

# SATURATION FROM NONLINEAR pQCD AT SMALL $x$ IN $ep$ AND $eA$ PROCESSES\*

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In this talk we discuss inclusive  $ep, eA$  scattering in the framework of nonlinear, small  $x$ , pQCD, in particular the natural emergence of nuclear shadowing within this framework through simple rescaling of the natural scaling variable  $\tau$ , in this approach, by  $A^\delta$ . We then compare this approach to other popular approaches to nuclear shadowing like the eikonal approximation or leading twist calculations.

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## 1. Introduction

The question of the onset of saturation/unitarization in inclusive QCD observables, in particular in the proton structure function  $F_2(x, Q^2)$ , has been the subject of active discussions for many years now (see a detailed discussion of this subject in [1,2] and references therein.). The most promising approach to properly include saturation/unitarization effects within the framework of QCD is the JIMWLK equation [1] which resums the leading  $\ln(1/x)$  terms in *all*  $N$ -point correlators functions of the participating fields in the process in question, *i.e.* not just the leading correlators as in  $k_\perp$ -factorization. This leads to non-linearities in a RGE for the generating functional for these  $N$ -point correlators which slows down the rapid growth of structure functions at small  $x$  due to the presents of an infrared stable fixpoint which bounds the equation from below. When linearizing this JIMWLK equation one obtains the well-known BFKL equation (see [3] for details), which, however, gives to strong a growth in  $F_2(x, Q^2)$ , precisely because it does *not* have an infrared fixpoint bounding the equation from

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below. Evolution in the nonlinear pQCD approach is among other things *target independent*. Target dependence arises only from the initial conditions, the distribution of gluons in the transverse plane, which will be  $A$  dependent. This allows us to compare  $ep$  and  $eA$  scattering.

The basic picture in the approach to small  $x$  resummation is quite similar in all the various approaches: The virtual photon, in the infinite momentum frame split into a  $q\bar{q}$  color-dipole (treated as eikonized Wilsonlines) of, in principle, arbitrary size, which then punches through, and interacts with, the target, which it sees as a pancake of infinitesimal width but with the targets, gluonic spectator fields spread densely in the transverse plane. The number of these gluons increases during evolution.

The kinematic situation underlying the evolution picture translates into a formula for the  $\gamma^*p$  cross section or alternatively for  $F_2$  (here for a proton target),

$$\sigma_{\text{tot}}(x, Q^2) = N \int d^2z \int_0^1 d\alpha |\psi_{\gamma^*}(z^2 \alpha(1-\alpha)Q^2)|^2 \sigma_{\text{dipole}}(z^2 Q_s(x, x_0, Q_0)) \quad (1)$$

with  $\mathbf{z} = \mathbf{x} - \mathbf{y}$ , where  $\psi_{\gamma^*}$  encodes the information about the color dipole and  $\sigma_{\text{dipole}}$  contains the information about the interaction of the dipole with the spectator fields and which is obtained through the JIMWLK equation. As mentioned above, the JIMWLK equation can be recast as an equation for a generating functional of the Fokker–Plank type which in turn can be easily rewritten as a Langevin equation with white noise, at least in leading order of  $\ln(1/x)$ . This type of equation can be discretized and solved numerically [4].  $Q_s = (x/x_0)^\lambda Q_0$  is a saturation scale and  $\lambda$  can be obtained from the numerical solution of the JIMWLK equation.  $N$  is a normalization constant which has to be fitted to data at a  $x_0$  and  $Q_0$ .

## 2. Nuclear shadowing within nonlinear pQCD and other approaches

Nuclear shadowing can be easily incorporated in this approach by realizing that the impact parameter integral will yield an  $A$  dependence via a simple rescaling of  $\mathbf{z}$  and  $Q$  by  $A^\delta$  where  $\delta = 1/3$  if the distribution of spectator partons in the target were homogeneous. Since this is a priori not necessarily the case, we leave this parameter to be determined by data. Eq. (1) then turns into

$$F_2^A(\tau) = N(A) \left(\frac{x}{x_0}\right)^{2\lambda} F_2^p(x_0, \tau(x, Q^2, A, Q_0^2)), \quad (2)$$

with  $\tau = \left(\frac{x}{x_0}\right)^{2\lambda} \left(\frac{Q}{A^\delta Q_0}\right)^2$  and where the normalization  $N(A)$  has to be determined for some  $x_0$  and  $Q_0$  from data. In fact normalizing the  $F_2^p$  one can use the same  $x_0$  and  $Q_0$  as for the normalization of  $F_2^p$ . Thus we have an unambiguous prediction for  $F_2^A$  which says that not only can the observed nuclear shadowing be explained by a simple rescaling of the variable  $\tau$  with  $A$ , but also that, after proper normalization, all  $F_2^A$  data should, plotted *vs.*  $\tau$ , lie on the same line as the data for  $F_2^p$ . Thus we also predict geometric scaling in inclusive  $eA$  scattering.

**The eikonal approach:**

The main equation of this approach is

$$\text{Im } A_{e-A} \propto |\psi_{\gamma^* \rightarrow q\bar{q}}|^2 \otimes \exp\left(\sigma_{q\bar{q}-N}^{\text{tot}}\right) \tag{3}$$

which basically says the following two things:

- (a) the  $q\bar{q} - N$  interaction does *not* change the transverse size or momentum fractions of the dipole and
- (b) that no higher Fock components like  $q\bar{q}g$  contribute to the cross section.

