FUTURE AND PERSPECTIVES OF QCD*

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I discuss various areas of perturbative QCD where there is much current activity and which are likely to lead to significant developments over the next few years.

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1. Introduction

It is hard to believe now that a few years ago we would still discuss QCD in terms of a "candidate theory" of strong interactions. More recently we have been passing from the era of "testing QCD" to that of how to extract the most precise information possible from this universally accepted theory.

We have just witnessed the end of LEP, we are passing from HERA 1 to the upgraded HERA 2, likewise from Run 1 to Run 2 at the Tevatron and not too far away (I hope) is the LHC. At each of these colliders, much of the physics is described by perturbative QCD. In this talk I would like to focus on a few areas of intense activity by QCD theorists which have a direct impact on collider physics. In each case there is still more work to be done and so these areas are likely to continue as hot-spots for some time.

2. Next-to-Next-to-Leading Order (NNLO)

Leading Order (LO) QCD results were followed in a matter of a few years by Next-to-Leading Order (NLO) results. The quest for NNLO results has taken a good deal longer and this is an indication of the huge effort required to carry out the vast programme of calculations — so we should first remind ourselves why such a programme is necessary.

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First and foremost it is matter of precision. In several instances, the theoretical uncertainty on a measured quantity is as large or more than any other uncertainty. For example, in extracting α_S from inclusive jet production at the Tevatron, the theoretical uncertainty is typically the same as the pdf uncertainty [1], former being indicated by the result of varying the renormalisation scale by a factor 2 either way. We also know that sometimes the NLO result can be as much as a 50% correction to the LO result. Computing the NNLO result predicts precisely the result of varying the scale at NLO (as indeed does the NLO for the scale at LO) and the hope is that in going LO $\rightarrow$ NLO $\rightarrow$ NNLO there is a "convergence" to a stable result. This hope has been justified by recent NNLO results, Drell–Yan and Higgs production. The NNLO coefficient functions were calculated some time ago [2] for the Drell–Yan and DIS processes and the NNLO corrections to the Higgs cross-section at the hadron colliders very recently [3]. The NNLO pdf's need to be estimated in order to use these results and that implies knowledge of the 3-loop $O(\alpha_S^3)$ splitting functions. Up to the $N = 14$ anomalous dimensions have now been computed [4] and a range of expectations for the relevant splitting functions extracted [5]. In this way a NNLO analysis [6] of DIS data generates a set of NNLO pdf's. From Fig. 1 there appears to be a con- vergence as the order is increased giving confidence in the final result. At the LHC, the W cross section will be measured to great precision, the ratio $\sigma(W^-)/\sigma(W^+)$ having an uncertainty of perhaps 1 per mil. and the integrated cross section providing a precise monitor of the machine luminosity.

Impressive progress has been made over the last 2–3 years in computing the NNLO corrections to the hadronic 2 jet cross section. The same graphs enter in computing the NNLO corrections to $e^+e^- \rightarrow 3$ jets with a different kinematic limit where one leg is off-shell. With such knowledge, the uncertainty in determining α_S at LEP could come down to 1%. In computing the NNLO virtual corrections, one needs to calculate: (a) the 2-loop, 2-parton final state (f.s.), (b) the $|\text{1-loop}|^2$, 2-parton f.s., (c) the 1-loop, 3-parton f.s. (or 2+1 parton f.s), (d) the tree, 4-parton f.s. (or 3+1, or even 2+2 f.s). Here $i + j$ parton f.s. means the j partons are unresolved soft or collinear partons. Of course the problems involved in ensuring the cancellation between n and $n + 1$ or $n + 2$ partons in the soft or collinear limit is the difficult task. The finite answer is the required goal but the poles which appear in the minimal subtraction analysis (*i.e.* $1/\varepsilon^k$ with $k = 1, 2, 3, 4$) have coefficients whose values can be cross-checked with those predicted in the elegant work of Catani in 1998 [7]. A large number of dedicated people have put a great deal of effort into this programme, reducing a huge number of graphs to manageable number of master integrals en route, which finally produced the complete structure of all the singularities involved to-

