INVESTIGATION OF NEXT-TO-LEADING EFFECTS IN CCFM*

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The effect of formally next-to-leading contributions to the CCFM evolution equation are discussed.

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1. Introduction

The CCFM [1] evolution equation in the framework of k_{t}-factorization and its practical realization in the Monte Carlo program Cascade [2] has been shown to be very successful in describing a bulk of experimental measurements [2–5], which were not described in the collinear approach. However, as BFKL, the CCFM equation was derived in the high energy approximation keeping only the singular terms (*i.e.* $1/z$ and $1/(1-z)$) in the splitting function P_g. The question arises, whether the other terms, which are present in the DGLAP splitting function, are already small enough to be neglected at the energies of present colliders. Also the scale in the running α_{s} was originally treated differently for the small and large z parts.

2. Next-to-leading effects

The splitting of $k_{i-1} \to k_i p_i$, where k (p) are the four-momentum vectors of the propagator (emitted) gluon, respectively, with momentum fractions x_{i-1}, x_i and the splitting variable $z = x_i/x_{i-1}$, is described by the splitting function P_g. The original CCFM [1] splitting function P_g was given by

$$\tilde{P}_g\left(z, q^2, k_{\mathrm{t}}^2\right) = \frac{\bar{\alpha}_{\mathrm{s}}\left(q^2(1-z)^2\right)}{1-z} + \frac{\bar{\alpha}_{\mathrm{s}}\left(k_{\mathrm{t}}^2\right)}{z} \Delta_{\mathrm{ns}}\left(z, q^2, k_{\mathrm{t}}^2\right) , \tag{2.1}$$

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where $q = p_{\rm t}/(1-z)$ and the non-Sudakov form factor $\Delta_{\rm ns}$ was defined as

$$\log \Delta_{\rm ns}(z,q^2,k_{\rm t}^2) = -\bar{\alpha}_{\rm s} \int_z^1 \frac{dz'}{z'} \int \frac{dq^2}{q^2} \Theta(k_{\rm t}-q)\Theta(q-z'q)\,. \tag{2.2}$$

Here only the singular terms $1/z$ and $1/(1-z)$ were included and for simplicity the scale in the running $\alpha_{\rm s}$ was not treated in the same manner for the small and large z part.

In the high energy approximation, the inclusion of the non-singular terms in the splitting function P_g as well as changes in the scale of $\alpha_{\rm s}$ are considered as next-to-leading effects. In the following we investigate the numerical importance of these effects at present collider energies.

2.1. Scale of $\alpha_{\rm s}$

Due to the complicated structure of the CCFM splitting function, for simplicity the transverse momentum of the propagator gluon, $k_{\rm t}$, was used as the scale in the running $\alpha_{\rm s}$, whereas next-to-leading order calculations suggest, that the proper scale is the transverse momentum of the emitted gluon, $p_{\rm t}$, for full range of z (for a summary of the arguments see [4]).

Changing the scale in $\alpha_{\rm s}$ from $k_{\rm t}$ to $p_{\rm t}$ (with $p_{\rm t} \sim q$ for $z \to 0$), also the non-Sudakov form factor needs to be properly changed, resulting in

$$\tilde{P} = \frac{\bar{\alpha}_{\rm s}(q(1-z))}{1-z} + \frac{\bar{\alpha}_{\rm s}(q)}{z}\Delta_{\rm ns}(z,q,k_{\rm t})\,, \tag{2.3}$$

$$\log \Delta_{\rm ns} = -\int_0^1 \frac{dz'}{z'} \int \frac{dq'^2}{q'^2}\alpha_{\rm s}(q')\Theta(k_{\rm t}-q')\Theta(q'-z'q)\,, \tag{2.4}$$

which leads to

$$\log \Delta_{\rm ns} = -\int_0^1 \frac{dz'}{z'} \int_{(z'q)^2}^{k_{\rm t}^2} \frac{dq'^2}{q'^2} \frac{1}{\log\left(\frac{q'}{\Lambda_{\rm QCD}}\right)}\,. \tag{2.5}$$

Due to the angular ordering constraint $q' > z'q$, q' can become very small and even $q' < \Lambda_{\rm QCD}$ at small values of z'. Thus a cutoff is required. For $z'q < q_0 = 0.71$ GeV we fix $\alpha_{\rm s}(q_0) = 0.5$, but keep the full angular ordering constraint in the integral.

In Fig. 1 we compare the new non-Sudakov form factor $\Delta_{\rm ns}$ with the standard one for three different values of $k_{\rm t}/q_{\rm t}$. It is interesting to note, that everywhere the very small z values are highly suppressed.

