HARD DIFFRACTION IN HADRON–HADRON COLLISIONS*

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Breakdown of factorization observed recently in the diffractive dijet production in deep inelastic lepton induced and hadron induced processes is explained using the Good-Walker picture of diffraction dissociation. Numerical estimates agree with the recent data.

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1. Diffractive production of hard jets has been recently measured by the CDF collaboration [1]. When compared with the hard diffraction observed earlier at HERA [2, 3], these measurements revealed a strong violation of Regge factorization. The measured diffractive structure function is about one order of magnitude smaller than that predicted from factorization [1,4,5].

In the present note I would like to suggest that

(a) The observed effect can be understood in terms of the Good–Walker picture [6] in which the diffractive dissociation is treated as a consequence of absorption of the particle waves[1].

(b) The magnitude of the factorization breaking can be *quantitatively* estimated from the data on proton–proton elastic scattering.

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[1] Another version of this idea (rather different from the one presented here) was recently discussed in [4]. Other mechanisms proposed to explain the discrepancy were: *(i)* exchange of "soft" gluons which destroy the rapidity gap in hadron–hadron collisions [7, 8]; *(ii)* renormalization of the "Pomeron flux" to prevent violation of unitarity [9].

2. In the Good–Walker formulation of diffraction dissociation the incident particle state $|\psi\rangle$ is expanded into a complete orthonormal set of "diffractive eigenstates" $|\psi_n\rangle$ which are eigenstates of the scattering operator T:

$$T|\psi_n\rangle = t_n|\psi_n\rangle\,, \tag{1}$$

where the eigenvalues t_n are positive numbers[2], not greater than 1.

To calculate the amplitude for the transition from the incident state $|\psi\rangle$ to a final state $|\psi'\rangle$ (orthogonal to $|\psi\rangle$) one expands also $|\psi'\rangle$ into the set $|\psi_n\rangle$. Then the amplitude for the transition from $|\psi\rangle$ to $|\psi'\rangle$ can be expressed in terms of the expansion coefficients and the eigenvalues t_n.

This relation takes a particularly simple form [10, 11] if the expansion of the observed states into the diffractive states is quasi-diagonal, *i.e.* if we consider only small quantum fluctuations:

$$|\psi\rangle = |\psi_1\rangle + \varepsilon|\psi_2\rangle + ...\ ; \quad |\psi'\rangle = -\varepsilon^*|\psi_1\rangle + |\psi_2\rangle + \dots\,, \tag{2}$$

where ε, the probability amplitude for the fluctuation, is a small number (we shall neglect ε^2) [3]. The relation between the expansion coefficients of $|\psi\rangle$ and $|\psi'\rangle$ follows from the orthogonality condition.

Using (2) we obtain (keeping only the terms linear in ε)

$$\begin{aligned}\langle\psi'|T|\psi\rangle = \varepsilon\,(t_2 - t_1) \; &= \; \varepsilon\,(\langle\psi_2|T|\psi_2\rangle - \langle\psi_1|T|\psi_1\rangle) \\ &= \; \varepsilon\left(\langle\psi'|T|\psi'\rangle - \langle\psi|T|\psi\rangle\right)\,.\end{aligned} \tag{3}$$

This formula, discussed in a similar context already long time ago [10, 11], is the starting point of our further discussion.

To give a definite physical meaning to the Good–Walker picture we have to define the diffractive eigenstates. Following [12] (see also [4, 13]) we assume that the diffractive eigenstates are states with a fixed parton number and configuration in the transverse (impact parameter) space. This is a natural choice since the partons, being elementary, cannot be excited and, at high energy, their transverse configuration is expected to remain unchanged during the collision.

