

## MULTIPLICITIES AND CORRELATIONS AT LEP\*

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A brief review on recent charge multiplicity and correlation measurements at LEP is given. The measurements of unbiased gluon jet multiplicity are discussed. Recent results on charged particle Bose–Einstein and Fermi–Dirac correlations at LEP1 are reported. New results on two-particle correlations of neutral pions are given. Correlations of more than two particles (high-order correlations) obtained using different methods are performed. Recent Bose–Einstein correlation measurements at LEP2 are discussed.

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**1. Introduction**

The number of hadrons, or *multiplicity*, is one of the most important observables in particle production processes [1]. The distribution of multiplicity is a sensitive characteristics of a collision event. However, the multiplicity distribution tells us just about average, integrated numbers, while deeper information comes from moments of the distribution, which measure particle *correlations*, *i.e.* probe the dynamics of the interaction [2].

Here, I report on recent results on multiplicity and correlation measurements at LEP. The statistics of hadronic events collected by each of four CERN LEP Collaborations at the  $Z^0$  peak exceeds four million events and gives an unique opportunity to study details of the theory of strong interactions, Quantum Chromodynamics (QCD), and its applicability to (“soft”) hadron production processes. Understanding of correlations at LEP2 is also crucial for ongoing Standard Model measurements.

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## 2. Definitions and notations [1, 2]

The multiplicity distribution, or density  $\rho_n$ , of  $n$  particles with kinematic variables  $p_1, p_2, \dots, p_n$  is defined by inclusive probability spectrum,

$$\rho_n(p_1, p_2, \dots, p_n) = \frac{1}{N_{\text{ev}}} \frac{dn(p_1, p_2, \dots, p_n)}{dp_1 dp_2 \cdots dp_n},$$

where  $N_{\text{ev}}$  is the number of events.

As it follows from this formula, the single particle distribution  $\rho_1(p_1)$  gives an *average* multiplicity,  $\int \rho_1(p_1) dp_1 = \langle n \rangle$ , while integration of the  $q$ -particle density leads to the unnormalised  $q$ -th order *factorial moments*,

$$\int \rho_1(p_1, p_2 \dots p_n) dp_1 dp_2 \cdots dp_n = \langle n(n-1) \cdots (n-q+1) \rangle \equiv \langle n^{[q]} \rangle = f_q.$$

The normalised moments,  $F_q = f_q / \langle n \rangle^q$  have been extensively used to study the intermittency phenomenon [2].

The  $q$ -particle densities give us a way to study particle correlations described by  $q$ -particle correlation functions, (factorial) *cumulants*,  $C_q(p_1, \dots, p_q)$ . The cumulants vanish whenever one of their arguments is statistically independent, *i.e.* these functions measure *genuine*  $q$ -particle correlations.

The cumulants are constructed from multiplicity densities, *e.g.*

$$\begin{aligned} C_1(p_1) &= \rho_1(p_1), \\ C_2(p_1, p_2) &= \rho_2(p_1, p_2) - \rho_1(p_1)\rho_1(p_2), \\ C_3(p_1, p_2, p_3) &= \rho_3(p_1, p_2, p_3) - \sum_{(3)} \rho_1(p_1)\rho_2(p_2, p_3) + 2\rho_1(p_1)\rho_1(p_2)\rho_1(p_3). \end{aligned} \tag{1}$$

These functions, being properly normalised, are used to study *multi*-particle correlations in different kinematic variables.

## 3. Multiplicity of unbiased gluon jet

In this section, I consider recent results on unbiased gluon jet multiplicity studies [3, 4]. This analysis provides a direct check of the QCD multiplicity predictions for quark jet *versus* gluon jet. The approach used allows to select “unbiased” gluon jet in 3-jet events, *i.e.* it is independent of jet-finding algorithm which were usually applied in earlier studies [1, 3].

