F_2^{ep} IN THE DEEP SEA AND DGLAP*

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The consistency of the DGLAP equations is tested in the deep sea region using the HERA F_2 data.

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1. Introduction

The theory of strong interactions is now 30 years old. Lepton–nucleon experiments have contributed decisively to the understanding of QCD and to the structure of the proton. During the last decade the experiments at the *ep*-collider HERA have extended the available phase space to very high values of Q^2 in the valence region and have opened at values of Q^2 below 100 GeV^2 a hitherto unexplored region, the *deep sea*, *i.e.* x <0.001. The observed strong rise of F_2 at low values of x was unexpected and so was the successful inclusion of the low-x data into global QCD analysis [1] without loosing apparently in fit quality.

A phenomenological study of the F_2 data in the deep sea revealed two prominent features, when plotting the data in terms of the variable $q = \log_{10}(1 + Q^2/Q_0^2)$ [2] (with $Q_0^2 = 0.5\,\mathrm{GeV}^2$) rather than the usual $\ln Q^2$ *(i)* Within the experimental precision the data [3] are well represented by $F_2(x, q) = u_0(x) + u_1(x)\ (q - \langle q\rangle)$. For $x < 0.001$ the linear extrapolation to $q = 0$ satisfies $F_2(x, 0) = 0$ as required by the conservation of the electromagnetic current, while for $x > 0.001$ the valence contribution gets increasingly important and makes a linear extrapolation inappropriate. *(ii)* The data covering the range above $Q^2 = 0.05\,\mathrm{GeV}^2$ do not indicate any change of behavior in the transition region from non-perturbative to perturbative physics. This empirical fact [3] challenges the question of how the linear behavior of F_2 in q is brought about as a result of intrinsic properties of the kernels in the validity region of the DGLAP equations.

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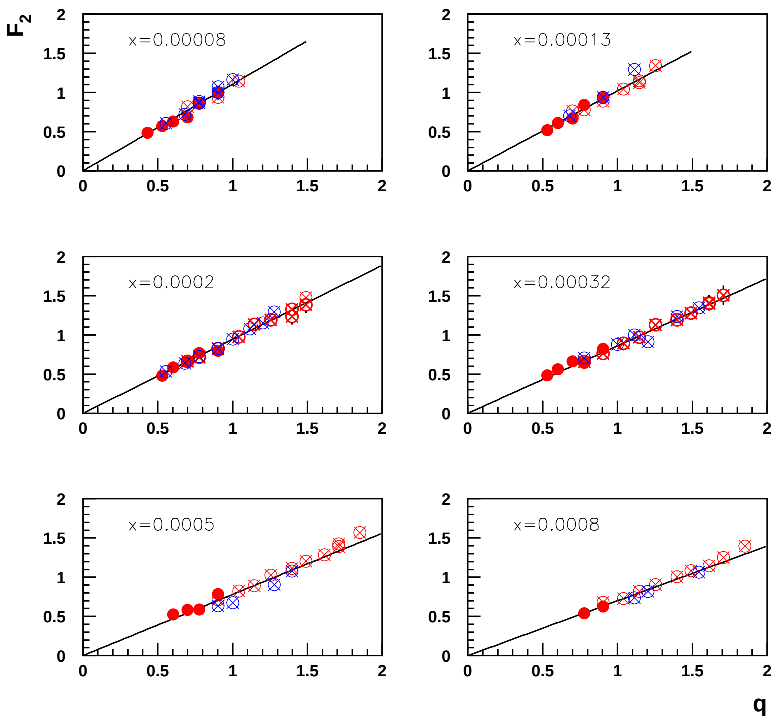

Fig. 1. F_2 data from H1 and ZEUS for 6 fixed x-bins *versus* q.