3. Consider first the photon-induced reaction: $|\gamma^*\rangle \to |\mathrm{jets}\rangle$. We write

$$|\gamma^*\rangle = |0\rangle + \varepsilon|\mathrm{hard}\rangle\,; \qquad |\mathrm{jets}\rangle = -\varepsilon^*|0\rangle + |\mathrm{hard}\rangle\,,\,, \tag{4}$$

where $|0\rangle$ denotes the state with no partons and $|\mathrm{hard}\rangle$ a state containing some hard partons (decaying into the large transverse momentum jets in the final state).

[2] We use the convention in which the high-energy elastic amplitudes (in impact parameter representation) are real.

[3] $\dots$ denote other possible small terms of the order ε. They do not affect our argument.

Substituting (4) into (3) we obtain

$$\langle \mathrm{jets}|T|\gamma^*\rangle = \varepsilon\left(\langle \mathrm{hard}|T|\mathrm{hard}\rangle - \langle 0|T|0\rangle\right) = \varepsilon\langle \mathrm{hard}|T|\mathrm{hard}\rangle \tag{5}$$

because $\langle 0|T|0\rangle = 0$. Eq. (5) is well known since the early discussion of vector dominance model [14].

4. Consider now the production of jets in diffractive proton–proton collisions[4], *i.e.* the transition $|P\rangle \to |P' + \mathrm{jets}\rangle$, where $|P\rangle$ denotes the incident proton and $|P' + \mathrm{jets}\rangle$ contains the soft proton remnants (P') and hard jets observed in the final state.

We thus write

$$\begin{aligned} |P\rangle &= |\mathrm{soft}\rangle + \varepsilon|\mathrm{soft}' + \mathrm{hard}\rangle\,, \\ |P' + \mathrm{jets}\rangle &= -\varepsilon^*|\mathrm{soft}\rangle + |\mathrm{soft}' + \mathrm{hard}\rangle\,. \end{aligned} \tag{6}$$

When introduced into (3) this gives

$$\langle P' + \mathrm{jets}|T|P\rangle = \varepsilon\left(\langle \mathrm{soft}' + \mathrm{hard}|T|\mathrm{soft}' + \mathrm{hard}\rangle - \langle \mathrm{soft}|T|\mathrm{soft}\rangle\right). \tag{7}$$

To exploit this formula we have to estimate the elastic amplitudes in the r.h.s. To this end we first find that up to first order in ε

$$\langle \mathrm{soft}|T|\mathrm{soft}\rangle = \langle P|T|P\rangle\,. \tag{8}$$

To estimate $\langle \mathrm{soft}' + \mathrm{hard}|T||\mathrm{soft}' + \mathrm{hard}\rangle$ we observe that it represents the amplitude for scattering of a system composed of two objects: the *soft* partons from the incident proton and the *hard* partons which decay into the observed final jets. We can thus apply the Glauber prescription [15] and write[5]

$$\begin{aligned} &\langle \mathrm{soft}' + \mathrm{hard}|T|\mathrm{soft}' + \mathrm{hard}\rangle = \langle \mathrm{soft}'|T|\mathrm{soft}'\rangle \\ &+\langle \mathrm{hard}|T|\mathrm{hard}\rangle - \langle \mathrm{hard}|T|\mathrm{hard}\rangle\langle \mathrm{soft}'|T|\mathrm{soft}'\rangle\,. \end{aligned} \tag{9}$$

Assuming, furthermore, that

$$\langle \mathrm{soft}'|T|\mathrm{soft}'\rangle \approx \langle \mathrm{soft}|T|\mathrm{soft}\rangle \tag{10}$$

we see that the soft amplitudes in (7) cancel and we obtain

$$\langle P' + \mathrm{jets}|T|P\rangle = \varepsilon\langle \mathrm{hard}|T|\mathrm{hard}\rangle\left(1 - \langle P|T|P\rangle\right), \tag{11}$$

where we have used (8).

[4] The same argument applies for any hadron–hadron collision.
[5] This idea was already proposed in [11].

