

# STRONG COUPLING CONSTANT FROM THE PHOTON STRUCTURE FUNCTION\*

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We extract the value of the strong coupling constant  $\alpha_s$  from a single-parameter pointlike fit to the photon structure function  $F_2^\gamma$  at large  $x$  and  $Q^2$  and from a first five-parameter full (pointlike and hadronic) fit to the complete  $F_2^\gamma$  data set taken at PETRA, TRISTAN, and LEP. In next-to-leading order and the  $\overline{\text{MS}}$  renormalization and factorization schemes, we obtain  $\alpha_s(m_Z) = 0.1183 \pm 0.0050(\text{exp.})_{-0.0028}^{+0.0029}(\text{theor.})$  [pointlike] and  $\alpha_s(m_Z) = 0.1198 \pm 0.0028(\text{exp.})_{-0.0046}^{+0.0034}(\text{theor.})$  [pointlike and hadronic]. We demonstrate that the data taken at LEP have reduced the experimental error by about a factor of two, so that a competitive determination of  $\alpha_s$  from  $F_2^\gamma$  is now possible.

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## 1. Introduction

In these proceedings, we demonstrate that new TRISTAN and LEP data, which extends to high  $\langle Q^2 \rangle \leq 780\text{GeV}^2$ , improves the sensitivity of  $F_2^\gamma$  to  $\alpha_s$  significantly, yielding a fitted  $\alpha_s$  that is consistent with the world average and has competitive experimental and theoretical errors.

## 2. General procedure

We work in a fixed flavor number scheme with three active quark flavors ( $u, d, s$ ), and include the heavy quark contribution via the  $\mathcal{O}(\alpha)$  expression

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for the Bethe–Heitler process  $\gamma^*(Q^2)\gamma \rightarrow h\bar{h}$  [1]. Bottom and top quark contributions are numerically negligible, while charm is not. We adopt a charm quark mass of  $m_c = 1.5 \pm 0.1$  GeV [2]. We omit spurious higher order terms [3].

We use all published  $F_2^\gamma$  data collected at the high-energy  $e^+e^-$ -colliders PETRA [4–6], TRISTAN [7–9], and LEP [10–16]. If more than one set of statistically overlapping data exists, the most recent publication is used. We exclude from our fit the data published by the TPC/2 $\gamma$  Collaboration at PEP [17, 18], since several data points, mainly at low  $x$ , are inconsistent with measurements published by PLUTO [5], L3 [12], and OPAL [15] in the range  $1.9 < Q^2 < 5.1$  GeV<sup>2</sup>. Data where the charm component has been subtracted are also discarded. Statistical uncertainties and correlations between data points due to the experimental unfolding are taken into account as provided by the experiments, while systematic uncertainties are assumed to be uncorrelated, so on average  $\chi^2/\text{DF}$  is expected to be slightly less than unity. For asymmetric errors, the data points are taken at the center of the full error interval. We neglect  $P^2$  in this analysis, since usually  $P^2 \ll Q^2$ .

### 3. Pointlike fit

For our pointlike fit, we set  $Q_0 = \Lambda$ , so that the hadronic input vanishes automatically and only a single parameter ( $\Lambda$ , or equivalently  $\alpha_s(m_Z)$ ) has to be fitted. This is only justified at large  $x$  and  $Q^2$ , where the residue of the pointlike singularity is expected to be small. Thus we perform our single-parameter pointlike fit only to a subset of data points with  $x \geq 0.45$  and  $Q^2 \geq 59$  GeV<sup>2</sup>. Very similar results are obtained with  $Q_0 = 0.5 \dots 0.6$  GeV [19–22], while choosing  $Q_0 = 1$  GeV significantly increases the value of  $\chi^2/\text{DF}$ ; two-parameter pointlike fits of  $\alpha_s$  and  $Q_0$  are driven to  $Q_0 \simeq \Lambda$ . In the first three lines of Table I we list the  $\chi^2/\text{DF}$  and  $\alpha_s(m_Z)$  values

TABLE I  
 $\chi^2/\text{DF}$  and  $\alpha_s(m_Z)$  values obtained in LO and NLO in the  $\overline{\text{MS}}$  and DIS $_\gamma$  factorization schemes with a single-parameter fit of the pointlike photon structure function  $F_2^\gamma$ . Also shown are the results obtained without LEP data and with very high  $Q^2$  data.

