DIFFRACTION AND $\sigma_{\gamma^* p}$* **

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The empirical scaling law, wherein the total photo-absorption cross section depends on the single variable $\eta = (Q^2 + m_0^2)/\Lambda^2(W^2)$, provides empirical evidence for saturation in the sense of $\sigma_{\gamma^* p}(W^2, Q^2)/\sigma_{\gamma p}(W^2) \to 1$ for $W^2 \to \infty$ at fixed Q^2. The total photo-absorption cross section is related to elastic diffraction in terms of a sum rule. The excess of diffractive production over the elastic component is due to inelastic diffraction that contains the production of hadronic states of higher spins. Motivated by the diffractive mass spectrum, the Generalized Vector Dominance/Color Dipole Picture (GVD/CDP) is extended to successfully describe the DIS data in the full region of $x \leq 0.1$, all $Q^2 \geq 0$, where the diffractive two-gluon exchange mechanism dominates.

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In the present talk, I wish to concentrate on the relation between the total photo-absorption cross section, $\sigma_{\gamma^* p}(W^2, Q^2)$, at low $x \cong Q^2/W^2 \leq 0.1$ and diffractive production, $\gamma^* p \to Xp$ [1].

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The experimental data [2–6] on $\sigma_{\gamma^* p}(W^2, Q^2)$ at $x \leq 0.1$ and all $Q^2 \geq 0$, including photoproduction ($Q^2 = 0$), lie on a single curve [7]

$$\sigma_{\gamma^* p}\left(W^2, Q^2\right) = \sigma_{\gamma^* p}\ \eta\left(W^2, Q^2\right)\ , \tag{1}$$

if plotted against the low-x scaling variable

$$\eta\left(W^2, Q^2\right) = \frac{Q^2 + m_0^2}{\Lambda^2\left(W^2\right)}, \tag{2}$$

where $\Lambda^2(W^2)$ is a slowly increasing function of W^2 and $m_0^2 \cong 0.16\,\mathrm{GeV}^2$. Compare Fig. 1 for a plot of $\sigma_{\gamma^* p}(W^2, Q^2)$ against η. The function $\Lambda^2(W^2)$ may be represented, alternatively, by a power law or by a logarithm

$$\Lambda^2\left(W^2\right) = \begin{cases} C_1\left(W^2 + W_0^2\right)^{C_2}, \\ C_1' \ln\left(\frac{W^2}{W_0^2} + C_2'\right). \end{cases} \tag{3}$$

We refer to Refs. [7, 8] for the numerical values of the fit parameters in $\Lambda^2(W^2)$.

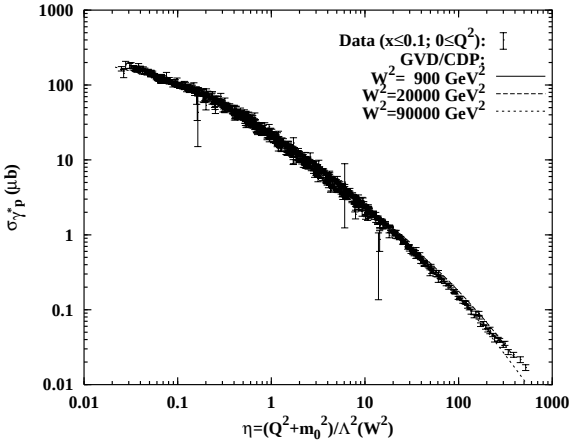


Fig. 1. The experimental data [2–6] for the total photo-absorption cross section, $\sigma_{\gamma^* p}(W^2, Q^2)$ as a function of $\eta(W^2, Q^2)$ compared with the predictions from the GVD/CDP.

The empirical model-independent finding (1) is interpreted in the generalized vector dominance/color dipole picture [7, 8] that rests on the generic structure of the two-gluon exchange virtual photon forward Compton scattering amplitude. Evaluation of this amplitude in the $x \to 0$ limit and transition to transverse position space implies [9]:

$$\sigma_{\gamma^*_{\mathrm{T,L}} p}\left(W^2,Q^2\right)=\int dz\int d^2r_\perp \sum_{\lambda,\lambda'=\pm 1}\left|\psi^{(\lambda,\lambda')}_{\mathrm{T,L}}\left(\vec{r}_\perp,z,Q^2\right)\right|^2\sigma_{(q\bar{q})p}\left(\vec{r}_\perp^{\,2},z,W^2\right), \tag{4}$$