When compared to (5), this formula explains the breakdown of the factorization between the (virtual)photon-induced and hadron-induced processes. The factor $(1 - \langle P|T|P\rangle)$ is usually interpreted as "absorption" of the initial state particles. One sees, however, from its derivation that it is actually a result of rather subtle cancellations between the interactions in the initial and final states.

5. Using (3) and the formula for (2×2) scattering [16], it is also not difficult to derive the result for the process of double diffraction dissociation. Up to first order in amplitudes involving hard scattering it reads (for the *symmetric* situation)

$$\langle P'_{\rm L} + J_{\rm L}, P'_{\rm R} + J_{\rm R}|T|P_{\rm L}, P_{\rm R}\rangle \approx 2\varepsilon^2 \langle {\rm hard}|T|{\rm hard}\rangle [1 - \langle P|T|P\rangle] \,. \qquad (12)$$

Comparing this with (5) and (11) one sees that the breaking of factorization should be about four times less effective in the double diffraction dissociation than the single one[6]. This result seems not too far from the recent experimental findings [17].

6. To estimate the size of the discussed effect we have taken the elastic pp amplitude in the form suggested in [18] from which one can calculate the impact parameter representation needed in (11). The product $\varepsilon \times \langle {\rm hard}|T|{\rm hard}\rangle$ was taken as a Gaussian $\sim \exp(-b^2/2B)$ where B is the slope of the *cross-section* in the (virtual)photon-induced process (3).

The hadron-induced diffraction dissociation *cross-section* can then be expressed as

$$\sigma(P \to P' + {\rm jets}) = R\, \sigma_{\rm factorized}(P \to P' + {\rm jets}) \,, \qquad (13)$$

where $\sigma_{\rm factorized}$ denotes the cross-section extrapolated from the deep inelastic scattering data, and R is calculated using (11).

R depends on one unknown parameter, B. For the inclusive diffraction at HERA one finds $B \approx 7{\rm GeV}^2$ [19]. For this value of B the numerical estimate of R (using $\sigma^{p\bar{p}}_{\rm tot} = 71.7$ mb [20]) gives $R = 0.10$. For production of heavy vector mesons $B \approx 4$ GeV^{-2} [21]. One can speculate that this is a lower limit for B which may be approximately valid for production of jets with a small mass (large β). For this value of B one obtains $R = 0.09$ Both numbers are in reasonable agreement with the recent phenomenological estimates [5].

[6] The factor 2 in the amplitude becomes 4 in the cross-section.

6. Some comments are in order.

(i) One sees from the discussion in Section 3 that the *uncorrected* formula (5) is valid independently of the virtuality of the incident photon: The same formula applies to photoproduction and to deep inelastic scattering. This emphasizes the (already mentioned) point: the effect we consider cannot be simply identified with absorption in the initial state of the process.

(ii) Using the cross-sections at other energies, one can investigate the energy dependence of the correction factor R. Taking $\sigma_{\rm tot}(630)= 63$ mb, one finds that $R(630)/R(1800)$ varies from ~ 1.5 ($B = 4$ GeV^{-2}) to ~ 1.2 ($B = 10$ GeV^{-2}), in a reasonable agreement with recent data from the CDF Collaboration [22].

(iii) In the numerical estimate of Section 5 we have assumed that the dipole corresponding to the two jets is created at the same impact parameter as the incident proton. This assumption seems rather natural but some deviations cannot be excluded. They would increase somewhat the correction factor R.

(iv) Our result given in Eq. (11) resembles, to some extent, the "renormalization" of the Pomeron flux, proposed in [9]. One should keep in mind, however, that the Eq. (11) refers to impact parameter space and thus it can be at best only approximately interpreted as the (corrected) Regge formula.

7. In conclusion, we have shown that the breakdown of Regge factorization between the diffractive production of hard jets observed at HERA and at FERMILAB is naturally explained in the Good–Walker picture of diffraction dissociation. The correction to the factorization formula is explicitely given in terms of the elastic $p\bar{p}$ amplitude at small momentum transfers. The numerical estimates seem to be consistent with the experimental findings.