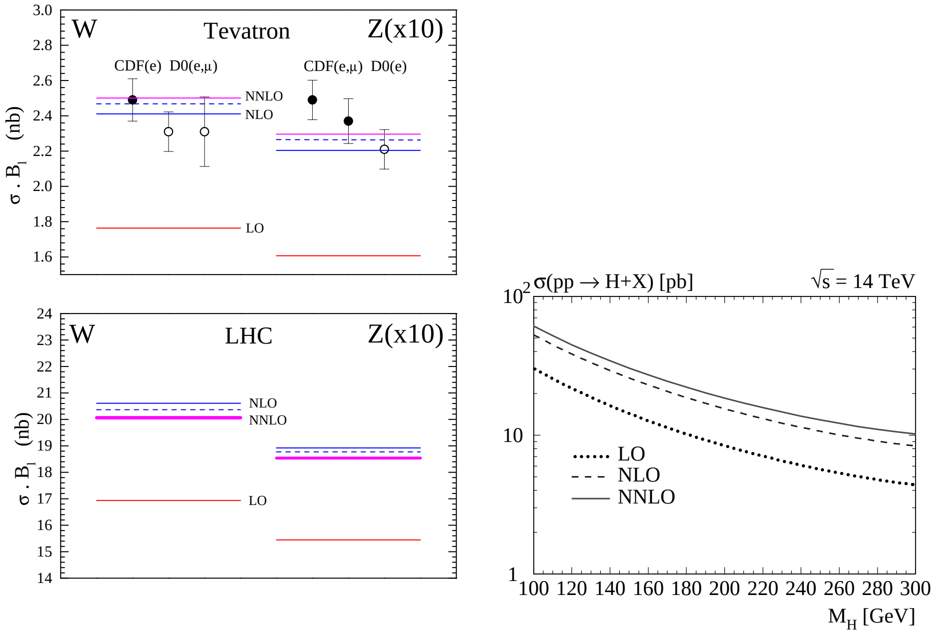

Fig. 1. The LO, NLO and NNLO Drell–Yan cross sections for W and Z production [6] at the Tevatron and LHC (left hand side). The LO, NLO, and NNLO Higgs cross sections at the LHC [3] (right hand side).

gether with an understanding of the subtle cancellations involved. The culmination is a remarkable paper [8] where the 2-loop matrix elements for $e^+e^- \to 3$ jets are evaluated.

Great though this achievement is, there is the problem of implementing the results into a numerical evaluation of these NNLO calculations. The stumbling block here is that, as yet, the parton-level Monte Carlo programs to handle cancellation of singularities with the contributions from the real graphs do not exist at NNLO. At NLO, there are various well-tried approaches using phase-space slicing procedures or variations on subtraction methods. So far these have not been extended to handle the $1/\varepsilon^3$ and $1/\varepsilon^4$ divergences arising in this case. An interesting proposal [9] is to use purely numeric calculations (in the Coulomb gauge) to do the necessary integrations. The claim is that by effectively reversing the order of summing graphs involved and doing the momentum integrals, the singularities cancel between different cuts. It has been tested in the $\mathrm{O}(\alpha_\mathrm{S}^2)$ 3-jet like variables and it is claimed to be simple, flexible and, most importantly, capable of being applied to the NNLO case.

3. Next-to-Leading Order (NLO)

Progress is being made on two fronts here. Again there is the quest for increased precision on matrix elements for important processes previously calculated only at LO. Not only should the signals for important physics at the colliders be calculated to higher order, the backgrounds should also be determined as accurately as possible. For example, for a Higgs mass above about $2M_W$ the crucial background is $p\bar{p} \rightarrow W + 2$ jets. Recently this has been computed to NLO [10] and again we see the expected improved stability with respect to variation of the renormalisation scale (the factorisation scale, as usual, being set equal in value). Thus we find that

$$\frac{\sigma(W + 2\,\mathrm{jets}; \mu = \frac{1}{2}M_W)}{\sigma(W + 2\,\mathrm{jets}; \mu = 2M_W)} = 1.7\,(\mathrm{LO}) \longrightarrow 1.1\,(\mathrm{NLO}). \tag{1}$$

In the course of computing this process at NLO, one has to evaluate the real and virtual corrections which involves the cancellation of divergences leaving the finite answer as accurately as possible. As discussed above one does this with an NLO parton-level Monte Carlo procedure.

