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LARGE GEOMETRIC SCALING AND QCD EVOLUTION*

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We study the impact of the QCD DGLAP evolution on the geometric scaling of the gluon distributions which is expected to hold at small x within the saturation models. To this aim we solve the DGLAP evolution equations with the initial conditions provided along the critical line $Q^2 = Q_{\mathrm{s}}^2(x)$ with $Q_{\mathrm{s}}^2(x) \sim x^{-\lambda}$ and satisfying geometric scaling. Both fixed and running coupling cases are studied. We show that in the fixed coupling case the geometric scaling at low x is stable against the DGLAP evolution for sufficiently large values of the parameter λ and in the double logarithmic approximation of the DGLAP evolution this happens for $\lambda \geq 4N_{\mathrm{c}}\alpha_{\mathrm{s}}/\pi$. In the running coupling case geometric scaling is found to be approximately preserved at very small x. The residual geometric scaling violation in this case can be approximately factored out and the corresponding form-factor controlling this violation is found.

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Theoretical QCD expectations concerning the region of small values of the Bjorken parameter x can be briefly summarised as follows:

1. At very small x the **linear** (DGLAP or BFKL) evolution generates strong increase of the gluon distributions for $x \to 0$ which eventually violates unitarity.

2. Unitarity is restored by including the non-linear screening corrections [1–5].

3. Those non-linear effects lead to emergence of the **saturation scale** $Q_{\mathrm{s}}^2(x)$ $(Q_{\mathrm{s}}^2(x) \sim x^{-\lambda})$.

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4. Non-linear effects are small (eventually negligible) for $Q^2 > Q_s^2(x)$.

5. Non-linear effects are very strong and lead to saturation for $Q^2 < Q_s^2(x)$.

These theoretical QCD expectations have been implemented phenomenologically within the saturation model by Golec-Biernat and Wüsthoff [6]. In this model the $\gamma^* p$ total cross-section $\sigma_{\gamma * p}(Q^2, x)$ is driven by the cross-section $\sigma_{dp}(x,r)$ describing interaction of the colour $q\bar{q}$ dipoles, which the virtual photon fluctuates to, with the proton. Here r denotes the transverse size of the dipole. In the saturation model the cross section $\sigma_{dp}(x,r)$ is the function of the single variable $r^2 Q_s^2(x))$, where $Q_s^2(x) \sim x^{-\lambda}$ is the saturation scale, *i.e.*

$$\sigma_{dp}(x,r) = f\left(r^2 Q_s^2(x)\right) . \tag{1}$$

This property of $\sigma_{dp}(x,r)$ is called geometric scaling. The function $f(r^2 Q_s^2(x))$ has the following properties:

- $f(r^2 Q_s^2(x)) \sim r^2 Q_s^2(x)$ for small values of $r^2 Q_s^2(x)$ that corresponds to colour transparency.
- $f(r^2 Q_s^2(x)) \sim \sigma_0$ for large values of $r^2 Q_s^2(x)$ that corresponds to saturation of the dipole-proton cross section.

The dipole-proton cross-section is related to the gluon distribution:

$$\sigma_{dp}(x,r) \sim r^2 \alpha_s \left(\frac{1}{r^2}\right) xg\left(x, \frac{1}{r^2}\right) . \tag{2}$$

Saturation of $\sigma_{dp}(x,r)$ does, therefore, imply saturation of $xg(x, 1/r^2)$.

The fact that the dipole cross-section exhibits geometric scaling implies similar properties of the cross-section $\sigma_{\gamma * p}(Q^2, x)$, *i.e.*

$$\sigma_{dp}(x,r) = f\left(r^2 Q_s^2(x)\right) \rightarrow$$

$$\sigma_{\gamma * p}(Q^2, x) = h(\tau) , \tag{3}$$

where

$$\tau = \frac{Q^2}{Q_s^2(x)} . \tag{4}$$

It has been found that geometric scaling is very well confirmed by the HERA data [7]. Our aim is to understand possible effects of the QCD evolution on geometric scaling. We shall summarise results obtained in [8] where all the details can be found. Let us assume that:

- For $Q^2 < Q_s^2(x)$ the linear evolution is strongly perturbed by the nonlinear effects which generate geometric scaling for the dipole cross section $\sigma_{dp}(x,r)$ and for the related quantities.