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REFERENCES

[1] CDF Collab., T. Affolder *et al.*, *Phys. Rev. Lett.* **84**, 5043 (2000).

[2] H1 Collab., T. Ahmed *et al.*, *Phys. Lett.* **B348**, 681 (1955); C. Adloff *et al.*, *Z. Phys.* **C76**, 613 (1997); *Eur. Phys. J.* **C20**, 29 (2001).

[3] ZEUS Collab., M. Derrick *et al.*, *Z. Phys.* **C68**, 569 (1995); *Phys. Lett.* **B356**, 129 (1995); *Eur. Phys. J.* **C6**, 43 (1999).

[4] A.B. Kaidalov, V.A. Khoze, A.D. Martin, M.G. Ryskin, *Eur. Phys. J.* **C21**, 521 (2001).

[5] F.-P. Schilling, *Acta Phys. Pol.* **B33**, 3419 (2002), these proceedings.

[6] M.L. Good, W.D. Walker, *Phys. Rev.* **120**, 1857 (1960).

[7] W. Buchmueller, T. Gehrmann, A. Hebecker, *Nucl. Phys.* **B487**, 283 (1997); A. Hebecker, *Acta Phys. Pol.* **B30**, 3777 (1999); A. Edin, G. Ingelman, J. Rathsman, *Z. Phys.* **C75**, 57 (1997).

[8] V.A. Khoze, A.D. Martin, M.G. Ryskin, *Eur. Phys. J.* **C18**, 167 (2000); R. Enberg, G. Ingelman, N. Timneanu, *Phys. Rev.* **D64**, 114015 (2001) and references quoted there.

[9] K. Goulianos, *Phys. Lett.* **B358**, 379 (1995) and references quoted there.

[10] M. Ross, L. Stodolsky, *Phys. Rev.* **149**, 1172 (1966);

[11] J. Pumplin, *Phys. Rev.* **D4**, 3482 (1971); *Phys. Rev.* **D7**, 795 (1973); A. Bialas, W. Czyz, A. Kotanski, *Ann. Phys.* **73**, 439 (1972).

[12] L. Van Hove, K. Fialkowski, *Nucl. Phys.* **B107**, 211 (1976); L. Van Hove, *Acta Phys. Pol.* **B7**, 339 (1976); H.I. Miettinen, J. Pumplin, *Phys. Rev.* **D18**, 1696 (1978).

[13] A. Bialas, R. Peschanski, *Phys. Lett.* **B378**, 302 (1996).

[14] L. Stodolsky, *Phys. Rev. Lett.* **18**, 135 (1967); K. Gottfried, D.R. Yennie, *Phys. Rev.* **182**, 1585 (1969).

[15] R.J. Glauber, *Phys. Rev.* **100**, 242 (1955).

[16] W. Czyz, L. Maximon, *Ann. Phys. NY* **52**, 59 (1969).

[17] CDF Collab., T. Affolder *et al.*, *Phys. Rev. Lett.* **87**, 141802 (2001), and private communication from K. Goulianos.

[18] A. Donnachie, P.V. Landshoff, *Phys. Lett.* **B296**, 227 (1992); *Phys. Lett.* **B387**, 637 (1996).

[19] ZEUS Collab., J. Breitweg *et al.*, *Eur. Phys. J.* **C1**, 81 (1998).

[20] E811 Collab, C. Avila *et al.*, *Phys. Lett.* **B445**, 419 (1999).

[21] For a review, see J.A. Crittenden, *Exclusive Production of Neutral Vector Mesons at the Electron–Proton Collider HERA, Springer Tracks in Modern Physics*, Vol.140, Springer, Berlin, Heidelberg 1997.

[22] CDF Collab., D. Acosta, *Phys. Rev. Lett.* **88**, 151802 (2002).