ANALYTIC DESCRIPTION OF PROTON STRUCTURE FUNCTIONS IN THE WHOLE KINEMATIC REGION*

A.A. Petrukhin and D.A. Timashkov

Moscow Engineering Physics Institute
Moscow, 115409, Russia
e-mail: timashkov@nevod.mephi.ru

(Received July 1, 2002)

A new approach based on a search of analytic solutions for the main limiting cases and of linking functions describing intermediate region is developed. In frame of this work, analytical formulas for proton form factors in the limiting cases (photoproduction, quasielastic scattering and low-x limit) have been obtained. Connections and transitions between these limiting cases have been studied and linking functions have been found. The obtained dependencies of proton structure functions for the whole kinematic region of variables Q^2 and x contain practically no free parameters but they are in a good agreement with experimental data.

PACS numbers: 13.60.–r, 13.60.Hb

In many cases simple and convenient formulas for proton structure functions which describe their behavior in the whole kinematic region are very useful. At present, for these purposes various fits [1,2] of experimental data are used. These fits contain many free parameters determined from experiments[1]. However, when new experimental data appear these parameters are changed. One of the main reasons is that in these fits the limiting dependencies usually are not regarded. The purpose of this work is to obtain phenomenological description of F_2 in a whole kinematic region, taking into account structure function behavior in limiting cases.

There are three main limiting cases for inelastic scattering: photoproduction, quasielastic limit, low $x_{\rm B}$[2] limit (high energies) and for each of them corresponding model exists: vector dominance [3], standard DGLAP equations [4], Pomeron conception [5].

* Presented at the X International Workshop on Deep Inelastic Scattering (DIS2002) Cracow, Poland, 30 April–4 May, 2002.

[1] Usually only the results of F_2 investigations are fitted, since another function, F_1, is related with F_2 by Callan-Gross relation or on the basis of QCD considerations.

[2] $x_{\rm B}$ is a Bjorken variable.

For the first limiting case (low Q^2 limit) the close connection between structure functions and photoproduction cross section was used [6]. In the frame of vector meson dominance model, the cross section of virtual photon may be written in a following form:

$$\sigma_{\gamma^* p}(Q^2,\nu) = \alpha_e \int\limits_{m_0^2}^{\infty} \frac{\rho\left(m^2,\nu\right)}{\left(1+Q^2/m^2\right)^2}\, dm^2 \,. \tag{1}$$

Here ρ is dynamic density of vector meson states. The lower limit in the integral has a sense of squared mass of the lightest state in meson spectrum:

$$m_0^2 = \left(m_\rho - \frac{\Gamma_\rho}{2}\right)^2 = 0.483\ \mathrm{GeV}^2. \tag{2}$$

Basing on Regge theory and dimensional reasons, the density function is chosen as

$$\rho\left(m^2,W^2\right) = \frac{\pi^3}{m^4}\left(\frac{W^2}{m^2}\right)^{\alpha-1} . \tag{3}$$

Here α is effective intercept, which provides the transition from reggeon intercept $\alpha_{\mathbb{R}} = 0.5$ for low energies to Pomeron intercept $\alpha_{\mathbb{P}} \sim 1$ for high energies with logarithmic dependence on energy:

$$\alpha(W^2) \;=\; \alpha_{\mathbb{R}} + (\alpha_{\mathbb{P}} - \alpha_{\mathbb{R}})\, \frac{f(W^2)}{1+f(W^2)}\,, \tag{4a}$$

$$f(W^2) \;=\; \ln\sqrt{1+\frac{W^2-M^2}{m^2}}\,. \tag{4b}$$

Substituting (3) into (1), and taking into account the connection between F_2 and $\sigma_{\gamma^* p}$ one can obtain the following expression for F_2 at low Q^2:

$$F_2\left(x_{\mathrm{B}},Q^2\right) = \frac{Q^2}{4\pi^2\alpha_e}\sigma_{\gamma^* p} = \frac{\pi}{4}x_{\mathrm{T}} \int\limits_{u_0}^{\infty} \frac{u^{1-\alpha}du}{(u+x_{\mathrm{T}})^2}\,, \quad u = \frac{m^2}{Q^2}x_{\mathrm{T}}\,, \tag{5}$$

$$x_{\mathrm{T}} = \frac{x_{\mathrm{B}}}{1+x_{\mathrm{B}}M^2/Q^2}\,. \tag{6}$$

The comparison with data [7] is shown in Fig. 1.

