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STATUS OF LATTICE STRUCTURE FUNCTION CALCULATIONS*

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Lattice QCD allows computations of moments of structure functions from first principles. An overview of the present status of the calculations is given. Recent results and future perspectives are discussed.

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1. Introduction

The moments of structure functions can be calculated from first principles, without making any model assumptions, using the non-perturbative techniques of lattice QCD. Numerous results have been obtained from lattice computations in the last years. They include the calculation of the lowest moments of the unpolarized structure functions, of the spin-dependent g_1 and g_2 structure functions and of the h_1 transversity structure function. Some higher-twist matrix elements have been studied as well.

The novelties of the last couple of years are the first computations of structure functions done in full QCD (unquenched), and new proposals for the extrapolations to the chiral limit using chiral perturbation theory. A lot of work, however, still needs to be done in order to control other systematic uncertainties like the continuum limit or the non-perturbative renormalization.

2. Structure functions on the lattice

It is not possible to compute the structure functions directly on the lattice, as they describe the physics close to the light cone, and Monte Carlo simulations are instead done in Euclidean space. One can, however, calculate their moments via OPEs which have the general form

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$$\int_0^1 x^n \mathcal{F}_i\left(x, Q^2\right) \sim C_{i,n}\left(\frac{Q^2}{\mu^2}\right) \langle h | O_n(\mu) | h \rangle . \tag{1}$$

The Wilson coefficients $C_{i,n}$ contain the short-distance physics, calculable in continuum perturbation theory. The long-distance physics is contained in the hadronic matrix elements, which can be computed doing Monte Carlo simulations and then performing a proper lattice renormalization.

The expansions of Eq. (1) are dominated by operators of twist two. The moments of the unpolarized quark distributions, of the spin-dependent structure function g_1, of the d_2 component of g_2 and of the h_1 transversity structure function are respectively measured by the following operators

$$\langle x^n \rangle \sim \langle h | \overline{\psi} \gamma_{\{\mu} D_{\mu_1} \cdots D_{\mu_n\}} \psi | h \rangle , \tag{2a}$$

$$\langle (\Delta x)^n \rangle \sim \langle h | \overline{\psi} \gamma_5 \gamma_{\{\mu} D_{\mu_1} \cdots D_{\mu_n\}} \psi | h \rangle , \tag{2b}$$

$$\langle x^n \rangle_{d_2} \sim \langle h | \overline{\psi} \gamma_5 \gamma_{[\mu} D_{\{\mu_1} D_{\mu_2]} \cdots D_{\mu_n\}} \psi | h \rangle , \tag{2c}$$

$$\langle (\delta x)^n \rangle \sim \langle h | \overline{\psi} \gamma_5 \sigma_{\mu\{\mu_1} D_{\mu_2} \cdots D_{\mu_n\}} \psi | h \rangle . \tag{2d}$$

The moments of the unpolarized gluon distribution

$$\langle x^n \rangle_g \sim \left\langle h \middle| \sum_\rho \mathrm{Tr}\left(F_\mu{}^\rho D_{\mu_2} \cdots D_{\mu_n} F_{\rho\mu_1}\right) \middle| h \right\rangle , \tag{3}$$

which in the flavor singlet case mix with the moments in Eq. (2a), have been quite hard to compute on the lattice up to now, and for this reason only non-singlet quark distributions are generally considered.

Structure functions calculations on the lattice have been done with Wilson fermions which, however, break chiral symmetry. This causes an additive mass renormalization even for vanishing bare masses, and a heavy lattice pion ($m_\pi \sim 500$ MeV). Therefore, one needs to perform extrapolations to the chiral limit, and this systematic error has to be controlled[1].

Extrapolations to the continuum limit $a \to 0$ need also to be done. To get a faster convergence to the continuum and decrease the systematic error arising from the finiteness of the lattice spacing, $O(a)$ improvement has been implemented in most cases: the contributions of $O(a)$ are removed so that one has

$$\left\langle p \middle| \widehat{\mathcal{O}}_L \middle| p' \right\rangle_{\mathrm{MC}} = a^d \left[\left\langle p \middle| \widehat{\mathcal{O}} \middle| p' \right\rangle_{\mathrm{phys}} + O(a^2) \right] . \tag{4}$$

[1] Recent formulations of chiral fermions on the lattice, called Ginsparg–Wilson (in particular: overlap, domain-wall and fixed-point fermions), possess exact chiral symmetry also at finite lattice spacing and thus do not suffer from this problem.

