

DGLAP EVOLUTION IN THE SATURATION MODEL* **

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(Received June 26, 2002)

A modification of the saturation model of deep inelastic scattering at small x which includes the Altarelli-Parisi (DGLAP) evolution is presented. Significant improvement of the description of the structure function F_2 at large Q^2 is achieved and a good description of diffractive data is preserved.

PACS numbers: 13.60.Hb, 12.38.Bx

1. Introduction

The saturation model [1] has provided a successful description of HERA deep inelastic scattering data. This includes both the F_2 data, with the transition into the nonperturbative photoproduction region, and the DIS diffractive data [2]. Whereas the formulae are particularly appealing through their simplicity, they also have an attractive theoretical background: the idea of parton saturation. Despite its success, the model suffers from the lack of proper scaling violation, *i.e.* at larger values of Q^2 it does not exactly match with QCD evolution described by the DGLAP equations. This becomes clearly visible in the region $Q^2 > 20 \text{ GeV}^2$ where the model predictions are below the data on F_2 . Therefore, one expects that QCD evolution should enhance the cross section in this region.

* Presented at the X International Workshop on Deep Inelastic Scattering (DIS2002) Cracow, Poland, 30 April–4 May, 2002.

** Supported by the Polish State Committee for Scientific Research (KBN) grant No. 5 P03B 144 20 and by the Deutsche Forschungsgemeinschaft.

We present a modification of the saturation model which preserves its success in the low- Q^2 (transition) region, while incorporating QCD evolution in the large- Q^2 domain [3]. Since the energy dependence in the large- Q^2 region is mainly due to the behaviour of the dipole cross section at small dipole sizes r , our changes will affect mostly the small- r region. At the same time, we leave the large- r behaviour of the dipole cross section practically unchanged which allows to retain a good description of DIS diffractive cross section. Recent attempts [4] along the same lines indicate that diffraction provides a highly nontrivial restriction on possible modifications of the saturation model.

2. The saturation model and its modification

We start with a brief review of the saturation model [1]. Within the dipole formulation of the γ^*p scattering, the cross section

$$\sigma_{T,L}^{\gamma^*p}(x, Q^2) = \int d^2r dz \psi_{T,L}^*(Q, r, z) \hat{\sigma}(x, r) \psi_{T,L}(Q, r, z), \quad (1)$$

where $\psi_{T,L}$ are the virtual photon wave functions with the transverse and longitudinal polarisation. In the saturation model the following form of the dipole cross section $\hat{\sigma}$ is proposed

$$\hat{\sigma}(x, r) = \sigma_0 \left\{ 1 - \exp\left(-\frac{r^2}{4R_0^2(x)}\right) \right\}, \quad (2)$$

where $R_0(x)$ is the saturation scale which decreases when $x \rightarrow 0$,

$$R_0^2(x) = \frac{1}{\text{GeV}^2} \left(\frac{x}{x_0}\right)^\lambda. \quad (3)$$

In order to be able to study the formal photoproduction limit, the Bjorken variable $x = x_B$ was modified to be

$$x = x_B \left(1 + \frac{4m_q^2}{Q^2}\right) = \frac{Q^2 + 4m_q^2}{W^2}, \quad (4)$$

where m_q is an effective quark mass and W denotes the γ^*p center-of-mass energy. The parameters of the model, $\sigma_0 = 23$ mb, $\lambda = 0.29$ and $x_0 = 3 \cdot 10^{-4}$ (for fixed $m_q = 140$ MeV), were found from a fit to small- x data [1].

As it is well known [5], in the small- r region the dipole cross section is related to the gluon density obeying the DGLAP evolution

$$\hat{\sigma}(x, r) \simeq \frac{\pi^2}{3} r^2 \alpha_s xg(x, \mu^2), \quad (5)$$

where the scale $\mu^2 \simeq C/r^2$ for $r \rightarrow 0$. The equation (5) is valid in the double logarithmic approximation in which the constant C is not determined. Expanding the exponent in Eq. (2) for $r \ll 2R_0(x)$, we find the gluon density in the saturation model

$$xg(x, \mu^2) = \frac{3}{4\pi^2 \alpha_s} \frac{\sigma_0}{R_0^2(x)}. \quad (6)$$

For fixed α_s this gluon density is clearly scale independent. Thus, we have to modify the small- r behaviour of the dipole cross section to include the DGLAP evolution and, at the same time, keep the large- r behaviour unchanged. This will preserve the idea of saturation, which reflects unitarity, and allows a good description of the diffractive cross section.