In this approach the formula for photoproduction cross section becomes very simple (in this case $W^2 = s = M^2 + 2ME_\gamma$):

$$\sigma_{\gamma p} = \frac{\pi^3\alpha_e}{s}\int\limits_{u_0}^{\infty}\frac{du}{u^{1+\alpha(u)}}\,, \qquad u = \frac{m^2}{s}\,. \tag{7}$$

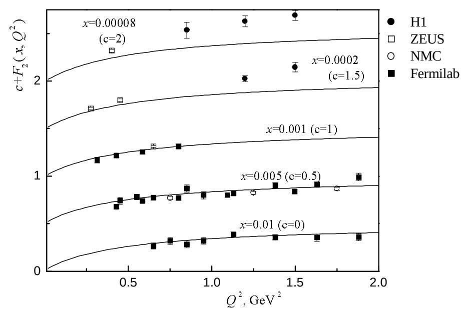

Fig. 1. Proton structure function in low Q^2 region. Experimental points from [7].

It is necessary to make an important remark. While value of $\alpha_{\mathbb{R}}$ is constant, the estimate of $\alpha_{\mathbb{P}}$ increases with energy explored in experiments. To describe this growth, effective Pomeron intercept may be used:

$$\alpha_{\mathbb{P}}^{\mathrm{eff}} = 1 + k_0\sqrt{\ln\left(\frac{s}{M^2}\right)}\,. \tag{8}$$

In particular, experimental data on photoproduction cross section presented by ZEUS collaboration [8] can be described with $k_0 = 0.028$. More detailed consideration of low Q^2 limit may be found in [9].

The second limiting case corresponds to quasielastic region with x_{B} close to unity. In this region the evolution equation for valence quarks distribution function q_v is usually used. As it is shown in [10] this equation can be written in a form analogous to cascade theory equation:

$$\frac{dq_v\left(x,Q^2\right)}{d\ln Q^2} = \frac{\alpha_S\left(Q^2\right)}{2\pi}\left[\int\limits_x^1 \frac{dy}{y} q_v(y) P_v\left(\frac{x}{y}\right) - q_v(x)\int\limits_0^1 dy P_v(y)\right]\,, \tag{9}$$

$$P_v(z) = \frac{4}{3}\,\frac{1+z^2}{1-z},$$

x corresponding to "energy" of particle and logarithm of Q^2 corresponding to "depth". The method of solution of such equations is known from cascade theory. After choosing initial conditions like $\sim x^k(1-x)^{n_0}$, exact analytic solution for F_2 in quasielastic limit has been obtained[3] [10]:

$$F_2\left(x,Q^2\right) = F_2\left(x,Q_0^2\right)(1-x)^{\tau}\, G_0(\tau), \tag{10}$$

$$G_0(\tau) = \frac{\Gamma\left(n_0+1\right)}{\Gamma\left(n_0+1+\tau\right)} \mathrm{e}^{(3/4-\gamma)\tau}, \quad \tau = \frac{16}{33-2n_f}\ln\frac{\ln Q^2/\Lambda^2}{\ln Q_0^2/\Lambda^2}. \tag{11}$$

[3] $n_0 = 2n_s - 1 = 3$; here n_s is number of spectators, for proton $n_s = 2$.