**The leading twist approach:**

Gribov observed that if  $R_{\text{hadronic}} \ll R_{N-N}$  in  $A$  then there is a *direct* relationship between nuclear shadowing in  $N - A$  collisions and  $\sigma_{N-N}^{\text{diff}}$ . Furthermore, there is a generalization [6] to calculate the leading twist component of nuclear shadowing for each nuclear parton distribution separately through the factorizable, *diffractive*  $F_2^{D(4)}$ :

$$\begin{aligned} \delta F_2^A = AF_2 - F_2^A = \frac{16\pi A(A-1)}{2} \text{Re} & \left[ \frac{(1-i\eta)^2}{1+\eta^2} \int d^2b \int_{z_1} dz_1 \int_x dz_2 \int dx_{\mathbb{P}} \right. \\ & \left. \times F_2^{D(4)}(\beta, Q^2, x_{\mathbb{P}}, k_{\perp}^2 = 0) \rho(b, z_1) \rho(b, z_2) e^{(x_{\mathbb{P}} m_N (z_1 - z_2))} \right]. \end{aligned} \tag{4}$$

In the following we compare the nonlinear pQCD and the leading twist approach with one another as well as with data (here only NMC data for  $x \leq 0.05$  was used). We will plot  $F_2^A(t)$  *vs.*  $\tau$ , normalized to  $F_2^p$ , where, for the leading twist approach [7], we choose the same value for  $\lambda$ ,  $x_0$  and  $Q_0$  as in the nonlinear pQCD one:  $\lambda \sim 0.2$  for definiteness,  $x_0 = 2 \times 10^{-4}$

$(x/x_0)^{2\lambda} F_2^A$  vs.  $\tau(x, x_0, Q^2, A)$

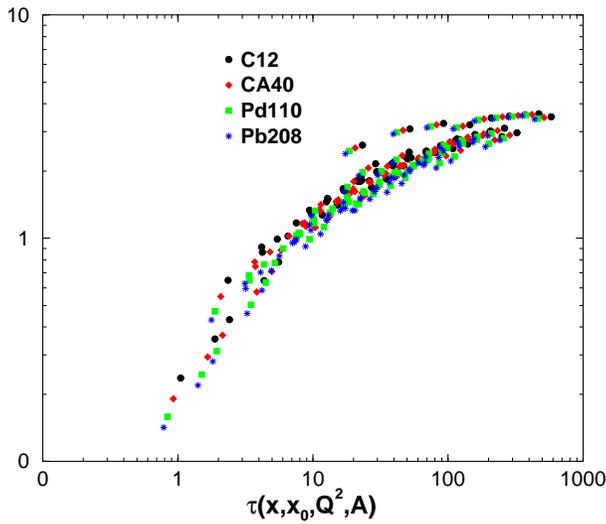


Fig. 1. Scaling plot of  $F_2^A$  from the leading twist approach.

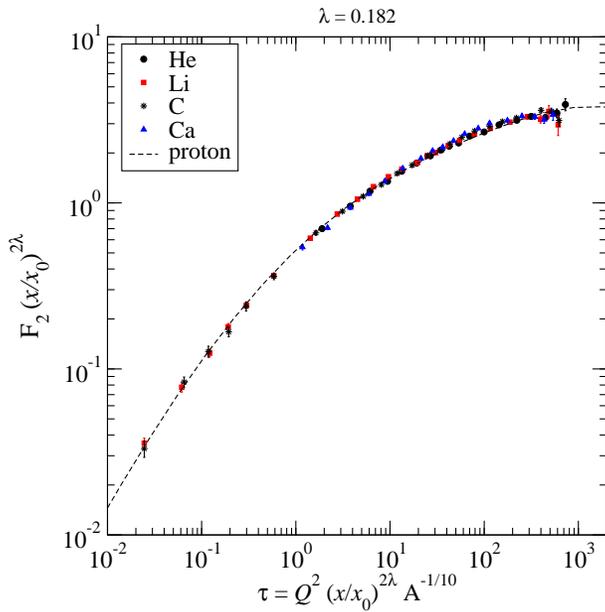


Fig. 2. Scaling plot of NMC  $F_2^A$  data.

and  $Q_0 = 1$  GeV.  $\delta$  was found from a fit to the data to be about  $\delta \simeq 0.05$ . This implies a weak  $A$  dependence in the data, implying a more granular structure of the nucleus. Note, however, that the bulk of the data on  $F_2^A$  is at small  $x$  and for light, not heavy, nuclei, thus it will be very interesting to see how the  $A$  dependence changes for larger nuclei at small  $x$ .

As can be seen from the Fig. 2, the  $F_2^A$  data indeed falls on the  $F_2^p$  curve as predicted by the nonlinear pQCD approach and thus scaling is a prominent feature of the data.

### 3. Conclusions

To summarize, the nonlinear pQCD approach makes an unambiguous prediction of the  $x$  behavior of  $ep$  and  $eA$  cross sections where only the normalization of the cross section has to be fitted to data. Furthermore, nuclear shadowing appears naturally in this approach through simple rescaling of  $\tau$  and it is predicted that the  $eA$  data lie, after proper normalization, on the same curve as the data for small  $x$   $ep$  scattering. Thus geometric scaling is predicted and indeed born out by a comparison with NMC data. A comparison with the leading twist approach, Fig. 1, shows a similar behavior in  $\tau$  but a certain scattering of points, a different normalization and in addition a steeper slope in  $\tau$  at small  $\tau$ . Thus the two approaches are qualitatively similar, however, details of saturation and initial conditions seem to be different.

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