This is achieved in on-shell matrix elements by adding a counterterm to the Wilson action, $\Delta S_I^f = c_{sw}\, \mathrm{i} g_0 a^4\, r/4a\, \sum_{x,\mu\nu} \overline{\psi}(x)\sigma_{\mu\nu} F_{\mu\nu}^{\rm clover}(x)\psi(x)$. The improvement coefficient c_{sw}, which has to be exactly tuned so as to cancel all $O(a)$ contributions coming from the action, is now well known. The various operators have to be improved as well, and this is achieved by adding as counterterms bases of higher-dimensional operators with the same symmetries as the original unimproved ones: $O^{\rm imp} = (1 + b_O\, am)\, O + a \sum_i c_i\, \widetilde{O}_i$ (with $\dim(\widetilde{O}_i) = \dim(O) + 1$), and determining the coefficients b_O and c_i.

After the matrix elements have been simulated with Monte Carlo algorithms, they need to be renormalized from the lattice to the continuum

$$\langle O_i^{\rm cont} \rangle = \sum_j \left(\delta_{ij} - \frac{g_0^2}{16\pi^2} \Big(R_{ij}^{\rm lat} - R_{ij}^{\rm cont} \Big) \right) \Big\langle O_j^{\rm lat} \Big\rangle \, . \tag{5}$$

Since Lorentz and (for Wilson fermions) chiral symmetry are broken on the lattice, mixing of operators under renormalization is more complicated than in the continuum. Much effort has been spent on computing perturbative renormalization factors, which are now known for the lowest three moments of all structure functions in the case of Wilson (often in the $O(a)$ improved theory) and of overlap fermions. For renormalization calculations see Ref. [1] and references therein. The relevant perturbative renormalization factors have been calculated also for some classes of higher-twist operators [2].

Non-perturbative renormalization has instead been quite hard to do in the standard Wilson case, where it is difficult to find a plateau for extracting the signal. This is not the case in the Schrödinger Functional approach, where one has a better control over many systematic errors, although the computation of the running of renormalization factors is quite tedious [3].

For reasons of computing power, structure functions have been studied mostly in the quenched approximation, and only recently full QCD computations have become feasible. On the lattice the (Grassmann) fermion variables are integrated out, and in full QCD one uses the partition function

$$Z = \int \mathcal{D}U \; \det(\not{D}[U] + m_q)\, \mathrm{e}^{-S_g[U]} \, , \qquad \Big(U_\mu = \mathrm{e}^{\mathrm{i} a g_0 A_\mu} \Big) \, . \tag{6}$$

Quenching instead amounts to putting $\det(\not{D}[U] + m_q) = 1$ in the simulations. In physical terms, this means that there are no sea quarks in the calculations: the internal quark loops are neglected. Although it looks quite drastic, in many cases this does not turn out to be a bad approximation.

3. Results

Various collaborations have been studying structure functions on the lattice in the last years: the QCDSF Collaboration, the LHPC Collaboration, and a collaboration which uses the Schrödinger Functional scheme. See Refs. [3–5] respectively for recent works and further references.

The major advancement in the last couple of years has been the computation of the structure functions in full QCD. The first studies with dynamical fermions have shown that for various structure functions of the u and d quarks there are no statistically significant differences between quenched and full QCD results. This was discovered by the LHPC Collaboration and then confirmed by the QCDSF Collaboration. We then have two calculations in full QCD with the same message: the results for quenched and unquenched moments are nearly equal, and the observed discrepancies of lattice results with experiment are still there. Quenching had been previously conjectured as the cause of these discrepancies, but evidently this is not the case, at least for the values of quark masses attainable at present, which are unfortunately quite large.