In theory the gluon jet multiplicity,  $N_g$ , is defined in gluon–gluon ( $gg$ ) jet systems, while experimentally the  $N_g$  multiplicity is obtained from 3-jet

$q\bar{q}g$  final states. To this end, one uses the formula which connects 3-jet multiplicity with  $q\bar{q}g$  and  $gg$  multiplicities,  $N_{q\bar{q}}$  and  $N_{gg}$ , respectively,

$$N_{q\bar{q}g} = N_{q\bar{q}}(L, k_{\perp, Lu}) + \frac{1}{2}N_{gg}(k_{\perp, Lu}), \quad (2)$$

$$N_{q\bar{q}g} = N_{q\bar{q}}(L_{q\bar{q}}, k_{\perp, Le}) + \frac{1}{2}N_{gg}(k_{\perp, Le}). \quad (3)$$

Here,  $L$  specifies the  $e^+e^-$  c.m.s. energy, while  $L_{q\bar{q}}$  the  $q\bar{q}$  system energy. The two expressions are given by two approaches to define the gluon jet energy w.r.t.  $q\bar{q}$  system by Lund and Leningrad groups. The important fact is that  $N_{gg}$  depends only on a single scale  $k_{\perp}$ , *i.e.* it is *unbiased* in contrast to  $N_{q\bar{q}}$  depending on the energy scale too.

Fig. 1 shows OPAL results on  $N_{gg}$  of charged particles as a function of the jet energy  $Q$  [3]. One can see that calculations using the Lund approach better describe the data and Monte Carlo predictions (the latter well reproduce the data) than that of Leningrad.

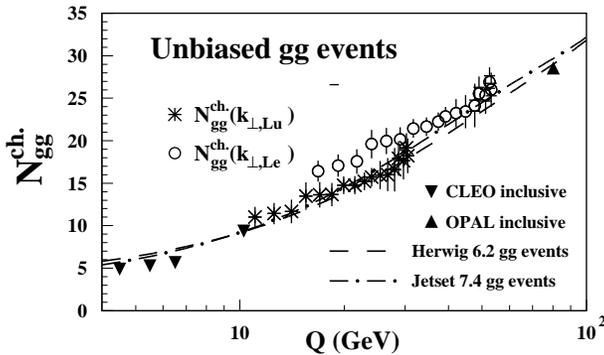


Fig. 1. The average charged particle multiplicity of unbiased  $gg$  events as a function of energy scale. Different  $k_{\perp}$  correspond to Eqs. (2) and (3), respectively. See text and Ref. [3] for more details.

The Lund formalism was proceed to obtain the ratios,  $r^{(j)} \equiv (d^j N_g/d\varepsilon)/(d^j N_q/d\varepsilon)$  of gluon and quark multiplicities. Here  $\varepsilon$  specifies the jet energy. The ratios were found [3] to satisfy the QCD prediction,  $r^{(0)} < r^{(1)} < r^{(2)} \rightarrow 2.25$  as  $Q \rightarrow \infty$ . From this it was obtained an effective value of QCD colour factors,  $C_A/C_F = 2.23 \pm 0.14$  being in a good agreement with the QCD value of 2.25. A similar value is preliminarily reported by DELPHI [4].

#### 4. Two-particle correlations in hadronic $Z^0$ decays

During last years, LEP Collaborations actively study two-particle correlations of bosons, Bose–Einstein correlations (BEC), and fermions, Fermi–Dirac correlations (FDC) [5]. To study two-particle correlations one needs

to measure  $\rho_2(p_1, p_2)/\rho_1(p_1)\rho_1(p_2)$ , where  $p_1$  and  $p_2$  are the 4-momenta of particles. Experimentally, one measures  $C(Q) = \rho_2(Q)/\rho_2^0(Q)$ , where  $Q^2 = -(p_1 - p_2)^2$ . The normalisation  $\rho_2^0$  of a reference sample has to be free of BEC/FDC and can be defined in different ways: Monte Carlo without such kind of correlations,  $\rho_2(Q)$  of unlike-sign hadrons, pairs with particles from different events, or from different hemispheres (mixings). Then the  $C(Q)$  function fit assuming the Gaussian source,

$$C(Q) = 1 \pm \lambda \exp(-Q^2 R^2), \quad (4)$$

gives  $\lambda$  to be a measure of the strength of the correlations while  $R$  is considered as emitter radius. In this approach  $\lambda = 1$  indicates completely incoherent emission. The “+” sign stands for BEC, and the “-” for FDC.

BEC of *charged* pions are well established at LEP, and the radius is obtained to vary between 0.5 and 1.0 fm depending on the reference sample.

Recently, L3 measured BEC of *neutral* pions [6]. It is found that in the same framework for charged and neutral pions,  $R(\pi^0\pi^0) < R(\pi^\pm\pi^\pm)$  (with  $2\sigma$  evidence). This is in qualitative agreement with the Lund string model.