Therefore, we propose the following modification of the model (2)

$$\hat{\sigma}(x, r) = \sigma_0 \left\{ 1 - \exp \left(- \frac{\pi^2 r^2 \alpha_s(\mu^2) xg(x, \mu^2)}{3 \sigma_0} \right) \right\}, \quad (7)$$

where the scale μ^2 is assumed to have the form

$$\mu^2 = \frac{C}{r^2} + \mu_0^2. \quad (8)$$

The parameters C and μ_0^2 will be determined from a fit to DIS data. The scale $\mu_0 \gg \Lambda$ allows to freeze the value of the gluon distribution for large r at a perturbative scale which leads to $\hat{\sigma}(x, r) \approx \sigma_0$, as in the original model. The transition between small and large r depends on x but in detail it might be different from the original formulation. Thus, the modified form mimics in a consistent way the saturated behaviour of the dipole cross section.

In a first approximation, $g(x, \mu^2)$ is evolved with the leading order DGLAP equation in which quarks are neglected in the spirit of the small- x limit. The starting gluon distribution at the initial scale $Q_0^2 = 1 \text{ GeV}^2$

$$xg(x, Q_0^2) = A_g x^{-\lambda_g} (1-x)^{5,6}, \quad (9)$$

where A_g and λ_g are another fit parameters. The exponent determining the large x behaviour is motivated by the recent MRST parameterisation [6].

For small r , the exponential in (7) can be expanded in powers of its argument, and relation (5) with the running $\alpha_s = \alpha_s(\mu^2)$ is found. In contrast to the model (2), the rise in $1/x$ now has become r -dependent. When inserting $\hat{\sigma}$ into (1) and convoluting with the photon wave function, the integrand peaks near $r \sim 2/Q$ for large Q^2 , and the argument of the gluon density turns into $\mu^2 \sim Q^2$. Consequently, with increasing Q^2 , DGLAP evolution will strengthen the rise in $1/x$, whereas in the original saturation model the power of $1/x$ had been constant.

3. Fit results and comparison with data

We performed a global fit to the DIS data with $x < 0.01$ in the range $0.1 < Q^2 < 500 \text{ GeV}^2$. For the HERA experiments the new 1996–97 data from H1 and ZEUS were used [7] together with the E665 experiment data [8] (in total 330 points) The statistical and systematic errors were added in quadrature. For a full discussion of fit details see [3]. With the fixed quark mass $m_q = 140 \text{ MeV}$, the value of $\chi^2/N_{\text{df}} = 1.18$ was found (for the original model re-fitted to the new data $\chi^2/N_{\text{df}} \simeq 3$) with the following fit parameters: $C = 0.26$, $\mu_0^2 = 0.52 \text{ GeV}^2$, $A_g = 1.2$ and $\lambda_g = 0.28$. In addition, for the agreement with the diffractive data we fix the normalisation of the dipole cross section to the original saturation model value $\sigma_0 = 23 \text{ mb}$.

The form of the new dipole cross section (7) is shown in figure 1 (solid lines) for different values of x . As expected the main modification in comparison to the model (2) (dashed lines) lies in the small- r region. In figure 2