To go further what one really wants in order to study detailed final state configurations with the experimental acceptance folded in, is a parton-showering Monte Carlo program at NLO — *i.e.* one containing the information of the NLO matrix elements directly. In considering a multi-jet final state, we could interpret a configuration either as a higher order matrix element or as a $q\bar{q}$ state plus parton showering. In trying to combine the virtues of both one must avoid double-counting. This is recognised as a high priority and several groups are attempting practical solutions [11]. The most recent and ambitious is **MC@NLO** of Frixione and Webber [12] which although so far applied only to a toy model scenario is very encouraging. While hard emissions are treated as in NLO computations, soft/collinear emissions are handled by Monte Carlo simulation. Only a small fraction of events end up with a negative weight and even these can be reduced by efficient choice of parameters for a given process.

4. Large *x*

There have been interesting developments in understanding the summations of potentially large logarithms at large x which we know phenomenologically is a region where Higher-Twist (HT) effects appear to be important. The large logs arise from phase space for the real emission of soft gluons being “squeezed” and we understand how these contributions can be resummed through exponentiation of the large (Sudakov) logs. This is a consequence of factorisation in pQCD [13]. The soft gluons in this case are emitted on-shell.

At the same time we know that coefficients of a perturbative QCD series invariably tend to increase like $n!$ which is related to the running of the strong coupling, so-called renormalon contribution. In contrast to above, the gluons here are off-shell and "dressed". The claim by Gardi [14] is that if factorisation holds *beyond* the perturbative level, the power corrections associated with renormalons *also* exponentiate. This "Dressed Gluon Exponentiation" (DGE) thus resume the entire perturbative series of log-enhanced terms that describe single gluon emission close to the threshold.

In DIS, we make the usual expansion for the n-th moment of the structure function with increasing twist,

$$M_N\left(Q^2\right) = \sum C_{T=2}\left(N,\frac{Q^2}{\mu^2}\right)\left\langle O_N^{T=2}(\mu^2)\right\rangle + \frac{1}{Q^2}\sum C_{T=4}\left(N,\frac{Q^2}{\mu^2}\right)\left\langle O_N^{T=4}(\mu^2)\right\rangle, \tag{2}$$

where μ is the renormalisation scale. As we vary μ^2, the operators on the rhs mix with each other, so that the overall expression is independent of the value of the scale. More specifically, at the level of the $\ln\mu^2$ divergence the $T = 4$ operators mix among themselves, while at the level of the μ^2 divergence, the $T = 2$ operators mix with the $T = 4$ ones, which is the way that renormalon ambiguity cancels within the OPE. The renormalon ambiguity at $T = 2$ is cancelled by the power corrections at $T = 4$ and assuming that this is the dominant source of the observed power corrections is the "renormalon dominance" model for the $1/Q^2$ behaviour. An interesting conjecture by Gardi *et al.*, [15] is that it is the most divergent part of *each* higher twist that dominates and thus mixes with the leading twist. So we understand "renormalon dominance" in terms rather of a more general concept — "ultraviolet dominance".

Quite independently, the structure of the HT simplifies as $x \to 1$. This follows from the fact that both leading twist and HT are kinematically driven by the production of a "narrow" quark jet. Formally this means that the quark–gluon correlation function is dominated by the region where the momentum carried by the gluon is extremely small thus approximating the quark density function which enters the leading twist expression. The resulting expression in moment space is then appealingly simple:

$$M_N\left(Q^2\right) = q_N^{T=2}\left(\mu^2\right)\ H\left(\frac{Q^2}{\mu^2}\right)\ J\left(\frac{Q^2}{N}\Big/\mu^2\right)\ J^{NP}\left(\frac{Q^2}{N}\Big/\Lambda^2\right). \tag{3}$$

