# A STUDY OF QCD COUPLING CONSTANT AND POWER CORRECTIONS IN THE FIXED TARGET DEEP INELASTIC MEASUREMENTS* 

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We reanalyze deep inelastic scattering data of BCDMS Collaboration by including proper cuts of ranges with large systematic errors. We perform also the fits of high statistic deep inelastic scattering data of BCDMS, SLAC, NM and BFP Collaborations taking the data separately and in combined way and find good agreement between these analyses. We extract the values of both the QCD coupling constant $\alpha_{\mathrm{s}}\left(M_{Z}^{2}\right)$ up to NLO level and of the power corrections to the structure function $F_{2}$.

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## 1. Introduction

The deep inelastic scattering (DIS) leptons on hadrons is the basical process to study the values of the Parton Distribution Functions (PDF) which are universal (after choosing of factorization and renormalization schemes) and can be used in other processes. The accuracy of the present data for deep inelastic Structure Functions (SF) reached the level at which the $Q^{2}$-dependence of logarithmic QCD-motivated terms and power-like ones may be studied separately (for a review, see [1] and references therein).

In the present paper we sketch the results of our analysis [2] at the Next-to-Leading Order (NLO) of perturbative QCD for the most known DIS SF $F_{2}\left(x, Q^{2}\right)$ taking into account experimental data [4-7] of SLAC, NM, BCDMS and BFP Collaborations. We stress the power-like effects, so-called twist-4 (i.e. $\sim 1 / Q^{2}$ ) contributions. To our purposes we represent the $\mathrm{SF} F_{2}\left(x, Q^{2}\right)$ as the contribution of the leading twist part $F_{2}^{\mathrm{pQCD}}\left(x, Q^{2}\right)$

[^0]described by perturbative QCD, when the target mass corrections are taken into account, and the nonperturbative part ("dynamical" twist-four terms):
\[

$$
\begin{equation*}
F_{2}\left(x, Q^{2}\right) \equiv F_{2}^{\mathrm{full}}\left(x, Q^{2}\right)=F_{2}^{\mathrm{pQCD}}\left(x, Q^{2}\right)\left(1+\frac{\tilde{h}_{4}(x)}{Q^{2}}\right) \tag{1}
\end{equation*}
$$

\]

where $\tilde{h}_{4}(x)$ is magnitude of twist-four terms.
In this paper we do not present exact formulae of $Q^{2}$-dependence of SF $F_{2}$ which are given in [2]. We note only that the PDF at some $Q_{0}^{2}$ is theoretical input of our analysis and the twist-four term $\tilde{h}_{4}(x)$ is considered as a set of free parameters (one constant $\tilde{h}_{4}\left(x_{i}\right)$ per $x_{i}$-bin): $\tilde{h}_{4}^{\text {free }}(x)=\sum_{i=1}^{I} \tilde{h}_{4}\left(x_{i}\right)$, where $I$ is the number of bins.

## 2. Fits of $\boldsymbol{F}_{\mathbf{2}}$

First of all, we choose the cut $Q^{2} \geq 1 \mathrm{GeV}^{2}$ in all our studies. For $Q^{2}<1 \mathrm{GeV}^{2}$, the applicability of twist expansion is very questionable. Secondly, we choose $Q_{0}^{2}=90 \mathrm{GeV}^{2}\left(Q_{0}^{2}=20 \mathrm{GeV}^{2}\right)$ for the nonsinglet (combine nonsinglet and singlet) evolution, i.e. quite large values of the normalization point $Q_{0}^{2}$ : our perturbative formulae should be applicable at the value of $Q_{0}^{2}$. Moreover, the higher order corrections $\sim \alpha_{\mathrm{s}}^{k}\left(Q_{0}^{2}\right)$ and $\sim\left(\alpha_{\mathrm{S}}\left(Q^{2}\right)-\alpha_{\mathrm{s}}\left(Q_{0}^{2}\right)\right)^{k}(k \geq 2)$ should be less important at these $Q_{0}^{2}$ values.

We use MINUIT program [8] for minimization of $\chi^{2}\left(F_{2}\right)$. We consider free normalizations of data for different experiments. For the reference, we use the most stable deuterium BCDMS data at the value of energy $E_{0}=200 \mathrm{GeV}$ ( $E_{0}$ is the initial energy lepton beam). Using other types of data as reference gives negligible changes in our results. The usage of fixed normalization for all data leads to fits with a bit worser $\chi^{2}$.

$$
\text { 2.1. } B C D M S{ }^{12} C+H_{2}+D_{2} \text { data }
$$

We start our analysis with the most precise experimental data [6] obtained by BCDMS muon scattering experiment at the high $Q^{2}$ values. The full set of data is 762 points.