2. The DGLAP equations and F_2^{ep}

The formalism describing the evolution of parton distributions is well known [4]. In order to take advantage of the properties of q, the coupled DGLAP equations for the singlet (S) and the gluon (G) distributions are expressed in this variable

$$\frac{\partial S(x,q)}{\partial q} = a(q)\Big(P_{qq}\otimes S(x,q) + P_{qg}\otimes G(x,q)\Big)\,, \tag{1a}$$

$$\frac{\partial G(x,q)}{\partial q} = a(q)\Big(P_{gq}\otimes S(x,q) + P_{gg}\otimes G(x,q)\Big)\,, \tag{1b}$$

where $a(q) = (\alpha_{\rm s}(Q^2)/2\pi)\ \big((Q^2+Q_0^2)/Q^2\big)$ ln10 plays the role of the QCD-coupling. The structure function F_2 in ep scattering evolves differently for the singlet part, $\varepsilon\, S(x)$, and nonsinglet part, $N(x)$. In the Quark–Parton Model $\varepsilon = \frac{1}{f}\sum_i^f e_i^2$ and $S(x) = \sum_i^f x(q_i(x) + \overline{q}_i(x))$, where e_i^2 are the QED coupling constants for the f active flavors. In QCD at next-to-leading order the parton distributions get Q^2-dependent. Choosing the $\overline{MS}$ renormalization scheme the expression for F_2^{ep} reads [4]: $F_2^{ep} = C_F \otimes N + \varepsilon\,(C_F \otimes S + C_G \otimes G)$. In the kinematic region of interest, the deep sea, $\varepsilon\, S(x,q) = F_2^{ep}(x,q)(1 + \mathcal{O}(\text{few }\%))$, as long as $Q^2 > 1$ GeV^2. In the calculations below the kernels are used at next-to-leading order with 3 flavors and the singlet function $\varepsilon\, S(x,q)$ is identified for $x < 0.001$ with F_2^{ep} itself, while for $x > 0.001$ S and $\partial S/\partial q$ are extended smoothly to the valence region in agreement with data. Eq. (1a) is equivalent to

$$a(q)P_{qg} \otimes G(x,q) = \frac{\partial S(x,q)}{\partial q} - a(q)P_{qq} \otimes S(x,q)\,. \tag{1c}$$

Now the r.h.s. $\hat{S}(x,q) \equiv (\partial/\partial q - a(q)P_{qq}\otimes)\, S(x,q)$ consists of known quantities: S, $\partial S/\partial q$ by experiment and P_{qq}, $\alpha_{\rm s}$ by theory, thus constraining the properties of the unknown gluon on the l.h.s. This information ought to be consistent with the second DGLAP equation (Eq. (1b)). A quantitative test in the deep sea is performed under the two hypotheses

1. *The singlet $S(x,q)$ is exactly linear in q in the deep sea;*
2. *The DGLAP equations are valid in the considered phase space region;*

using later on as test quantity

$$a(q)P_{qg} \otimes \frac{\partial G(x,q)}{\partial q}\,. \tag{2}$$

3. The first DGLAP equation

(a) $\hat{S}$: the term $\partial S/\partial q$ is given, in the deep sea, by the measured slopes of F_2^{ep}, *i.e.* $u_1(x)$ (see Fig. 1). The other term $a(q)P_{qq} \otimes S(x,q)$ involves the kernel P_{qq} and so a convolution with S over the full range from x until 1. Its effect is numerically small as shown in Fig. 2(a). The convolution with the lowest order kernel is also shown. The effect of the $1/x$-term in the NLO-part of P_{qq} gets prominent at low x. In conclusion, the r.h.s. of Eq. (1c) is well determined and is nearly Q^2-independent for $1 < Q^2 < 100$ GeV^2. The precise shape of S in the valence region is not relevant.

(b) The gluon function satisfying Eq. (1c) must have a strong dependence upon q, since both $q\,a(q)$ and $\hat{S}$ are weakly q-dependent. Fig. 2(b) shows $\hat{S}(x,q)$ for q=1. Its shape is dominated by the logarithmic behavior of $\partial S/\partial q = u_1(x) \sim \log(1/x)$ [3] with a strong suppression at large x and a small negative curvature at low x caused by $P_{qq} \otimes$

$S(x,q)$. Eq. (1c) can be approximately solved for the gluon function by noting the property of the kernel P_{qg}, which applied to a valence-like distribution produces a constant, while applied to a constant produces a logarithmic rise in the deep sea. The resulting gluon function for $q=1$ is displayed in Fig. 2(b). The decrease at low x accounts for the small negative curvature in $\hat{S}$. For verification both $\hat{S}(x,1)$ and $a(q)P_{qg}\otimes G(x,q)$ for $q=1$ using the reconstructed gluon $G(x,1)$ is also shown in Fig. 2(b) by the two curves, one displaced for better visibility.