Taking each contribution in turn:

$q_N^{T=2}(\mu^2)$ — this is the usual (moment of) pdf;
$H(\frac{Q^2}{\mu^2})$ — the hard scattering of quark and photon;
$J(\frac{Q^2}{N}/\mu^2)$ — propagation of the narrow quark jet.
These two last terms are calculable at $T = 2$ and include the resummed Sudakov $L = \ln N$ terms: $J^{NP}(\frac{Q^2}{N}/\Lambda^2)$ — this is now the dressed gluon exponentiation of the renormalon contribution and is written

$$J\left(\frac{Q^2}{N}/\mu^2\right) = \exp\left[-\frac{C_F}{\beta_0}\left\{\omega_1\left(\frac{N\Lambda^2}{Q^2}\right) + \omega_2\left(\frac{N\Lambda^2}{Q^2}\right)^2\right\}\right]. \tag{4}$$

The implication is clear; there is a close relation between the simultaneous resummation of both the renormalons and the Sudakov logarithms and the non-perturbative corrections. Thus large x is an area where one expects progress in the phenomenological description. It would appear that including non-leading Sudakov log terms and/or a $T = 4$ contribution can adequately describe the data on the derivatives of the large N moments [16].

A similar spirit drives the attempt to simultaneously resume two large logarithms which occur in studying the transverse momentum distributions of W, Z production at the colliders. Consider the vector boson of mass Q produced with transverse momentum Q_{T} by partons with momentum fractions x_1, x_2 of the initial hadrons. If $\tau = Q^2/s$ and $z = \tau/x_1x_2$ we have potentially large threshold log terms of the type

$$\alpha_{\mathrm{S}}^N \, \frac{\ln^{2N-1}(1-z)}{(1-z)} \quad \text{as } z \to 1\,,$$

and potentially large recoil log terms

$$\alpha_{\mathrm{S}}^N \ln^{2N-1}\left(\frac{Q^2}{Q_{\mathrm{T}}^2}\right) \quad \text{as } Q_{\mathrm{T}} \to 0\,.$$

Resummation of each of these contributions separately was demonstrated some years ago [17, 18], but the programme for jointly resumming the large logs to NLL has been successfully achieved only recently [19]. It involves inverting impact parameter transforms and reproduces correctly the individual single resummations. In addition to giving a good description of the observed Q_{T} distribution, the interesting thing is that it suggests, similarly to the discussion above, the functional form of the non-perturbative correction, which here takes a Gaussian form at small transverse momentum k_{T} of the soft radiation.

5. Small x

A very nice summary of the present status of phenomenology in the small x region was recently published by the "Small x Collaboration" [20]. Here the relative successes of the collinear factorisation versus the k_{T} factorisation approaches are studied. A rough conclusion is that while the rise of the inclusive cross section can be adequately described by the DGLAP evolution, several non-inclusive observables are much better described by the BFKL approach. Among these are forward jet production, particle spectra and photoproduction of D^*. However, there is a suggestion that even the description of the structure function F_2 at low x benefits from adding small contributions involving $\ln 1/x$. Thorne [21] finds that modifying the gg and qg splitting functions in the following way

$$P_{gg}(x) \longrightarrow P_{gg}(x) + 2\alpha_{\mathrm{S}}^4 \frac{1}{x}\left[\frac{1}{3!}\ln^3\frac{1}{x} - \frac{1}{2!}\ln^2\frac{1}{x}\right],$$

$$P_{qg}(x) \longrightarrow P_{qg}(x) + \frac{n_f}{3\pi}\alpha_{\mathrm{S}}^5 \frac{1}{x}\left[\frac{1}{3!}\ln^3\frac{1}{x} - \frac{1}{2!}\ln^2\frac{1}{x}\right],$$

maintains energy-momentum conservation and is enough to give a consistent description of the F_2 data both in the small *and* medium x ranges where there are problems for the conventional DGLAP description. Thus there is a suggestion that some resummation of large $\ln 1/x$ terms is required. A measurement much more likely to be sensitive to such resummation terms is, of course, that of F_L. Some of us have often begged in the past for a direct measurement at HERA of F_L but the importance of this seems passed unrecognised by those in charge of the physics programme.