It is well known that the original analyses given by BCDMS Collaboration itself (see also Ref. [9]) lead to quite small values $\alpha_{\mathrm{s}}\left(M_{Z}^{2}\right)=0.113$. Although in some recent papers (see, for example, $[10,11]$ ) more higher values of the coupling constant $\alpha_{\mathrm{S}}\left(M_{Z}^{2}\right)$ have been observed, we think that an additional reanalysis of BCDMS data should be very useful.

Based on study [12] we proposed in [2] that the reason for small values of $\alpha_{\mathrm{s}}\left(M_{Z}^{2}\right)$ coming from BCDMS data was the existence of the subset of the
data having large systematic errors. We studied this subject by introducing several so-called $Y$-cuts ${ }^{1}$ (see [2]):

$$
\begin{array}{lllll}
y \geq 0.14 & \text { when } & 0.3<x \leq 0.4, & y \geq 0.16 & \text { when }
\end{array} \quad 0.4<x \leq 0.5, ~ 子 Y_{\text {cut3 }} \quad \text { when } \quad 0.5<x \leq 0.6, \quad y \geq Y_{\text {cut4 }} \quad \text { when } \quad 0.6<x \leq 0.7,
$$

and several $N$ sets for the cuts at $0.5<x \leq 0.8$ :
TABLE I
The values of $Y_{\text {cut } 3}, Y_{\text {cut } 4}$ and $Y_{\text {cut } 5}$.

| $N$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $Y_{\text {cut3 }}$ | 0 | 0.14 | 0.16 | 0.16 | 0.18 | 0.22 | 0.23 |
| $Y_{\text {cut } 4}$ | 0 | 0.16 | 0.18 | 0.20 | 0.20 | 0.23 | 0.24 |
| $Y_{\text {cut5 }}$ | 0 | 0.20 | 0.20 | 0.22 | 0.22 | 0.24 | 0.25 |

From the Figs. 1 and 2 we can see that the $\alpha_{\mathrm{s}}$ values are obtained for $N=1 \div 6$ of $Y_{\text {cut } 3}, Y_{\text {cut } 4}$ and $Y_{\text {cut } 5}$ are very stable and statistically consistent.


Fig. 1. The study of systematics at different $Y_{\text {cut }}$ values in the fits based on nonsinglet evolution (i.e. when $x \geq 0.25$ ). The inner (outer) error-bars show statistical (systematic) errors.

[^1]

Fig. 2. All other notes are as in Fig. 1 with two exceptions: the fits based on combine evolution and the points $N_{Y c u t}=1,2,3,4,5$ correspond the values $N=1,2,4,5,6$ in Table I.

## 2.2. $S L A C, B C D M S, N M$ and BFP data

After these $Y$-cuts have been incorporated (with $N=6$ ) for BCDMS data, the full set of combine data is 1309 points. The results of the fits are compiled in the Summary.

## 3. Summary

We have demonstrated several steps of our study [2] of the $Q^{2}$-evolution of DIS structure function $F_{2}$ fitting all fixed target experimental data.

From the fits we have obtained the value of the normalization $\alpha_{\mathrm{S}}\left(M_{Z}^{2}\right)$ of QCD coupling constant. First of all, we have reanalyzed the BCDMS data cutting the range with large systematic errors. As it is possible to see in the Figs. 1 and 2, the value of $\alpha_{S}\left(M_{Z}^{2}\right)$ rises strongly when the cuts of systematics were incorporated. In another side, the value of $\alpha_{\mathrm{s}}\left(M_{Z}^{2}\right)$ does not dependent on the concrete type of the cut within modern statistical errors.