A decrease of  $C(Q)$  as  $Q \rightarrow 0$  for fermions was observed for  $\Lambda$  pairs [5], while OPAL preliminarily reported [7] on a depletion in antiproton pairs.

Combining measurements of emission radius of pions, kaons, Lambdas and antiprotons, the hadron mass hierarchy  $R_\pi > R_K > R_{\Lambda, \bar{p}}$  is obtained as shown in Fig. 2 [8]. This hierarchy can be explained by Heisenberg uncertainty principle model [8] or with correlation between space/time and

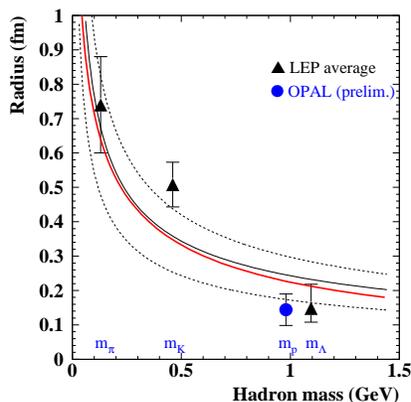


Fig. 2. The emitter radius  $R$  as a function of the hadron mass obtained from LEP [8]. The thin lines are from the Heisenberg uncertainty relations with the time scale values of  $10^{-24}$  (central thin line) and  $0.5 \times 10^{-24}$  and  $1.5 \times 10^{-24}$  (thin dashed lines). The thick line is given by the virial theorem with a general QCD potential. For more details, see Ref. [8].

momentum/energy of the particle in the hadroproduction process [9]. Meanwhile, the interpretation of  $R$  as the emitter radius leads to the very high emitter energy density at baryon mass,  $\sim 100 \text{ GeV}/\text{fm}^3$  [8].

### 5. High-order correlations at the $Z^0$ peak

Further step in understanding the hadroproduction process is to analyse the correlations of more than two particles, *i.e.* high-order correlations [2, 5].

At LEP, 3-particle BEC study has been recently performed by L3 [10], in addition to the earlier studies [5]. A key point in this analysis is to remove *non-genuine* 3-particle correlations of two- and single-particle product, see  $C_3$  in Eqs. (1). Then, similarly to the two-particle case analysis is carried out using  $Q_3$  variable along with no-BEC normalisation  $\rho_3^0(Q_3)$ . Extended Gaussian fit (4) of the genuine 3-particle correlation function with Hermite polynomials and used the Fourier transform of a source density, L3 concludes with fully incoherent pion production mechanism.

Genuine correlations up to the 4-th order are observed by OPAL for like-sign pions in terms of the normalised cumulants, Eqs. (1) [11]. BEC algorithm of PYTHIA Monte Carlo is found to reproduce multiparticle correlations in 1- to 3-dimensional phase space regions of rapidity, azimuthal angle and transverse momentum. Interrelation between higher-order and two particle correlations are obtained.

### 6. Bose–Einstein correlations at LEP2

In WW 4-quark hadronic decays, it is difficult to separate products of each  $W$  decay since separation of the decay vertices is  $\sim 0.1 \text{ fm}$ , while the hadronisation scale is of  $0.5\text{--}1 \text{ fm}$ . Therefore, due to possible *inter-W* BEC, the Standard Model measurement of the  $W$  mass is expected to be biased and needs BE effect between observed hadrons to be taken into account.

At LEP, we used the method of  $\Delta\rho(Q)$  function of two-particle densities,

$$\Delta\rho = \rho_2^{\text{WW}\rightarrow 4\text{q}} - (2\rho_2^{\text{W}\rightarrow 2\text{q}} + \rho_2^{\text{mix}}),$$

and  $D(Q)$  being the ratio of  $\rho_2^{\text{WW}\rightarrow 4\text{q}}$  to the sum in parentheses [12]. The latter represent faked 4q events of the hadronic part of semileptonic events and mixed events from two independent semileptonic events without leptons. If there is no inter-W BE effect, then  $\Delta\rho = 0$  and  $D = 1$ .

The  $\Delta\rho(Q)$  measurements show consistency with the absence of the inter- $W$  BEC, and  $D$  is found to be about 1 [13]. Further study is ongoing.

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