FRACTAL STRUCTURE OF THE PROTON AT LOW BJORKEN x*

Tomáš Laštovička

DESY Zeuthen, Platanenallee 6, D15738 Zeuthen, Germany
and
Charles University, Faculty of Mathematics and Physics
V Holešovičkách 2, 18000 Prague 8, Czech Republic

(Received September 16, 2002)

A description of the structure function $F_2(x, Q^2)$ at low Bjorken x reflecting a fractal self-similar proton structure is proposed. The specific parameterisation obtained provides an excellent description of all low x data extending in four momentum transfer squared from the non-perturbative region, $Q^2 \gtrsim 0.05$ GeV^2, into the deep inelastic domain of $Q^2 \lesssim 100$ GeV^2.

PACS numbers: 13.60.Hb, 14.20.Dh

1. Introduction

Fractality is an inherent property of many natural objects and processes. Such objects do not lose their characteristic structure even if investigated along a wide range of scales. While natural objects, like plants or clouds, are exhibiting fractal properties up to some limiting scale, mathematics can go further and model objects which are perfect fractals. Example of such an object is the Sierpinski gasket (see Fig. 1) [1, 2].

In general, fractal properties may be observed in a wide variety of topics, *e.g.* self-organized systems, cellular automats, local systems on lattice, signal analyses, chaos and many others. The most essential way how to describe them is to know the internal dynamics or iteration rules of the system. However, in many cases these are not known and global properties of the system are used instead. In the presented work the *fractal dimension* concept [2] is used to obtain a novel parameterisation of the proton structure function $F_2(x, Q^2)$ which reflects fractal behaviour. This parameterisation is then fitted to the recent HERA precision data.

* Presented at the X International Workshop on Deep Inelastic Scattering (DIS2002) Cracow, Poland, 30 April–4 May, 2002.

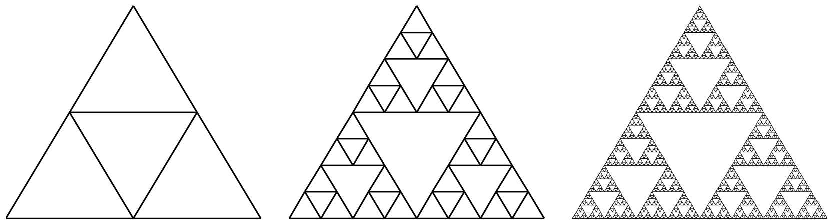

Fig. 1. Sierpinski gasket in iterations No. 1, 3 and 6 (from left). Iteration No. 1 corresponds to the *seed image* which is arbitrary while the iteration always converges to the same object.

2. $F_2(x, Q^2)$ parameterisation and fit to the HERA data

Referring to [3] for details, the proton structure function $F_2(x, Q^2)$ may be parametrised with the following expression reflecting self-similarity

$$F_2(x,Q^2) = \frac{\mathrm{e}^{\mathcal{D}_0}\, Q_0^2\, x^{-\mathcal{D}_2+1}}{1+\mathcal{D}_3-\mathcal{D}_1\log x} \times \left(x^{-\mathcal{D}_1 \log\left(1+\frac{Q^2}{Q_0^2}\right)} \left(1+\frac{Q^2}{Q_0^2}\right)^{\mathcal{D}_3+1} - 1 \right), \qquad (1)$$

where $\mathcal{D}_0$ is a normalisation parameter, $\mathcal{D}_2$ and $\mathcal{D}_3$ are the fractal dimensions with respect to $1/x$ and $1 + Q^2/Q_0^2$, respectively, while $\mathcal{D}_1$ is their dimensional correlation. Q_0^2 is a reference scale of the four momentum transfer squared Q^2.

The five parameters are determined using recent data from the HERA experiments H1 [4] and ZEUS [5] in the range $1.5 \leq Q^2 \leq 120$ GeV^2 (H1) and $0.045 \leq Q^2 \leq 0.65$ GeV^2 (ZEUS). Additionally a cut $x < 0.01$ has been applied to exclude the valence quark region. The fit result is shown in Fig. 2 as F_2/Q^2 in bins of W^2 and compared to ALLM97 parameterisation [6]. The χ^2/ndf is about 0.82 and it was calculated with total errors, adding the statistical and systematical errors in quadrature. As can be seen, the fit describes the new, preliminary H1 data in the intermediate Q^2 range [7] equally well. These data were not used in the fit.