Scheme	$\chi^2/\text{DF}$	$\alpha_s(m_Z)$
LO	7.9/ 19	$0.1260 \pm 0.0055(\text{ex})_{-0.0055}^{+0.0061}(\text{th})$
$\overline{\text{MS}}$	9.1/ 19	$0.1183 \pm 0.0050(\text{ex})_{-0.0028}^{+0.0029}(\text{th})$
DIS $_\gamma$	8.1/ 19	$0.1195 \pm 0.0051(\text{ex})_{-0.0028}^{+0.0031}(\text{th})$
w/o LEP	3.2/ 7	$0.1244 \pm 0.0126(\text{ex})_{-0.0032}^{+0.0033}(\text{th})$
high $Q^2$	11.9/ 8	$0.1159 \pm 0.0125(\text{ex})_{-0.0018}^{+0.0018}(\text{th})$

obtained in LO and NLO. The NLO fit is performed in two factorization schemes ( $\overline{\text{MS}}$  and  $\text{DIS}_\gamma$  [3]) with different treatment of the pointlike Wilson coefficient in  $F_2^\gamma$ , but the numerical variation is found to be small. The values of  $\chi^2/\text{DF}$  for the individual data sets (not shown) lie around unity or below. The experimental errors are determined by varying  $\alpha_s(m_Z)$  until the total value of  $\chi^2$  is increased by one unit. To estimate the theoretical error, we vary the charm quark mass as indicated above and vary the factorization and renormalization scales by factors of two about their central value, the physical scale  $Q$ . We then add these three individual errors in quadrature. In the fourth line of Table I, we list the result of a fit without the LEP data. The experimental error is more than doubled, showing that the LEP data have considerably increased the sensitivity of  $F_2^\gamma$  to  $\alpha_s$  at high  $x$  and  $Q^2$ . When data at all values of  $x$ , but very high  $Q^2$  ( $Q^2 \geq 284 \text{ GeV}^2$ ) are fitted, the central value of  $\alpha_s(m_Z)$  remains virtually unchanged (last line of Table I). At very high  $Q^2$ , the theoretical error drops by a factor of two, whereas the experimental error increases. Measurements of  $F_2^\gamma$  at a future linear  $e^+e^-$  or  $e\gamma$ -collider like TESLA at very high values of  $Q^2$  and with small experimental errors will therefore lead to even more precise determinations of  $\alpha_s$ .

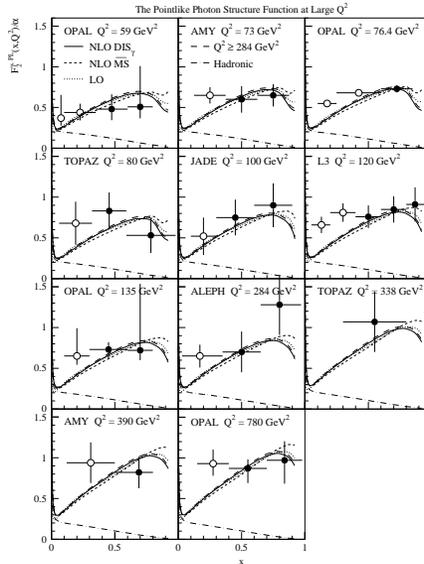


Fig. 1. Single-parameter fits of the pointlike photon structure function, compared to PETRA [4], TRISTAN [7,9], and LEP [10,12–14,16] data at large  $Q^2$ . The data points marked by open circles have not been used in the fits. Also shown is the hadronic contribution from a five-parameter NLO fit of the full photon structure function in the  $\text{DIS}_\gamma$  scheme.

The goodness of our pointlike fit may also be judged from Fig. 1, where the fitted data points are shown as full circles, and where the statistical and systematic errors have been added in quadrature. The LO and NLO fits differ only by small amounts. The choice of factorization scheme only affects the region outside the data. Also shown in Fig. 1 is the hadronic contribution from a five-parameter NLO fit of the full photon structure function in the  $\text{DIS}_\gamma$  scheme, and amounts to only a few percent in the region that has been used in the pointlike fit.

#### 4. Pointlike + hadronic fit

$F_2^\gamma$  is dominated by the  $u$ -quark density in the photon and is only sensitive to the combined density of  $d$ - and  $s$ -quarks, which is suppressed by the smaller  $d$ - and  $s$ -quark charges. The gluon contributes to  $F_2^\gamma$  in LO only through a rather weak coupling to the quark singlet density in the evolution equations. A consecutive fit of the  $u$ -quark,  $d$ - and  $s$ -quark, and gluon densities shows that only the first is well constrained by  $F_2^\gamma$  data and that the fit does not improve when more degrees of freedom are added. Therefore we set the gluon PDF to zero and assume that the hadronic fluctuations of the photon are insensitive to the quark charge, *i.e.* we identify the hadronic boundary conditions for  $u$ -quarks and  $d$ - and  $s$ -quarks at the starting scale  $Q_0$ . Together with  $\alpha_s(m_Z)$  and  $Q_0$ , we then fit the parameters  $N$ ,  $\alpha$ , and  $\beta$  of our ansatz  $f_{u,d+s}^\gamma(x, Q_0^2) = Nx^\alpha(1-x)^\beta$  to the full data set described above. In the first three lines of Table II we list the  $Q_0$ ,  $\chi^2/\text{DF}$ , and  $\alpha_s(m_Z)$

TABLE II

$Q_0$ ,  $\chi^2/\text{DF}$ , and  $\alpha_s(m_Z)$  values obtained in LO and NLO in the  $\overline{\text{MS}}$  and  $\text{DIS}_\gamma$  factorization schemes with a five-parameter fit of the hadronic photon structure function  $F_2^\gamma$ . Also shown are the results obtained without LEP data.