To use this analytic solution for description of large $x_{\rm B}$ region it is necessary to take into account mass corrections concerned with target mass. It appears as modification of scaling variable from $x_{\rm B}$ to another scaling variables: Feynman's or Nachtman's one. For light quarks they coincide:

$$x_{\rm F} = \xi = \frac{2x_{\rm B}}{1+\sqrt{1+4M^2x_{\rm B}^2/Q^2}}\,. \tag{12}$$

Although mass corrections do not allow to obtain exact solution like (10), we take them into account by means of direct substitution $x_{\rm F}$ for $x_{\rm B}$ in analytic solution. Results of calculations using formula (10) are compared with experimental points at large $x_{\rm B}$ in Fig. 2.

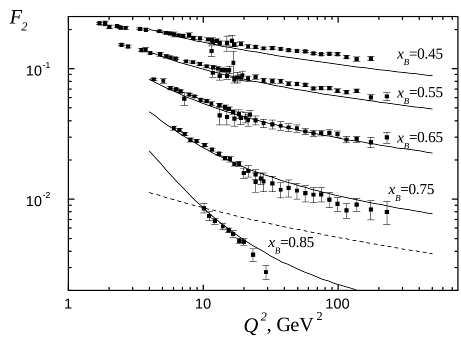

Fig. 2. Structure function for high $x_{\rm B}$ with (solid curves) and without (dashed curve) mass corrections. Experimental data from BCDMS and SLAC (see [7]).

To connect these two main limiting cases, one can use (5) as initial condition for (10). But there are two problems: different variables x in photoproduction limit and quasielastic one, and the absence of the function G_0 at low Q^2. To solve the first problem, we have supposed that two variables x_F and $x_{\rm T}$ are particular cases of some unified variable which can be written in the following form:

$$x_{\rm P} = \frac{2x_{\rm B}}{1+\sqrt{1+4M^2x_{\rm B}^{1+x_{\rm B}}/Q^2}}\,. \tag{13}$$

This expression gives smooth transition from (6) at $x_{\rm B}\to 0$ to (12) at $x_{\rm B}\to 1$.

To solve the second problem we used a linking function. This function has been chosen in the simplest linear form which is suitable for satisfactory description of F_2 in intermediate region as it is shown below:

$$G\left(x_{\rm B},\tau\right) = G_0(\tau)x_{\rm B} + \left(1-x_{\rm B}\right). \tag{14}$$

As a result, proton structure function assumed a following form:

$$F_2\left(x_{\mathrm{B}}, Q^2\right) = \frac{\pi}{4} G\left(x_{\mathrm{B}}, \tau\right) x_{\mathrm{P}} (1 - x_{\mathrm{P}})^{n_0 + \tau} \int\limits_{u_0}^{\infty} \frac{u^{1-\alpha(u)} du}{\left(u + x_{\mathrm{P}}\right)^2} . \tag{15}$$

Here τ is determined in (11), x_{P} is given by (13), $G(x_{\mathrm{B}}, \tau)$ is described by (14) and $u = m^2 x_{\mathrm{P}}/Q^2$. Before comparing this formula with experimental data, it is necessary to consider the limit of $x_{\mathrm{B}} \to 0$.

Low x_{B} limit, which corresponds to large transferred energies ν, may be divided in two regions: high (or medium) Q^2 region (perturbative) and low Q^2 region (non-perturbative). Usually, the first region, where sea quarks give the main contribution to structure function and provide significant growth of F_2, is considered. To describe this region various formulas containing double-logarithmic terms (as $\sim \ln Q^2 \ln 1/x$) [11] were proposed. But the same growth has been observed in high energy limit for photoproduction in the second (non-perturbative) region, that was described by growth of Pomeron intercept (8). It is obvious, that transition between perturbative and non-perturbative regions must be smooth. Therefore, considering expression (8) as boundary condition and taking into account proposed earlier formulas [12], effective Pomeron intercept may be written as:

$$\alpha_{\mathbb{P}}^{\mathrm{eff}} = 1 + k_0 \left(1 + \ln \sqrt{1 + \frac{Q^2}{m_0^2}}\right) \sqrt{\ln\left(1 + \frac{1}{x_{\mathrm{P}}} \frac{Q^2}{M^2 + Q^2}\right)} . \tag{16}$$