Fitting SLAC, BCDMS, NM and BFP data, we have found in [2] that at $Q^{2} \geq 10 \div 15 \mathrm{GeV}^{2}$ the formulae of pure perturbative QCD (i.e. twist-two approximation together with target mass corrections) are in good agreement with all data. When we have added twist-four corrections, we have very good agreement between QCD (i.e. first two coefficients of Wilson expansion) and the data starting already with $Q^{2}=1 \mathrm{GeV}^{2}$, where the Wilson expansion should begin to be applicable. The results for $\alpha_{s}\left(M_{Z}^{2}\right)$ are very similar (see [2]) for the both types of analyses and have the following form:

$$
\alpha_{\mathrm{s}}\left(M_{Z}^{2}\right)=0.1177 \pm 0.0007 \text { (stat) } \pm 0.0021 \text { (syst) } \pm 0.0009 \text { (norm) },(3)
$$

where the symbols "stat", "syst" and "norm" mark the statistical error, systematic one and the error of normalization of experimental data.

We would like to note that we have good agreement also with the analysis [11] of combined H1 and BCDMS data, which has been given by H1 Collaboration very recently. Our results for $\alpha_{\mathrm{S}}\left(M_{Z}^{2}\right)$ are in good agreement also with the average value for coupling constant, presented in the recent studies (see [10, 14] and references therein) and in famous reviews [15].

At the end of our paper we would like to discuss the contributions of higher twist corrections. In our study [2] we have reproduced well-known $x$-shape of the twist-four corrections at the large and intermediate values of Bjorken variable $x$ (see the Fig. 3 and $[2,9]$ ). Note that there is a small- $x$


Fig. 3. The values of the twist-four terms. The black and white points correspond to the small- $x$ asymptotics $\sim x^{-\omega}$ of sea quark and gluon distributions with $\omega=0$ and $\omega=0.18$, respectively. The statistical errors are displayed only.
rise of the magnitude of twist-four corrections, when we use flat parton distributions at $x \rightarrow 0$. As we have discussed in Ref. [2], there is a strong correlation ${ }^{2}$ between the small- $x$ behavior of twist-four corrections and the type of the corresponding asymptotics of the leading-twist parton distributions. The possibility to have a singular type of the asymptotics leads (in our fits) to the appearance of the rise of sea quark and gluon distributions as $\sim x^{-0.18}$ at low $x$ values, that is in full agreement with low $x$ HERA data. At this case the rise of the magnitude of twist-four corrections is completely cancelled. This cancellation is in full agreement with theoretical and phenomenological studies (see [2]).

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## REFERENCES

[1] M. Beneke, Phys. Rep. 317, 1 (1999).
[2] A.V. Kotikov, V.G. Krivokhijine, hep-ph/0108224; hep-ph/0206221.
[3] K. Long et al., hep-ph/0109092.
[4] L.W. Whitlow et al. [SLAC Collaboration], Phys. Lett. B282, 475 (1992); SLAC report 357 (1990).
[5] M. Arneodo et al. [NM Collaboration], Nucl. Phys. B483, 3 (1997).
[6] A.C. Benevenuti et al. [BCDMS Collaboration], Phys. Lett. B223, 485 (1989); B237, 592 (1990); B195, 91 (1987).
[7] P.D. Mayers et al. [BFP Collaboration], Phys. Rev. D34, 1265 (1986).
[8] F. James, M. Ross, "MINUIT", CERN Computer Center Library, D 505, Geneve, 1987.
[9] M. Virchaux, A. Milsztajn, Phys. Lett. B274, 221 (1992).
[10] S.I. Alekhin, Eur. Phys. J. C10, 395 (1999); Phys. Rev. D59, 114016 (1999).
[11] C. Adloff et al. [H1 Collaboration], Eur. Phys. J. C21, 33 (2001).
[12] V. Genchev et al., in Proc. Int. Conference of Problems of High Energy Physics (1988), Dubna, Vol. 2., p. 6; A. Milsztaijan et al., Z. Phys. C49, 527 (1991).
[13] ZEUS Collaboration]: S. Chekanov et al., Eur. Phys. J. C21, 443 (2001).
[14] G. Dissertori, hep-ex/0105070; T. Affolder et al., Phys. Rev. Lett. 88, 042001 (2002); P.A. Movilla Fernandez et al., Eur. Phys. J. C22, 1 (2001).
[15] G. Altarelli, hep-ph/0204179; S. Bethke, J. Phys. C26, R27 (2000).


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[^1]:    ${ }^{1}$ Hereafter we use the kinematical variable $Y=\left(E_{0}-E\right) / E_{0}$, where $E$ is scattering energies of lepton.

[^2]:    ${ }^{2}$ This correlation comes because of very limited numbers of experimental data used here lie at the low $x$ region. Indeed, only the NMC experimental data contribute there. We hope to incorporate the HERA data $[11,13]$ in our future investigations.