- Geometric scaling for the dipole cross-section implies geometric scaling for $\alpha_s(Q^2)xg(x,Q^2)/Q^2$, where $g(x,Q^2)$ denotes the gluon distribution. This follows from the LO relation between the dipole cross section and the gluon distribution, *i.e.* $\sigma(x,r^2) \sim r^2\alpha_s(1/r^2)xg(x,1/r^2)$.
- Geometric scaling for $\alpha_s(Q^2)xg(x,Q^2)/Q^2$ holds at the boundary $Q^2 = Q_s^2(x)$.
- For $Q^2 > Q_s^2(x)$ the non-linear screening effects can be neglected and evolution of parton densities is governed by the DGLAP equations.

One should, therefore, analyse solution of the DGLAP equation starting from the gluon distribution provided along the x dependent critical line $Q^2 = Q_s^2(x)$

$$Q_s^2(x) = Q_0^2 x^{-\lambda} \tag{5}$$

and satisfying geometric scaling along this line.

$$\alpha_s\left(Q^2\right) xg(x,Q^2)\Big|_{Q^2=Q_s^2(x)} = \text{const.}\,\frac{Q_s^2(x)}{Q_0^2}\,. \tag{6}$$

In order to analyse the solution of the DGLAP equation with the boundary conditions provided along the critical line instead at (x independent) value Q_0^2 it is convenient to go to the moment space where the solution of the DGLAP equation for the moment function $g_\omega(Q^2)$ of the gluon distribution reads

$$g_\omega\left(Q^2\right) = g_0(\omega)\exp\left[\gamma_{gg}(\omega)\xi\left(Q^2\right)\right]\,, \tag{7}$$

where

$$\gamma_{gg}(\omega) = \int_0^1 dz\, z^\omega P_{gg}(z) \tag{8}$$

and

$$\xi(Q^2) = \int_{Q_0^2}^{Q^2} \frac{dq^2}{q^2}\frac{\alpha_s\left(q^2\right)}{2\pi}\,. \tag{9}$$

Boundary condition along the x dependent critical line can be transformed into integral equation for $g_0(\omega)$ (see [8] for the details). The leading singularity of $g_0(\omega)$ generated by this equation controls the small x behaviour of the gluon distributions. We have considered both the fixed and running coupling cases and found the following results:

(a) Fixed coupling $\alpha_{\rm s}$:
Geometric scaling is found to be stable against the DGLAP evolution at small x provided

$$\lambda > 4N_{\rm c}\frac{\alpha_{\rm s}}{\pi}\,. \tag{10}$$

(b) Running coupling:
Geometric scaling is in general violated but this violation can be approximately factored out in the form of the form factor $VF(x)^{-1}$

$$VF(x) = \left[\frac{\ln\left(\frac{Q^2}{Q_{\rm s}^2(x)}\right)}{\ln\left(\frac{Q_{\rm s}^2(x)}{\Lambda^2}\right)}+1\right]^{1-b\gamma_{gg}(\lambda)}\,. \tag{11}$$

Geometric scaling still holds provided

$$\ln\left(\frac{Q^2}{Q_{\rm s}^2(x)}\right) \ll \ln\left(\frac{Q_{\rm s}^2(x)}{\Lambda^2}\right)\,, \tag{12}$$

where $VF(x)\sim 1$. The same condition has also been found in [9].

To summarise we would like to point out the following:

- Geometric scaling is expected to hold for $Q^2 < Q_{\rm s}^2(x)$.
- For $Q^2 > Q_{\rm s}^2(x)$ the non-linear effects are expected to be weak and parton distributions are expected to be controlled by the **linear** (BFKL or DGLAP) evolution.
- We solved the DGLAP evolution equations with the initial conditions provided along the critical line ($Q^2 = Q_{\rm s}^2(x)$).
- For fixed coupling geometric scaling is found to be stable against QCD DGLAP evolution provided $\lambda > 4N_{\rm c}\alpha_{\rm s}/\pi$.
- For the running coupling geometric scaling is found to be violated but this violation can be approximately factored out.
- In general geometric scaling is expected to hold even for $Q^2 > Q_{\rm s}^2(x)$ provided $\ln(Q^2/Q_{\rm s}^2(x)) << \ln(Q_{\rm s}^2(x)/\Lambda^2)$.

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