Two remarks about high energy region. **1**. The only coefficient k_0, which can be considered as a free parameter is not usual free parameter. Its value was evaluated from photoproduction data, but it describes the large Q^2 region in inelastic structure function very well, too. Therefore we hope that this parameter will be determined at further development of Pomeron theory. **2**. More general question is connected with Regge-like behavior of the cross section. Power dependence on energy contradicts Froissart bound [13] and can be used only in a limited interval of energy. But this problem exists not only for inelastic lepton scattering, and requires further investigations.

Expression (15) with effective Pomeron intercept (16) provides correct behavior of F_2 in the whole kinematic region (Fig. 3). It is important to note that this result was obtained practically without usual fitting procedures.

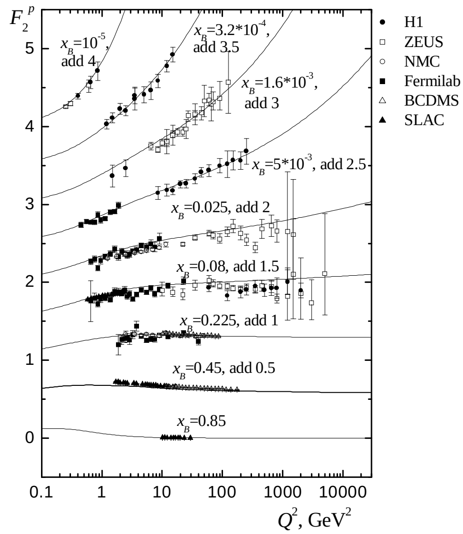

Fig. 3. Structure function in the whole kinematic region. Experimental data from [7].

Authors thank the organizers of DIS02 Conference for the partial support of participation.

REFERENCES

[1] CTEQ Collaboration, J. Pumplin *et al.*, `hep-ph/0201195`.

[2] A.D. Martin, R.G. Roberts, W.S. Stirling, R.S. Thorne, *Eur. Phys. J.* **C23**, 73 (2002).

[3] G. Cvetic, D. Schildknecht, A. Shoshi, *Acta Phys. Pol.* **B30**, 3265 (1999).

[4] V.N. Gribov, L.N. Lipatov, *Sov. J. Nucl. Phys.* **15**, 438 and 675 (1972); L.N. Lipatov, *Sov. J. Nucl. Phys.* **20**, 93 (1975); G. Altarelli, G. Parisi, *Nucl. Phys.* **B126**, 298 (1977); Yu.L. Dokshitzer, *Sov. Phys. JETP* **46**, 641 (1977).

[5] A.B. Kaidalov, `hep-ph/0103011`.

[6] L.N. Hand, *Phys. Rev.* **129**, 1834 (1963).

[7] Particle Data Group, *Eur. Phys. J.* **C15**, 220 and referencies therein.

[8] C. Amelung, *Nucl. Phys. B (Proc. Suppl.)* **79**, 176 (1999).

[9] A.A. Petrukhin, D.A. Timashkov, preprint MEPhI 001-2001, to be published in *Sov. J. Nucl. Phys.*

[10] S.R. Kelner, D.A. Timashkov, *Sov. J. Nucl. Phys.* **64**, 1802 (2001).

[11] E. Gotsman *et al.*, *Nucl. Phys.* **B539**, 535 (1999).

[12] D. Haidt, *Nucl. Phys. B (Proc. Suppl.)* **79**, 186 (1999).

[13] M. Froissart, *Phys. Rev.* **123**, 1053 (1961).