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COMBINING MATRIX ELEMENTS AND THE DIPOLE CASCADE MODEL*

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An algorithm is presented in which the Colour–Dipole Cascade Model as implemented in the Ariadne program is corrected to match the fixed order tree-level matrix elements for $e^+e^- \to n$ jets. For the results presented here, matrix elements were used up to second order in α_{s}, but the scheme is applicable also for higher orders.

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1. Introduction

Perturbative QCD has been very successful in describing many features of multi-particle production in high energy collisions. There are, however, several problems which have not yet been solved, mostly related to the transition between the perturbative and non-perturbative description of the theory. Observables involving a few widely separated jets are in principle well described with fixed-order perturbative matrix elements (MEs) for producing a few partons. But to make precision comparisons with experiments, it is important to understand the transition of these partons to observable hadrons. Our best knowledge of this transition comes from hadronization models which describes how multi-parton states are transformed into multi-hadron ones. But for these models to work reliably one needs also a description of the soft and collinear partons describing the internal structure of widely separated jets and the soft partons between the jets.

To describe soft and collinear partons it is not feasible to use fixed-order perturbation theory. Not only do the MEs for many-parton states become extremely complicated but, since the partons are no longer widely separated, the increase in phase space introduces large logarithms which compensates

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the smallness of α_s and makes the whole perturbative expansion ill-behaved. To describe the inner structure of jets, a more practical approach is to use a parton shower (PS) procedure. Here the large logarithms are resummed to all orders at the expense of only keeping the leading logarithmic behaviour of the full matrix elements.

To get a near complete description of multi-particle production it would be desirable to combine the generation of a few widely separated partons according to fixed-order MEs with the evolution of these states according PSs and finally the transition into hadrons using a hadronization model. To do this is, however, highly non-trivial and so far there exist no general procedure which is entirely satisfactory. The main problem is that one needs a resolution scale to separate the ME generation from the PS one. This scale needs to be small enough to benefit from the full ME description, but if it becomes too small the final result is spoiled by non-physical large logarithms involving the separation scale.

In this talk I will briefly describe a new algorithm which combines fixed order tree-level matrix MEs for $e^+e^- \to n$ jets with the colour-dipole cascade model [1,2] as implemented in the ARIADNE [3] event generator. A complete description of the algorithm can be found in reference [4].

The basic idea is the same as was previously presented by Catani *et al.* in [5] where tree-level MEs were combined with the PS of HERWIG [6]. There the n-parton states generated with the AMEGIC [7] matrix element generator were reweighted with Sudakov form factors, with evolution scales reconstructed by the $k_\perp$-algorithm [8, 9]. The same scales were used in the running of α_s, and the generated states were allowed to shower according to a special *vetoed* version of the HERWIG PS. In this way they can show that the dependence on the resolution scale used in the ME generator is canceled to next-to-leading logarithmic accuracy.

The algorithm presented here differs not only in that another parton cascade is used, but also in the way the jet reconstruction is performed and how the Sudakov form factors are obtained. Rather than using the $k_\perp$-algorithm, a modified version of the DICLUS [10, 11] algorithm is used to reconstruct, not only the evolution scales, but a complete dipole cascade *history*, *i.e.* a sequence of dipole cascade emissions which would result in the n-parton state obtained from the ME generator. The Sudakov form factor is then interpreted strictly as a no-emission probability in exactly the same way as in the subsequent dipole cascade and are obtained by a special veto algorithm.