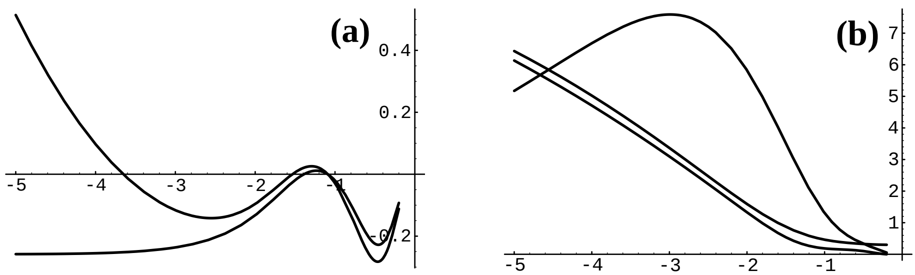

Fig. 2. (a): $a(q)P_{qq}\otimes S(x,q)$ for q=1 *vs* log(x) at NLO (upper) and LO (lower); (b): Display of the reconstructed gluon function $G(x,1)$; the lower curve represents $\hat{S}(x,1)$ and the curve shifted upward for visibility by 0.3 verifies that $G(x,1)$ approximately satisfies Eq. (1c).

4. Consistency test for q=1

The test quantity $T(x,q)=a(q)P_{qg}\otimes\partial G(x,q)/\partial q$ Eq. (2) is evaluated for $q=1$ in two ways. It appears as one of the terms, denoted by $T_{\rm I}$, when forming the derivative of the first DGLAP equation w.r.t. q

$$T_{\rm I}=\frac{\partial\hat{S}(x,q)}{\partial q}-\frac{\partial\ln a(q)}{\partial q}\;\hat{S}(x,q)-\frac{\partial\ln\alpha_{\rm s}(q)}{\partial q}\;a(q)(P_{qg}-P_{qg}^{\rm LO})\otimes G(x,q)\,.$$

On the other hand, substituting in T for $\partial G(x,q)/\partial q$ directly the second DGLAP equation (1b) yields

$$T_{\rm II}=a(q)P_{qg}\otimes a(q)\left(P_{gq}\otimes S(x,q)+P_{gg}\otimes G(x,q)\right).$$

The very low x behavior is different for $T_{\rm I}$ and $T_{\rm II}$, since the second one consists of a product of two kernels, while the first one involves only one kernel.

With the gluon distribution function satisfying the first DGLAP equation for q=1 one obtains the following numbers for $T_{\rm I}$ and $T_{\rm II}$ at 3 x-values

x	$T_{\rm I}$	$T_{\rm II}$
10^{-3}	3.4	3.3
10^{-4}	5.6	9.3
10^{-5}	7.8	18.5

5. Results

A transparent analysis has been carried out confronting the observed form of the structure function F_2^{ep} at low x with the form implied by the DGLAP kernels. No evolution is performed, but rather the interplay of the derivative w.r.t. q and the convolution is investigated locally. The two main results are

- In the deep sea region the linear q-dependence of F_2^{ep} is inconsistent with the DGLAP equations.
- $a(q)P_{qg} \otimes G(x,q)$ varies very little with Q^2 for $1 < Q^2 < 100$ GeV^2.

The first hypothesis regarding the linearity is not strictly satisfied. Indeed, the mere measurement uncertainties of the data do not exclude a small departure from linearity in q, which, however, is too small to invalidate the large deviation of the ratio $T_{\rm I}/T_{\rm II}$ from unity. Furthermore, this ratio is insensitive to the assumptions made in the analysis.

The observed inconsistency is hidden in global fits [1], since the majority of the F_2 data is in the valence dominated phase space region and only the small fraction of the HERA samples in the deep sea probe the critical $1/x$ terms in the DGLAP kernels. As Q^2 becomes smaller than 100 GeV^2 the low-x behavior affects the fits increasingly and unavoidably induces large gluon driven curvatures $\partial^2 F_2^{ep}/\partial q^2$ in conflict with the predominant linearity of F_2^{ep} in q borne out by the data.

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