where the Fourier representation of the color–dipole cross section

$$\begin{aligned}\sigma_{(q\bar{q})p}\left(\vec{r}_\perp^{\,2},z,W^2\right) &= \int d^2l_\perp\tilde{\sigma}_{(q\bar{q})p}\left(\vec{l}_\perp^{\,2},z,W^2\right)\left(1-\mathrm{e}^{-i\vec{l}_\perp\vec{r}_\perp}\right)\\ &= \sigma^{(\infty)}\begin{cases}\frac{1}{4}\vec{r}_\perp^{\,2}\langle\vec{l}_\perp^{\,2}\rangle_{W^2,z}\,, & \text{for } \vec{r}_\perp^{\,2}\langle\vec{l}_\perp^{\,2}\rangle_{W^2,z}\to 0,\\ 1\,, & \text{for } \vec{r}_\perp^{\,2}\langle\vec{l}_\perp^{\,2}\rangle_{W^2,z}\to\infty,\end{cases}\end{aligned} \tag{5}$$

contains "color transparency" in the limit of $\vec{r}_\perp^{\,2}\langle\vec{l}_\perp^{\,2}\rangle_{W^2,z}\to 0$, as well as hadronic unitarity, provided

$$\sigma^{(\infty)}\equiv\pi\int d\vec{l}_\perp^{\,2}\tilde{\sigma}\left(\vec{l}_\perp^{\,2},z,W^2\right) \tag{6}$$

has decent high-energy behavior. The average or effective gluon transverse momentum, $\langle\vec{l}_\perp^{\,2}\rangle_{W^2,z}$, in (5) is given by

$$\langle\vec{l}_\perp^{\,2}\rangle_{W^2,z}\equiv\frac{\int d\vec{l}_\perp^{\,2}\vec{l}_\perp^{\,2}\tilde{\sigma}_{(q\bar{q})p}\left(\vec{l}_\perp^{\,2},z,W^2\right)}{\int dl_\perp^2\tilde{\sigma}_{(q\bar{q})p}\left(\vec{l}_\perp^{\,2},z,W^2\right)}\,. \tag{7}$$

It is a characteristic feature of the $x\to 0$ limit of the two-gluon exchange amplitude that the representation (4) factorizes into the product of the photon wave function, $|\psi|^2$, that describes the photon coupling to the $q\bar{q}$ state and its propagation, and the color–dipole cross section, $\sigma_{(q\bar{q})p}$, that describes the forward scattering of the color dipole from the proton. The scattering is "diagonal" in the variables $\vec{r},z$, since these variables remain fixed during the scattering process.

The empirical scaling law (1) is embodied in the representation (4) by requiring the dipole cross section (5) to depend on the product $\vec{r}^{\,2}\cdot\Lambda^2(W^2)$. This implies that $\langle\vec{l}_\perp^{\,2}\rangle_{W^2,z}$ be proportional to $\Lambda^2(W^2)$. In the GVD/CDP, we approximate the distribution in the gluon transverse momentum $\vec{l}^{\,2}$ in (5) by a δ-function situated at the effective gluon transverse momentum $\langle\vec{l}_\perp^{\,2}\rangle_{W^2,z}$

$$\tilde{\sigma}_{(q\bar{q})p}\left(\vec{l}_\perp^{\,2},z(1-z),W^2\right)=\sigma^{(\infty)}\frac{1}{\pi}\,\delta\left(\vec{l}_\perp^{\,2}-\Lambda^2\left(W^2\right)z(1-z)\right)\,. \tag{8}$$

The proportionality factor $z(1-z)$ in (8) is a model assumption that improves the high-Q^2 behavior. With (5) and (8), and the Fourier representation of the wave function inserted, the expression for the cross section (4)

may be evaluated analytically in momentum space [7]. We only note the approximate final expression

$$\sigma_{\gamma^* p}\left(W^2, Q^2\right) = \frac{\alpha R_{e^+e^-}}{3\pi}\sigma^{(\infty)} \begin{cases} \ln\left(\frac{1}{\eta}\right), & \text{for } \eta \to \dfrac{m_0^2}{\Lambda^2\left(W^2\right)}, \\ \frac{1}{2}\eta, & \text{for } \eta \gg 1. \end{cases} \tag{9}$$

and refer to Ref. [7] for details.

According to (9), at any fixed value of Q^2, for sufficiently large W, a soft, logarithmic energy dependence is reached for $\sigma_{\gamma^* p}$. The GVD/CDP that rests on the generic structure of the two-gluon exchange from QCD, and contains hadronic unitarity and scaling in η, leads to the important conclusion that

$$\lim_{\substack{W^2\to\infty \\ Q^2=\text{const}}} \frac{\sigma_{\gamma^* p}\left(W^2, Q^2\right)}{\sigma_{\gamma p}(W^2)} = 1\,. \tag{10}$$

The behavior (9) may be called "saturation". Since the low x (HERA) data, according to Fig. 1, show evidence for the behavior (9) that implies (10), we may indeed conclude that HERA yields evidence for "saturation". Needless to stress, future tests of scaling in η, by increasing W as much as possible, are clearly desirable to provide further evidence for the validity of the remarkable conclusion (10) that puts virtual and real photoproduction on equal footing at any fixed Q^2 in the limit of infinite energy.

