), it is necessary that all stages of the theoretical calculation (including for example the determination of the fragmentation function), be carried out at that same level of precision.

Another, general, consideration is the need for the Monte Carlo models to be reliable and accurate, whether they be used to reconstruct data or to estimate backgrounds. This is especially relevant in cases where the actual measurements are limited to corners of phase space or where large extrapolations are needed. In this context the recent advances in the extension of Monte Carlo models to NLO accuracy is a significant development, and in the medium term we should expect progress from the current 'proof-of-concept' implementations to a widespread availability of NLO-merged event generators.

To conclude, it could well be that a few years from now, many of the measurements and theoretical approaches discussed here will have made it to textbooks as 'standard' QCD. We look forward to future speakers on this topic have an equally varied (but different) range of 'until recently controversial' tests of QCD to discuss!

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REFERENCES

[1] E. Rodrigues [for the ZEUS collaboration], talk given at the XXXVIIth Rencontres de Moriond, 'QCD and high energy hadronic interactions,' March 2002.

[2] D.E. Groom *et al.* [Particle Data Group], *Eur. Phys. J.* **C15**, 1 (2000).

[3] S. Bethke, *J. Phys. G* **26**, R27 (2000).

[4] *e.g.* P. Tortosa, not submitted to the Proceedings.

[5] P. Bambade, M. J. Costa, J. Fuster, P. Tortosa, in 'Budapest 2001, High energy physics,' PRHEP-hep2001/005.

[6] F.P. Schilling [H1 collaboration], to appear in the proceedings of DIS2001, `hep-ex/0107002.`

[7] See for example J.C. Collins, D.E. Soper, G. Sterman, *Nucl. Phys.* **B308**, 833 (1988), and references therein.

[8] L. Trentadue, G. Veneziano, *Phys. Lett.* **B323**, 201 (1994); A. Berera, D.E. Soper, *Phys. Rev.* **D50**, 4328 (1994); M. Grazzini, L. Trentadue, G. Veneziano, *Nucl. Phys.* **B519**, 394 (1998); J.C. Collins, *Phys. Rev.* **D57**, 3051 (1998) [Erratum **D61**, 019902 (2000)].

[9] G. Ingelman, P.E. Schlein, *Phys. Lett.* **B152**, 256 (1985).

[10] V.A. Abramovsky, V.N. Gribov, O.V. Kancheli, *Yad. Fiz.* **18**, 595 (1973) [*Sov. J. Nucl. Phys.* **18**, 308 (1974)].

[11] C. Adloff *et al.* [H1 Collaboration], *Eur. Phys. J.* **C20**, 29 (2001).

[12] A. Edin, G. Ingelman, J. Rathsman, *Phys. Lett.* **B366**, 371 (1996); A. Edin, G. Ingelman, J. Rathsman, *Z. Phys.* **C75**, 57 (1997).

[13] W. Buchmuller, T. Gehrmann, A. Hebecker, *Nucl. Phys.* **B537**, 477 (1999).

[14] J. Bartels, H. Lotter, M. Wusthoff, *Phys. Lett.* **B379**, 239 (1996) [Erratum **B382**, 449 (1996)]; J. Bartels, H. Jung, M. Wusthoff, *Eur. Phys. J.* **C11**, 111 (1999).

[15] P. Laycock, *Acta Phys. Pol.* **B33** (2002), next issue; F.-P. Schilling, *Acta Phys. Pol.* **B33** (2002), next issue; P. Thompson, not submitted to the Proceedings.

[16] H. Jung, *Comput. Phys. Commun.* **86**, 147 (1995).

[17] S. Catani, M.H. Seymour, *Nucl. Phys.* **B485**, 291 (1997) [Erratum **B510**, 503 (1997)].

[18] D. Graudenz, `hep-ph/9710244`.

[19] M. Beneke, *Phys. Rep.* **317**, 1 (1999).

[20] Y.L. Dokshitzer, B.R. Webber, *Phys. Lett.* **B352**, 451 (1995).

[21] P. Nason, M.H. Seymour, *Nucl. Phys.* **B454**, 291 (1995).

[22] Y.L. Dokshitzer, A. Lucenti, G. Marchesini, G.P. Salam, *Nucl. Phys.* **B511**, 396 (1998); Y.L. Dokshitzer, A. Lucenti, G. Marchesini, G.P. Salam, *J. High Energy Phys.* **9805**, 003 (1998); M. Dasgupta, B.R. Webber, *J. High Energy Phys.* **9810**, 001 (1998); M. Dasgupta, L. Magnea, G. Smye, *J. High Energy Phys.* **9911**, 025 (1999); G.E. Smye, *J. High Energy Phys.* **0105**, 005 (2001).

[23] R. Akhoury, V.I. Zakharov, *Phys. Lett.* **B357**, 646 (1995); R. Akhoury, V.I. Zakharov, *Nucl. Phys.* **B465**, 295 (1996); Y.L. Dokshitzer, G. Marchesini, G.P. Salam, *Eur. Phys. J.* **C3**, 1 (1999) [Erratum **C1**, 1 (2001)].

[24] G.P. Salam, D. Wicke, *J. High Energy Phys.* **0105**, 061 (2001).

[25] V. Antonelli, M. Dasgupta, G.P. Salam, *J. High Energy Phys.* **0002**, 001 (2000); M. Dasgupta, G.P. Salam, *Phys. Lett.* **B512**, 323 (2001); M. Dasgupta, G.P. Salam, *Eur. Phys. J.* **C24**, 213 (2002); M. Dasgupta, G.P. Salam, *Acta Phys. Pol.* **B33**, 3311 (2002).