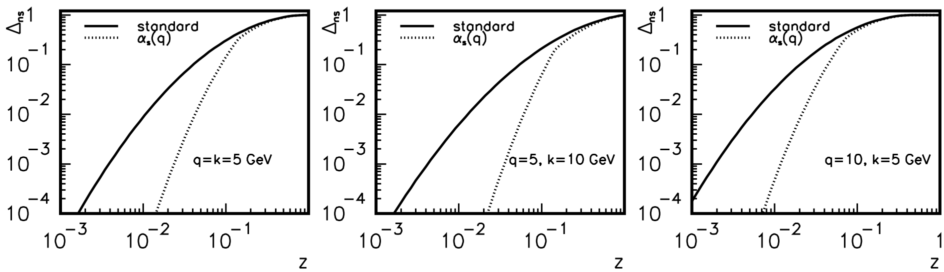

Fig. 1. The non-Sudakov form factor $\Delta_{\rm ns}$ for three different values of $k_{\rm t}/q_{\rm t}$ as a function of the splitting variable z according to Eq. (2.2) (solid) and Eq. (2.5) (dotted).

In Fig. 2 the splitting function P_g (dotted) is plotted as a function of z for three different values of $k_{\rm t}/q_{\rm t}$. Also shown for comparison is the splitting function without any suppression from the non-Sudakov form factor ($\Delta_{\rm ns} = 1$ dashed) and the standard version of the splitting function from Eqs. (2.1), (2.2) (solid). One can clearly see how the different non-Sudakov form factors suppress the small z region of P_g.

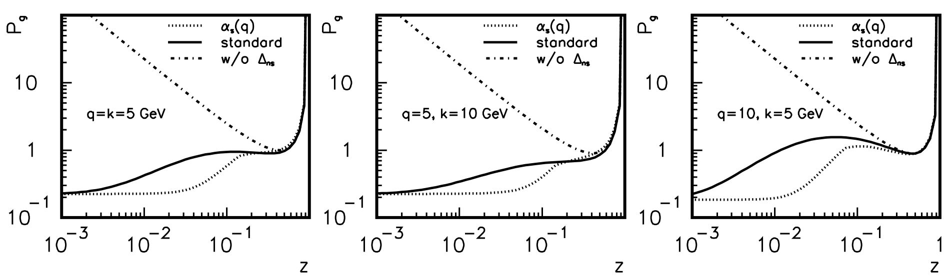

Fig. 2. The splitting function P_g for three different values of $k_{\rm t}/q_{\rm t}$ as a function of the splitting variable z according to Eqs. (2.1), (2.2) (solid) Eqs. (2.3), (2.5) (dotted), and with $\Delta_{\rm ns} = 1$ (dashed).

2.2. Non-singular terms in splitting function

Another source of next-to-leading-log corrections is the gluon splitting function itself. At very high energies, the $1/z$ term in P_{gg}, included in BFKL and CCFM, will certainly be dominant. However, the question is whether including just this term is sufficient at energies available at present colliders.

The implementation of the full DGLAP splitting function into CCFM is problematic. Naively one would simply replace $\frac{1}{1-z} \to \frac{1}{1-z} - 2 + z(1-z)$ in the CCFM splitting function. But this can lead to negative branching probabilities.

In [4] it was suggested to use

$$P(z,q,k) \;=\; \bar{\alpha}_{\mathrm{s}}\left(k_{\mathrm{t}}^{2}\right)\left(\frac{(1-z)}{z}+(1-B)z(1-z)\right)\Delta_{\mathrm{ns}}(z,q,k) \\ +\,\bar{\alpha}_{\mathrm{s}}\left((1-z)^{2}q^{2}\right)\left(\frac{z}{1-z}+Bz(1-z)\right)\,, \tag{2.6}$$

where B is a parameter to be chosen arbitrarily between 0 and 1, we take $B=0.5$. As a consequence of the replacement, the Sudakov form factor will change, but also the non-Sudakov form factor needs to be replaced by

$$\log\Delta_{\mathrm{ns}} \;=\; -\bar{\alpha}_{\mathrm{s}}\left(k_{\mathrm{t}}^{2}\right)\int\limits_{0}^{1}dz'\left(\frac{1-z}{z'}+(1-B)z(1-z)\right) \\ \times\int\frac{dq'^{2}}{q'^{2}}\Theta(k-q')\Theta(q'-z'q)\,. \tag{2.7}$$

In Fig. 3 we compare the new non-Sudakov form factor Δ_{ns} with the standard one for three different values of $k_{\mathrm{t}}/q_{\mathrm{t}}$.