We turn to diffractive production. The diagonal form (4) of $\sigma_{\gamma^*_{\mathrm{T,L}} p}$, or rather of the virtual forward Compton scattering amplitude, develops its full power when considering diffractive production, $\gamma^* p \to Xp$. The two-gluon exchange generic structure for $x \to 0$ implies [10]

$$\left.\frac{d\sigma_{\gamma^*_{\mathrm{T,L}} p \to Xp}(W^2, Q^2, t)}{dt}\right|_{t=0}$$

$$= \frac{1}{16\pi}\int_0^1 dz \int d^2 r_\perp \sum_{\lambda,\lambda'=\pm 1} \left|\psi_{\mathrm{T,L}}^{(\lambda,\lambda')}\left(r_\perp, z, Q^2\right)\right|^2 \sigma^2_{(q\bar{q})p}\left(\vec{r}_\perp^{\,2}, z, W^2\right)\,. \tag{11}$$

Note the close analogy of (11) to the simple ρ^0 dominance formula for photoproduction [11]

$$\left.\frac{d\sigma}{dt}\right|_{t=0}\left(\gamma p \to \rho^0 p\right) = \frac{1}{16\pi}\frac{\alpha\pi}{\gamma_\rho^2}\sigma^2_{\rho^0 p}\,. \tag{12}$$

Upon transition to the momentum–space representation in (11) and after integration over all variables with the exception of the mass M of the outgoing state X, one obtains the mass spectrum, $d\sigma_{\gamma^*_{\mathrm{T,L}} p \to Xp}/dt dM^2$ for forward

production that depends on W^2, Q^2 and M^2. A comparison of this mass spectrum with the integrand of the total cross section in (4) (obtained upon transition to momentum space and appropriate integration with the exception of one final integration over M^2), allows one to rewrite (4) as a sum rule that reads [1]

$$\sigma_{\gamma^* p}(W^2, Q^2) = \sqrt{16\pi}\sqrt{\frac{\alpha R_{e^+e^-}}{3\pi}} \times \int_{m_0^2} dM^2 \frac{M}{Q^2+M^2}\left[\sqrt{\left.\frac{d\sigma_{\gamma_{\rm T}^*}}{dtdM^2}\right|_{t=0}} + \sqrt{\frac{Q^2}{M^2}}\sqrt{\left.\frac{d\sigma_{\gamma_{\rm L}^*}}{dtdM^2}\right|_{t=0}}\right]. \quad (13)$$

The sum rule represents the total photo-absorption cross section in terms of diffractive forward production. It is amusing to note that (13) is the virtual photon analog of the photoproduction sum rule [11]

$$\sigma_{\gamma p}\left(W^2\right) = \sum_{V=\rho^0,\omega,\phi,\ldots} \sqrt{16\pi}\sqrt{\frac{\alpha\pi}{\gamma_V^2}}\sqrt{\left.\frac{d\sigma_{\gamma p\to V_0}}{dt}\right|_{t=0}}. \quad (14)$$

based on ρ^0, ω, ϕ dominance. Note, however, that (13) is a strict consequence of the generic two-gluon exchange structure evaluated in the $x \to 0$ limit that forms the basis of the GVD/CDP[1].

It is evident, even though apparently always ignored, that the diffractive production cross section (11) describes elastic and only elastic diffraction, where "elastic" is meant to denote diffractive production of hadronic states X that carry photon quantum numbers. Otherwise, the color dipole cross section under the integral in (11) could never be identical to the one in (4), and (13) could never follow from (4) and (11).

"Inelastic" diffraction, namely diffractive production of states with spins different from the projectile spin, subject to the restriction of natural parity exchange, is a well-known phenomenon in hadron physics [13]. Evidence for inelastic diffraction in DIS is provided by the decrease [14] of the average thrust angle ("alignment") with increasing mass of the produced state X. This observation implies production of hadronic states X that do not exclusively carry photon quantum numbers.

It is, accordingly, not surprising that the elastic diffraction obtained from (11) with the parameters employed for $\sigma_{\gamma^* p}$ underestimates the measured cross section considerably, in particular for high values of the mass M of the state X. Compare Fig. 2 taken from Ref. [1].

[1] The sum rule (13) is also obtained from GVD arguments by themselves [12].