2. Reconstruction of emissions

The colour-dipole cascade model describes the emission of a gluon in terms of dipole radiation from a colour dipole between two partons. The emissions are hence described as two partons going to three, rather than one going to two as in conventional parton shower models. This means that colour coherence is automatically taken into account and that the first gluon emission in e^+e^- annihilation trivially reproduces the full first order matrix element. But there are a couple of technical details which are particular to the dipole cascade. All partons are always on-shell at each step of the cascade. The conservation of energy and momentum is achieved since both emitting partons receives a recoil from the emitted gluon. The splitting of a gluon into a $q\bar{q}$-pair is also treated as if emitted from one of the dipoles connected to the gluon, and the parton in the other end will receive some recoil in order to conserve anergy and momentum [12]. Furthermore, the scale of an emission is defined in terms of a Lorentz-invariant $p_\perp$ of the emitted gluon with respect to the emitting partons. This is defined as

$$p_\perp^2 = \frac{(s_{12} - (m_1 + m_2)^2)(s_{23} - (m_2 + m_3)^2)}{s_{123}}, \tag{1}$$

where parton 2 is the emitted one and s_{ij} and s_{ijk} are the squared invariant masses of the two- and three-parton combinations.

The DICLUS algorithm can be thought of as the inverse of the dipole cascade. In each step the combination of three jets which have the smallest invariant $p_\perp$ are clustered together into two (massless) jets.

Rather than always selecting the three parton configuration which has the smallest invariant $p_\perp$ to be reconstructed, as is customary in jet algorithms, it is possible to reconstruct all possible dipole cascade histories. This is feasible since we are dealing with only a handful partons. The procedure will then be to choose randomly between these different histories weighted with the corresponding dipole splitting probabilities in analogy to the strategy in [13]. The splitting probabilities will not include a running α_s as in the normal dipole cascade, since a constant $\alpha_{\mathrm{s}0}$ was used in the generation of the parton state. Instead the whole event is reweighted by a factor

$$\frac{1}{\alpha_{\mathrm{s}0}^{n-2}} \prod_{i=1}^{n-2} \alpha_\mathrm{s}({p_{\perp i}}^2) \tag{2}$$

to get the running of α_s with the reconstructed scales.

3. The Sudakov veto algorithm

The reconstructed scales and states are also used to calculate the correction for the Sudakov form factors. Rather than using the approximate analytic expression as a weight, we can use the fact that it corresponds to the no-emission probability in a specific region of phase space.

Consider a three-parton state generated with the $\mathcal{O}(\alpha_s)$ ME, where the scale of the gluon emission has been reconstructed to $p_{\perp 1}^2$. The Sudakov form factor is then the probability of there being no emission from the initial $q\bar{q}$ state *before* the gluon was emitted, *i.e.* at a scale above $p_{\perp 1}^2$, and that there is no emission from the $qg\bar{q}$ state between the scale $p_{\perp 1}^2$ and the cutoff in the ME. By making two trial emissions with the dipole cascade, one from the reconstructed $q\bar{q}$ state, starting from the maximum scale, and one from the ME-generated $qg\bar{q}$ state starting from $p_{\perp 1}^2$ and rejecting *the whole event* if the first was at a scale above $p_{\perp 1}^2$ or the second was inside the ME cutoff, the probability of accepting the event is exactly equal to the Sudakov form factor. With this veto procedure the proper phase space region is taken into account rather than the approximate limits in the analytic form.

4. Results

With these ingredients we can now construct the algorithm described in detail in [4] which can be used together with basically any N-parton tree-level ME generator. Although the procedure is to add a dipole cascade to the 2-, 3-, ..., N-parton states from the N-parton matrix element generator, the result is that all final multi-parton states are distrubuted as if generated by the dipole cascade, except that if the $n-2$ (with $n \leq N$) hardest emissions are inside the resolution scale, y_0 of the ME generator, their distribution is described by the exact tree-level ME.

It is clear that there should only be a small dependence on the y_0 cut of the ME since the only change when going outside the cut is that the emissions are governed by the leading logarithmic expressions rather than the exact ME and these should be very similar for a small enough cut.