Usually only the lowest three or so moments of the various structure functions can be calculated, due to increasing computational difficulties and mixing problems. The largest discrepancies concern unpolarized quark distributions. The spin-dependent g_1 and g_2 structure functions have been extensively studied as well, and also the axial charge $g_{\rm A} = \Delta u - \Delta d$, whose result is not so far from experiment. The g_2 structure function with Wilson fermions presents a mixing problem due to chirality breaking[2]: there is a mixing with lower-dimensional operators, which gives rise to power divergences $\sim 1/a^n$ in the continuum limit. The h_1 transversity structure function, whose lowest moment is the tensor charge, has also been studied on the lattice. A recent lattice result for this quantity, which has never been measured, is $\delta u - \delta d = 1.21(4)$ [6]. An experimental measure of the h_1 transversity structure functions would therefore, be quite interesting.

The results coming from lattice QCD are getting more and more precise, so it would also be useful to have more precise measurements of parton distributions and above all a careful analysis of their errors. Some first studies in this direction have been presented at this conference [7].

Nevertheless, on current lattices there are still some practical limitations. One of them is given by the difficulty to compute disconnected diagrams (*i.e.* connected, but only by gluon lines). Only exploratory studies have been made so far, and these diagrams have not yet been included in lattice computations. They are, however, flavor independent and therefore, they

[2] With Ginsparg–Wilson fermions this mixing is thus forbidden.

do not contribute to the differences between u and d structure functions (for $m_u = m_d$, which is the case). This means that quantities like $g_{\rm A} = \Delta u - \Delta d$ and $\langle x^n \rangle_{u-d}$ do not receive contributions from disconnected diagrams.

Another limitation is given by the extrapolations to the chiral limit. Chiral and continuum extrapolations are performed using the fit formula

$$A + B\, m_\pi^2 + C\, a^2\,, \tag{7}$$

and doing just the naive chiral extrapolations in m_π^2 could also turn out to be an explanation for the discrepancies. The extrapolations to the continuum limit are quadratic in the lattice spacing when improved fermions are used.

There have also been quenched calculations which use the Schrödinger Functional. This is a finite volume scheme (with Dirichlet boundary conditions on the time direction) in which it is possible to do simulations at very small quark masses, and the pion is much lighter than in the standard Wilson case. The renormalization scale is identified with the inverse size of the lattice, $\mu = 1/L$, which allows one to apply finite-size scaling techniques; there is no need to go to the infinite volume limit. In the Schrödinger Functional calculations the renormalization of operators can be done non-perturbatively, at scales extending over some orders of magnitude. The results for the lowest moment of the unpolarized structure function of the pion, in the $\overline{\rm MS}$ scheme at the renormalization scale $\mu = 2.4$ GeV, is $\langle x \rangle = 0.30 \pm 0.03$, while the experimental number is $\langle x \rangle = 0.23 \pm 0.02$.

4. Chiral extrapolations

It has recently been suggested that extrapolations of the lattice data using chiral perturbation theory could solve the discrepancy with experiment.

It seems that the pion cloud of the nucleon is not adequately described by current lattices. The IR behavior of pion loops generates chiral logs $\sim m_\pi^2 \ln m_\pi^2$, and using chiral perturbation theory [8] one gets for the unpolarized case $\langle x^n \rangle_{u-d} \sim A_n \left[1 - (3g_{\rm A}^2+1)/(4\pi f_\pi)^2\, m_\pi^2 \, \ln m_\pi^2\right]$. Introducing in this non-analytic term a phenomenological cutoff Λ (the size of the source generating the pion cloud) one then arrives at the fit formula [9]

$$\langle x^n \rangle_{u-d} = A_n \left[1 - \frac{\left(3g_{\rm A}^2+1\right)}{\left(4\pi f_\pi\right)^2}\, m_\pi^2 \, \ln\left(\frac{m_\pi^2}{m_\pi^2 + \Lambda^2}\right)\right] + B_n\, m_\pi^2 + C_n\, a^2\,. \tag{8}$$

It looks like phenomenological extrapolations which use chiral perturbation theory could solve the discrepancy with experiment. However, they need to be better investigated, and although they have the potential to explain the discrepancy, it is still too soon to say something certain[3].