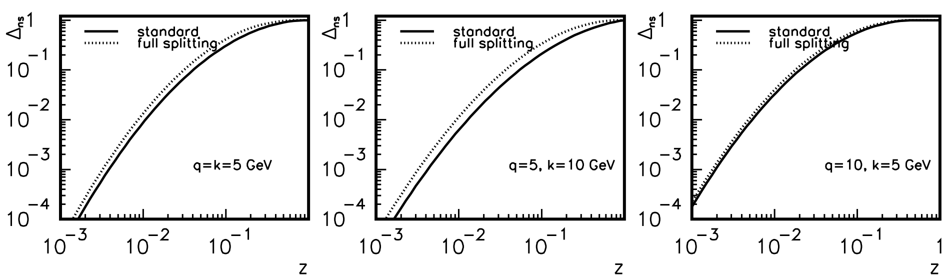

Fig. 3. The non-Sudakov form factor Δ_{ns} for three different values of $k_{\mathrm{t}}/q_{\mathrm{t}}$ as a function of the splitting variable z according to Eq. (2.2) (solid) and Eq. (2.7) (dotted).

In Fig. 4 the splitting function P_g (dotted) is plotted as a function of z for three different values of $k_{\mathrm{t}}/q_{\mathrm{t}}$. Also shown for comparison is the splitting function without non-Sudakov form factor ($\Delta_{\mathrm{ns}}=1$ dashed) and the standard version of the splitting function from Eqs. (2.1), (2.2) (solid). Here the effect of the different form of the splitting function P_g becomes obvious already at values of $z\sim 0.5$, whereas the non-Sudakov form factor is similar to the standard one. One should note that especially in the region of medium z, the new branching probability (including the non-singular terms) becomes smaller.

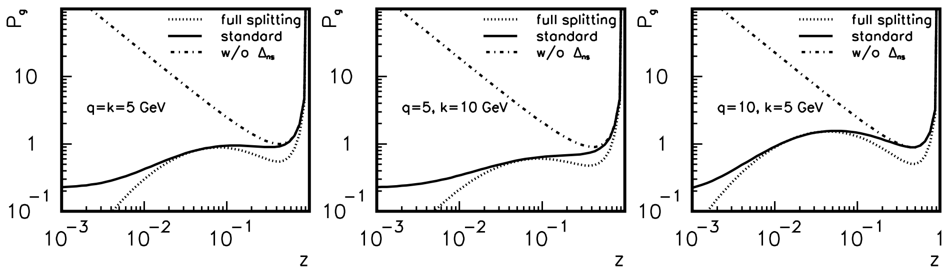

Fig. 4. The splitting function P_g for three different values of k_t/q_t as a function of the splitting variable z according to Eqs. (2.1), (2.2) (solid) Eqs. (2.6), (2.7) (dotted), and with $\Delta_{ns} = 1$ (dashed).

2.3. Consequences for forward jet production

In Fig. 5 we show the predictions for forward jet production at HERA [6] for the different scenarios discussed above. All cases have been re-fitted to the structure function F_2, with a similarly good χ^2/ndf. It becomes obvious, that the prediction for forward jet production is rather sensitive to the details of the gluon splitting function.

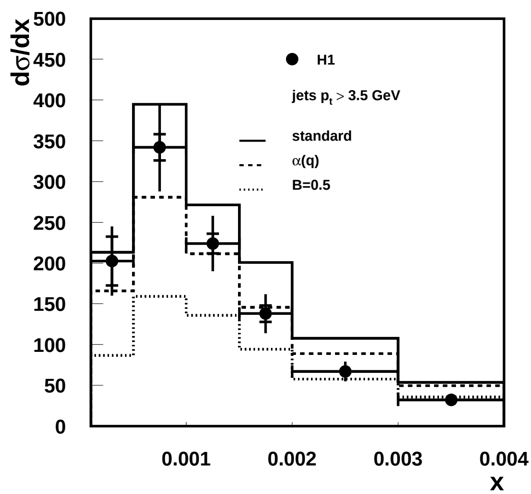

Fig. 5. The cross section for forward jet production as a function of x, compared to H1 data [6].

This paper is dedicated to the memory of Bo Andersson, who died unexpectedly from a heart attack on March 4th, 2002. I have learned so much from him. I am very grateful to G. Salam for all his ideas and advice concerning CCFM and the next-to-leading contributions, which formed the basis for this contribution.

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