Indeed when looking at standard event shapes from LEP, which are known to be reproduced at a satisfactory level by the standard ARIADNE program, there is only a small dependence on the y_0. An example is given in figure 1(a) where the result for the oblateness distribution is shown. In most cases the dependence is much smaller than the uncertainties due to hadronization parameters and the basic parameters in the dipole cascade, the cutoff $p_{\perp c}$ and Λ_{QCD}.

To really see the influence of the ME matching one must look at details in the correlations between jets. One example is the Bengtsson–Zerwas angle [14] which is not at all described by the standard ARIADNE program. In

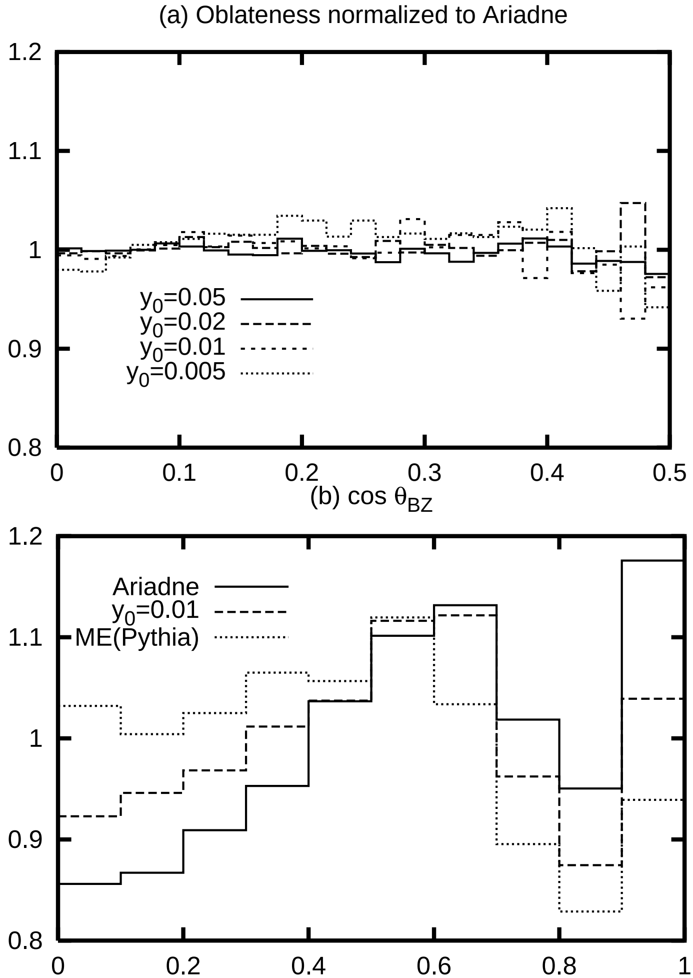

Fig. 1. (a) Ratios of the oblateness event shape on parton level at $E_{\rm CM} = 91$ GeV for the new ME matching algorithm using different values of Q_0 w.r.t. to the standard ARIADNE program. The full line is with $y_0 = Q_0^2/Q^2 = 0.05$, long-dashed: $y_0 = 0.02$, dashed: $y_0 = 0.01$ and dotted: $y_0 = 0.005$. (b) The distribution in the Bengtsson–Zerwas angle on parton level. The full line is standard ARIADNE, the dashed line is the new ME matching algorithm with $y_0 = 0.01$ and the dotted line is the tree-level $\mathcal{O}(\alpha_s^2)$ ME-only generator in PYTHIA with $y_0 = 0.01$.

figure 1(b) it is clear that the new matching procedure is closer than standard ARIADNE to the result from the pure $\mathcal{O}(\alpha_s^2)$ ME generator in PYTHIA. It does not, and should not, exactly reproduce the pure ME approach since the correlation is smeared by the subsequent soft radiation.

5. Conclusions

The algorithm presented here works well. So far it only works for $e^+e^-\to$jets but it should be possible to apply the strategy also for collisions with incoming hadrons. Investigations are underway to try to include also virtual corrections according to exact fixed-order MEs.

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