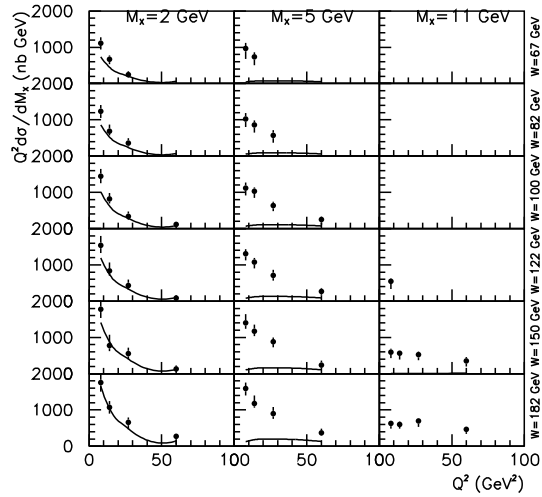


Fig. 2. The cross section for elastic diffractive production (GVD/CDP) as a function of Q^2 compared with ZEUS data from Ref. [15].

Theoretical approaches [16–19] to the description of high-mass diffractive production frequently introduce a quark–antiquark–gluon ($q\bar{q}g$) component in the incoming photon. As this component is usually ignored [16–18] in the treatment of the total cross section, I am afraid, there is the danger of an inconsistency, due to a violation of the optical theorem. A consistent inclusion of the $q\bar{q}g$ component in elastic diffraction is contained in Ref. [19], while an attempt for a consistent and unified treatment of inelastic and elastic diffraction and the total cross section, is provided in Ref. [20].

I return to the analysis of the total cross section. The above discussion of diffraction, in particular the sum rule (13), suggests to introduce an upper limit [1] in the integration over dM^2 in $\sigma_{\gamma^* p}(W^2, Q^2)$. At finite energy W, the diffractively produced mass spectrum is undoubtedly bounded by an upper limit that increases with energy. In our previous analysis [7, 8], we ignored such an upper limit, since the contribution of high masses seemed to be suppressed anyway. We have examined the effect of a cut-off, m_1^2, in the momentum space version of (4) or, equivalently, in (13). Putting

$$m_1^2 = (22\,\mathrm{GeV})^2 = 484\,\mathrm{GeV}^2\,, \tag{15}$$

that is the mass of the largest bin in the ZEUS data [14], we obtain an excellent description of all data with $x \leq 0.1$, all $Q^2 \geq 0$, as shown in Fig. 1. Putting $m_1^2 = \infty$ overestimates the cross section $\sigma_{\gamma^* p}$ significantly for $\eta \geq 10$, while values of m_1^2 smaller than the upper bound (15) yield results below the experimental ones at large η. It is gratifying that the

simple procedure of introducing a cut-off[2] that (approximately) coincides with the upper limit for diffractive production extends the GVD/CDP to the full region of $x \leq 0.1$, all $Q^2 \geq 0$, where diffraction dominates the virtual Compton forward scattering amplitude.

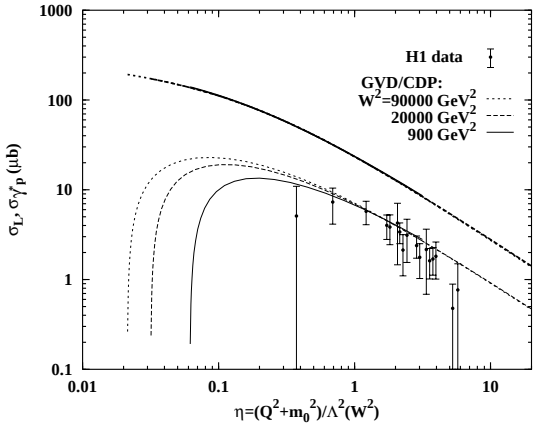


Fig. 3. The longitudinal part of the photo-absorption cross section as a function of η compared with H1 data.

In Fig. 3 we show the prediction [21] of the GVD/CDP for the longitudinal cross section in comparison with data from an H1 analysis [22].

In conclusion:

(i) Scaling, $\sigma_{\gamma^* p} = \sigma_{\gamma^* p}(\eta)$, in η yields $\sigma_{\gamma^* p}/\sigma_{\gamma p} \to 1$ for $W^2 \to \infty$ at fixed Q^2 and provides evidence for saturation.

(ii) Sum rules relate the elastic component in diffractive production to the total cross section, the terminology GVD/CDP being appropriate for low-x DIS.

(iii) The excess of diffractive production over the elastic $(q\bar{q})$ component is presumably due to higher spin components, and accordingly

(iv) any theory of diffraction has to discriminate between an inelastic and an elastic component and must be examined with respect to its compatability with the total cross section, $\sigma_{\gamma^* p}$.

[2] The simple cut-off procedure leads to a small violation of scaling in η for $\eta \geq 50$ (compare Fig. 1) that may presumably be avoided by a refined treatment.

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