The total hadronic cross section for $\gamma^*\gamma^* \to$ *hadrons* is regarded as a relatively clean probe of BFKL type resummation. For photons of virtuality Q_1^2, Q_2^2 we define $Q^2 = \sqrt{Q_1^2 Q_2^2}$ and the relevant large logarithm is $L = \ln(s/Q^2)$. We can write

$$\sigma_{\gamma^*\gamma^*} \sim \sum_{j=0}^{\infty} a_{0j}\alpha_{\mathrm{S}}^j + a_1\alpha_{\mathrm{S}}^2 \sum_{j=0}^{\infty}(\alpha_{\mathrm{S}}L)^j + a_2\alpha_{\mathrm{S}}^2\sum_{j=0}^{\infty}\alpha_{\mathrm{S}}(\alpha_{\mathrm{S}}L)^j\,. \tag{5}$$

The first sum is the box graph (with gluonic corrections); the second and third sums collect the contributions from only gluon exchange, the second (third) sum resumming the BFKL (N)L log corrections. Comparing with the LEP data (L3 and OPAL) there was a large discrepancy when only the a_{00}, a_{01} terms together with an asymptotic estimate of the leading BFKL term a_1 [22] are included. This discrepancy is much reduced when an *exact* calculation of a_{01} is done [23], suggesting that the four-parton final state is an important contribution. Meanwhile the flexibility of varying the scale in the NLO BFKL high energy cross section has been exploited by Brodsky *et al.*, [24]. Using the BLM choice of scale (resums the β_0 terms into the running coupling in all orders) they find (a) good agreement with the LEP measurements and (b) a much reduced sensitivity to the Regge scale s_0. I do not believe that the BLM scale choice is particularly relevant but it is clear that this cross section is still a candidate for the "golden" signature of BFKL.

Finally, in the context of the dynamics of small x physics, there is the interesting issue of saturation and whether one can hope to see the signals for non-linear effects in present and future data. We are still considering a weak coupling regime but the non-linear effects are enhanced by the energy being sufficiently high for overlap of the gluon densities due to the transverse size of the gluons growing $\sim 1/Q^2$. The saturation scale $Q^2_{\rm S}$ is expected to occur when

$$\text{the interaction probalility} \;\sim\; \frac{\alpha_s}{Q^2}\,\frac{1}{\pi R^2}\,xg\left(x,Q^2\right) \;\sim 1\,.$$

This amplification of interactions by high gluon densities suggests some form of "resummation" of these densities is required in this regime. An approach which attempts to do precisely this is the so-called Colour Glass Condensate [25] (CGC) which is an effective theory derived from QCD where the sources of classical colour fields are the small x saturated gluons. The degrees of freedom due to the other "fast" partons, whose mutual interactions are described by perturbative QCD in the LLA, are just integrated out. The non-linear effects of saturation thus appear in a classical context and provide a framework for carrying out exact calculations. Furthermore, the approach is not inconsistent with other approaches [26]. An interesting issue is whether geometric scaling [27] is a consequence of saturation — that it appears to persist to relatively large x and Q^2 seems to indicate a wider phenomenon. However, the CGC does appear to be a potentially useful approach combining intuitive ideas with a calculational framework and may be well suited for studying results from RHIC where nuclear gluon densities are likely to be significantly enhanced.

6. Conclusions

I have tried to select a few areas of perturbative QCD where I detect genuine excitement from recent results which represent significant achievement over the last few years. That the Tevatron and LHC will provide signals for new physics is of course everyone's hope but the ability to correctly interpret those signals depends on us having confidence in understanding QCD collider physics. In addition, QCD is itself a wonderfully rich theory from which we shall continue to extract intellectually rewarding discoveries for many years to come. Whatever the motivation, it is clear that QCD is a highly relevant subject with an exciting future.

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