So far we are able to reproduce the phenomenological results only at posteriori. The corresponding values of Λ lie between 300 MeV and 700 MeV, so Λ does not seem at present to have some predictive power. Moreover, the currently available lattice data do not even discriminate between naive chiral fits, Eq. (7), and chiral perturbation theory fits, Eq. (8). The problem is that in present simulations the pions are not sufficiently light. A smaller pion mass is needed ($m_\pi < 250$ MeV) in order that the parameters of the chiral expansions can be well determined on the lattice. For such determinations a computing power of about 8 Teraflops for one year is required [9]. The next generation of computers, which will come in a couple of years, should then be able to perform these calculations.

For the pion cloud of the proton to be adequately included in the lattice box and properly measured in Monte Carlo simulations, the pion correlation length should also be smaller than the lattice size, otherwise it will not be fully contained in the lattice. The length of a spatial dimension for currently avaliable lattices is about 2 fermi, so it would be very useful to do simulations also with larger physical volumes and minimize finite volume effects[4].

The pion cloud will be the focus of many future lattice investigations.

5. Higher twist

There are a few lattice results regarding higher-twist corrections for the pion and the nucleon [2]. They show that, for the twist-four contributions coming from 4-fermion operators like $\sum_{\rm A} \overline{\psi}\gamma_\mu\gamma_5\, t^{\rm A}\psi \cdot \overline{\psi}\gamma_\nu\gamma_5\, t^{\rm A}\psi$, the $1/Q^2$ power corrections are, at least in these particular cases, quite small.

The calculations give, for a particular contribution ($I = 2$) to the twist-4 matrix element of the first moment of the unpolarized pion distribution,

$$\int_0^1 dx\, F_2\left(x, Q^2\right)\Big|_{I=2} = 1.67(64)\ \alpha_{\rm s}\left(Q^2\right)\ \frac{f_\pi^2}{Q^2}\,, \tag{9}$$

[3] A recent calculation [10] seems also to show that in the polarized case the next order corrections give a fit formula which deviates very little from the naive extrapolations.

[4] We should also mention that the lattice results for elastic form factors seem to have discrepancy problems similar to the structure functions, particularly in the case of the magnetic form factor. It is likely that in this case too the pion cloud is not being properly simulated and finite-size effects are present.

and for the proton, for a particular class of operators (flavor **27**, $I=1$),

$$\int_0^1 dx\, F_2\left(x,Q^2\right)\Big|_{\mathbf{27},I=1} = -0.0006(5)\ \alpha_{\rm s}\left(Q^2\right)\frac{m_P^2}{Q^2}\,. \qquad (10)$$

These special flavor and isospin combinations are chosen to avoid mixing with lower-dimensional operators. The above twist-4 numbers are much smaller than the twist-2 lattice matrix element (at $Q^2=4\,\mathrm{GeV}^2$) of the pion, $\int_0^1 dx\, F_2(x,Q^2) = 0.152(7)$, and of the proton, which is about 0.14.

We point out that these studies, contrary to leading-twist operators, are not complete and systematic. Mixing problems have so far limited the calculations only to a restricted class of operators.

6. Perspectives

The lattice calculations of structure functions are getting more and more refined. However, the physics of the pion cloud must be properly included, and simulations on lattices of large physical volumes as well as proper extrapolations to the chiral limit will need to be done.

Ginsparg–Wilson fermions possess exact chiral symmetry and could help to study the chiral limit. Calculations with the Schrödinger Functional will also be useful for a better understanding of the approach to the chiral limit.

Contributions of higher-twist operators to moments of parton distributions seem to be quite small, but they have not been systematically studied. They will continue to be a challenge for lattice QCD for a few more years.

Disconnected diagrams will have to be included in the simulations.

An almost virgin territory for the lattice is the study of the momentum and spin distributions of the gluon, which so far have been plagued in Monte Carlo simulations by large statistical errors. It would be quite interesting to know them. The lattice distributions of sea quarks are also unknown.

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