Scheme	$Q_0/\text{GeV}$	$\chi^2/\text{DF}$	$\alpha_s(m_Z)$
LO	$0.79 \pm 0.18$	121/129	$0.1475 \pm 0.0074(\text{ex})_{-0.0072}^{+0.0141}(\text{th})$
$\overline{\text{MS}}$	$0.83 \pm 0.09$	118/129	$0.1198 \pm 0.0028(\text{ex})_{-0.0046}^{+0.0034}(\text{th})$
$\text{DIS}_\gamma$	$0.85 \pm 0.09$	115/129	$0.1216 \pm 0.0028(\text{ex})_{-0.0050}^{+0.0033}(\text{th})$
w/o LEP	$0.46 \pm 0.10$	37/ 38	$0.1147 \pm 0.0047(\text{ex})_{-0.0033}^{+0.0282}(\text{th})$

values obtained with this five-parameter fit in LO and NLO. The starting scale  $Q_0$  is perturbatively stable and is found to be close to the masses of the light vector mesons  $\rho$ ,  $\omega$ , and  $\phi$ . The individual values of  $\chi^2/\text{DF}$  lie around unity or below. The  $\chi^2$  value for the four TPC/2 $\gamma$  points at  $Q^2 = 2.8 \text{ GeV}^2$ , which have not been used in the fits, is 18.0. The gluon density, generated

with  $f_g^\gamma(x, Q_0^2) = 0$ , is in good agreement with recent H1 dijet data [23]. Due to the larger number of data points in the full fit, the experimental error turns out much smaller than in the pointlike fit. When the full fit is performed without the LEP data (last line of Table II), the experimental error is almost doubled, *i.e.* the impact of the LEP data is again impressive. A fit to LEP data only leads to almost identical results as the full fit. The theoretical error in LO and without the LEP data gets a large asymmetric contribution from doubling the factorization scale, which is highly correlated with an increase in the fitted value of  $Q_0$  and which is drastically reduced in the full NLO fit. Similar results as those listed in Table II are obtained, when only  $u$ -quarks are assigned a hadronic boundary condition.

In Fig. 2 we compare our results to the fitted  $F_2^\gamma$  data in the region of low  $x$  and  $Q^2$ . This region is clearly dominated by the hadronic contribution and by the impact of the LEP data. A fit without the LEP data results in a rise of  $F_2^\gamma$  at low  $x$ , which is much too steep. The fits are perturbatively stable and the data are described almost equally well in the  $\overline{\text{MS}}$  and  $\text{DIS}_\gamma$  scheme.

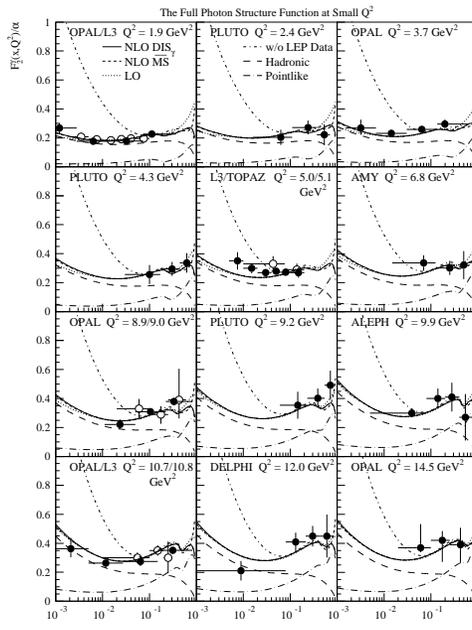


Fig. 2. Five-parameter fits of the full photon structure function, compared to data from PETRA [5], TRISTAN [8, 9], and LEP [10–13, 15] at small  $Q^2$ . The data points marked by open circles refer to the second experiment and/or  $Q^2$  value. Also shown are the hadronic and pointlike contributions to the NLO fit in the  $\text{DIS}_\gamma$  scheme.

## 5. Summary

Since the total error on  $\alpha_s(m_Z)$  is smaller in the full fit than in the pointlike fit due to the larger number of data points, we adopt as our final result

$$\alpha_s(m_Z) = 0.1198 \pm 0.0054 \quad (5.1)$$

in NLO and the  $\overline{\text{MS}}$  scheme, where the larger theoretical error has been added to the experimental error in quadrature. While our total error is slightly larger than those obtained in  $Z$ -boson- and  $\tau$ -decays at LEP, it is comparable to the errors obtained in deep-inelastic scattering at HERA and heavy quarkonium decays. This encourages us to combine our result with the current world average of  $0.1172 \pm 0.0014$  [24] to a new world average

$$\alpha_s(m_Z) = 0.1175 \pm 0.0014, \quad (5.2)$$

where the errors are assumed to be uncorrelated.

## 6. Conclusion

Our analysis proves that the available  $F_2^\gamma$  data contribute significantly to a precise determination of  $\alpha_s$  and that future measurements of  $F_2^\gamma$  at linear colliders will have a large impact.

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