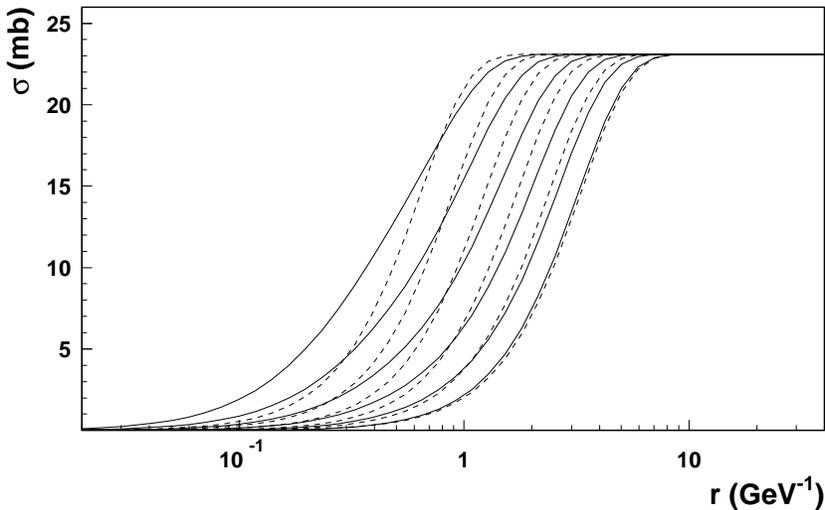


Fig. 1. The dipole cross section as a function of the dipole size r for different values of $\log_{10} x = -2, -3, \dots, -7$ (from the left to the right). The original saturation model: dashed lines and the improved model: solid lines.

the global characteristic of the data description is shown. Namely, we plot the effective slope $\lambda(Q^2)$, obtained from the parameterisation of F_2 at small x , $F_2 \sim x^{-\lambda(Q^2)}$, for fixed Q^2 . For the shown curves, λ was computed using the relation $F_2 = Q^2 / (4\pi^2 \alpha_{em}) \sigma_{\text{T+L}}^{\gamma^*p}$ with the two discussed forms of dipole cross sections. As expected, the inclusion of the DGLAP evolution for small r significantly improves agreement with the data at large Q^2 while at small Q^2 the results are practically the same.

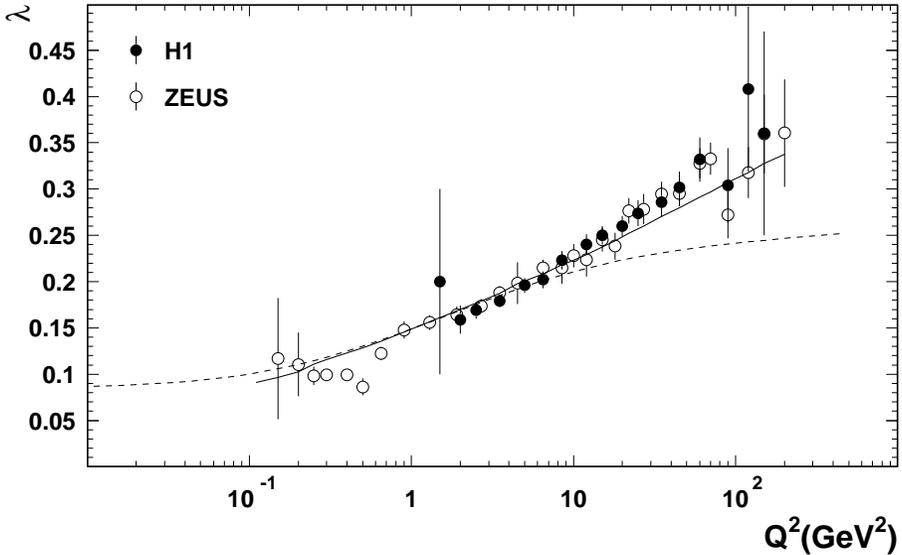


Fig. 2. The effective slope λ as a function of Q^2 . The original saturation model (dashed line) and the improved model (solid line) are shown against the data.

An important aspect of the dipole models is their straightforward description of diffractive processes. In particular, the constant ratio of the inclusive over diffractive cross sections as a function of x finds a natural explanation in the saturation model [1,2]. In DIS diffraction the cross section is dominated by the contribution from large dipole sizes r . Since the large- r part of the dipole cross section is practically unchanged in our modification, the description of diffractive data is as good as in the original saturation model. The only change introduced is a different treatment of the colour factors for the $q\bar{q}g$ component, see [3] for more details.

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