[26] C. Adloff *et al.* [H1 Collaboration], *Eur. Phys. J.* **C14**, 255 (2000) [Addendum **C18**, 417 (2000)].

[27] P.A. Movilla Fernandez, S. Bethke, O. Biebel, S. Kluth, *Eur. Phys. J.* **C22**, 1 (2001).

[28] G.J. McCance [for the H1 and ZEUS Collaborations], `hep-ex/0008009`, talk given at the 35th Rencontres de Moriond: QCD and High Energy Hadronic Interactions, France, March 2000.

[29] Y.L. Dokshitzer, B.R. Webber, *Phys. Lett.* **B404**, 321 (1997).

[30] G.P. Korchemsky, G. Sterman, *Nucl. Phys.* **B437**, 415 (1995); G.P. Korchemsky, G. Oderda, G. Sterman, presented at 5th International Workshop on Deep Inelastic Scattering and QCD (DIS 97), Chicago, IL, April 1997, `hep-ph/9708346`; G.P. Korchemsky, G. Sterman, *Nucl. Phys.* **B555**, 335 (1999);

[31] E. Gardi, J. Rathsman, *Nucl. Phys.* **B609**, 123 (2001); E. Gardi, J. Rathsman, `hep-ph/0201019`.

[32] R.P. Feynman, *Photon Hadron Interactions*, W.A. Benjamin, New York 1972.

[33] A. Banfi, G. Marchesini, Y. L. Dokshitzer, G. Zanderighi, *J. High Energy Phys.* **0007**, 002 (2000); A. Banfi, Y.L. Dokshitzer, G. Marchesini, G. Zanderighi, *Phys. Lett.* **B508** (2001) 269; A. Banfi, Y.L. Dokshitzer, G. Marchesini, G. Zanderighi, *J. High Energy Phys.* **0105**, 040 (2001);
A. Banfi, G. Marchesini, G. Smye, G. Zanderighi, *J. High Energy Phys.* **0108**, 047 (2001);

[34] D. Wicke, contributed to 37th Rencontres de Moriond on QCD and Hadronic Interactions, March 2002, `hep-ph/0205299`.

[35] B. Abbott *et al.* [D0 Collaboration], *Phys. Lett.* **B487**, 264 (2000).

[36] S. Frixione, in 'Budapest 2001, High energy physics,' PRHEP-hep2001/025, `hep-ph/0111368`.

[37] I. Abt *et al.* [HERA-B Collaboration], `hep-ex/0205106`.

[38] A. de Roeck, *Acta Phys. Pol.* **B33**, 2749 (2002).

[39] E.L. Berger, B.W. Harris, D.E. Kaplan, Z. Sullivan, T.M. Tait, C.E. Wagner, *Phys. Rev. Lett.* **86**, 4231 (2001).

[40] J. Binnewies, B.A. Kniehl, G. Kramer, *Phys. Rev.* **D58**, 034016 (1998); Also, discussions in P. Nason *et al.*, in 'Standard model physics (and more) at the LHC,' CERN 1999, `hep-ph/0003142`.

[41] M. Cacciari, P. Nason, `hep-ph/0204025`.

[42] P. Nason, B.R. Webber, *Phys. Lett.* **B395**, 355 (1997).

[43] M. Cacciari, M. Greco, P. Nason, *J. High Energy Phys.* **9805**, 007 (1998).

[44] P. Nason, C. Oleari, *Nucl. Phys.* **B565**, 245 (2000).

[45] A. Heister *et al.* [ALEPH Collaboration], *Phys. Lett.* **B512**, 30 (2001).

[46] C. Peterson, D. Schlatter, I. Schmitt, P.M. Zerwas, *Phys. Rev.* **D27**, 105 (1983).

[47] V.G. Kartvelishvili, A. K. Likhoded, V.A. Petrov, *Phys. Lett.* **B78**, 615 (1978).

[48] D. Acosta *et al.* [CDF Collaboration], *Phys. Rev.* **D65**, 052005 (2002).

[49] Yu.L. Dokshitzer, private communication.

[50] B. Abbott *et al.* [D0 Collaboration], *Phys. Rev. Lett.* **85**, 5068 (2000).

[51] S. Frixione, M.L. Mangano, *Nucl. Phys.* **B483**, 321 (1997).

[52] See *e.g.* M.H. Seymour, *Nucl. Phys.* **B436**, 163 (1995).

[53] F. Krauss, *Acta Phys. Pol.* **B33**, 3165 (2002); L. Lönnblad, *Acta Phys. Pol.* **B33**, 3171 (2002).

[54] S. Frixione, B.R. Webber, *J. High Energy Phys.* **0206**, 029 (2002).

[55] H. Baer, M.H. Reno, *Phys. Rev.* **D44**, 3375 (1991); H. Baer, M.H. Reno, *Phys. Rev.* **D45**, 1503 (1992).

[56] B. Potter, *Phys. Rev.* **D63**, 114017 (2001); B. Potter, T. Schorner, *Phys. Lett.* **B517**, 86 (2001).

[57] M. Dobbs, *Phys. Rev.* **D65**, 094011 (2002).

[58] J.C. Collins, X. m. Zu, *J. High Energy Phys.* **0206**, 018 (2002); *J. High Energy Phys.* **0204**, 041 (2002); J. Collins, *Phys. Rev.* **D65**, 094016 (2002). Y. Chen, J.C. Collins, N. Tkachuk, *J. High Energy Phys.* **0106**, 015 (2001). J.C. Collins, *J. High Energy Phys.* **0005**, 004 (2000).

[59] S. Mrenna, `hep-ph/9902471`.