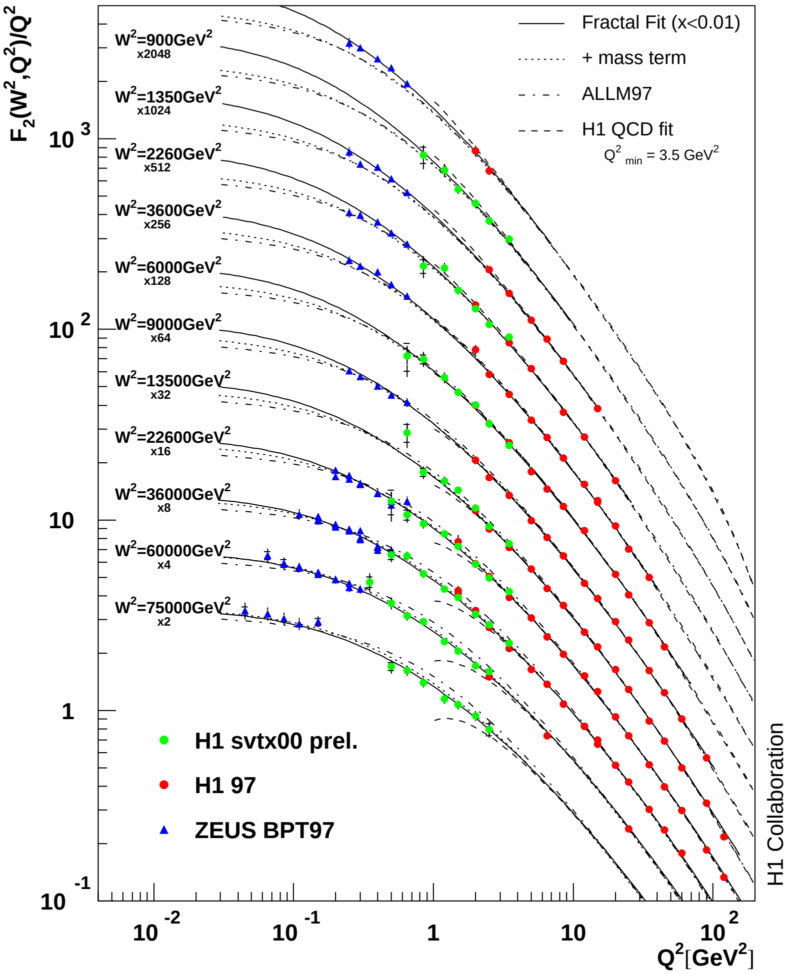

Fig. 2. Virtual photon–proton cross-section $\sigma_{\gamma^\star p} \propto F_2(W^2,Q^2)/Q^2$ as a function of Q^2 in W^2 bins, where W is the mass of the $\gamma^\star p$ system. H1 (points) and ZEUS (triangles) measurements are shown along with the fit to four parameters (full line), the fit with the mass term included (dotted line), H1 QCD fit (dashed line) and with ALLM97 parameterisation (dot-dashed line). The fractal fit to all five parameters is indistinguishable from the four parameter fit in the region of measured data.

2.1. Photoproduction limit

The ratio $F_2(W^2,Q^2)/Q^2$ is proportional to the virtual photon–proton cross-section $\sigma_{\gamma^\star p}(W^2,Q^2)$. In the limit $Q^2 \to 0$ and fixed W^2 it can be shown [3] that the parametrisation (1) behaves like Q^2 only for $\mathcal{D}_2 = 1$. Indeed, in the fit with $\mathcal{D}_2$ as a free parameter a value very close to 1 is obtained. Fixing $\mathcal{D}_2 = 1$ reduces the number of parameters to four but the corresponding fit gives an equally good value of $\chi^2/\mathrm{ndf} \doteq 0.82$ as the fit to all five parameters.

In the photoproduction limit one needs to introduce a mass term in the calculation of Bjorken x, which is negligible at high W^2:

$$x \to x + \frac{m^2}{W^2}, \tag{2}$$

where m is an effective mass of a proton constituent which is fitted as another parameter in the fit to the data. In such a case, the ratio $F_2(W^2, Q^2)/Q^2$ always converges when $Q^2 \to 0$ and the photon–proton cross-section $\sigma_{\gamma^\star p}$ has a power law behaviour in W^2 variable. The result of the fit with the mass term (involving also the photoproduction data) is shown in Fig. 2 as dashed line. The behaviour of the fit at low W^2 is similar to the ALLM97 parametrisation while at high W^2 it approaches the fractal fit without the mass term.

3. Summary

The proton structure at low Bjorken x exhibits fractal properties. A parameterisation of the proton structure function $F_2(x, Q^2)$, reflecting self-similarity, describes very well the low x HERA data, both in the non-perturbative and the deep-inelastic domain including a recent precision measurement of the inclusive deep inelastic cross section in the transition region near $Q^2 \sim 1$ GeV^2 by the H1 collaboration [7]. The introduced formalism uniquely defines the x and Q^2 dependence of parton densities. Thus it is applicable also to other measures of proton structure, like the longitudinal structure function F_{L}, the diffractive structure function F_2^{D} or the spin structure function g_1. In the context of related approaches involving self-organized criticality in particle production [8] and in low x gluon dynamics [9], the fractal mathematics and scale relativity [10] may become a powerful tool in exploration and description of various high energy physics phenomena.

REFERENCES

[1] W. Sierpinski, *Compt. Rendus Acad. Sci. Paris* **160**, 302 (1915); M.F. Barnsley, *Fractals Everywhere*, 2nd Edition, Academic Press Professional, 1993.

[2] B.B. Mandelbrot, *The Fractal Geometry of Nature.* W.H. Freeman, New York 1977.

[3] T. Laštovička, *Eur. Phys. J.* **C24**, 529 (2002).
[4] H1: C. Adloff *et al., Eur. Phys. J.* **C21**, 33 (2001).
[5] ZEUS: J. Breitweg *et al., Phys. Lett.* **B487**, 53 (2000).
[6] H. Abramowicz, A. Levy, `hep-ph/9712415`.
[7] T. Laštovička, *Acta Phys. Pol.* **B33**, 2835 (2002).
[8] A. Paramonov, A. Rostovtsev, `hep-ph/0208233`.
[9] M. Ta-chung, R. Rittel, K. Tabelow, `hep-ph/9905538`.
[10] M.N. Celerier, L. Nottale, `hep-th/0112213`.