COLOUR DIPOLES
AND DEEPLY VIRTUAL COMPTON SCATTERING*

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I report on an analysis of Deeply Virtual Compton Scattering (DVCS) within the dipole model, done in collaboration with M. McDermott and G. Shaw. The two models considered here are distinct in their structure and implications. They both agree with the available cross-section data on DVCS from HERA (see M. McDermott, R. Sandapen, G. Shaw, *Eur. Phys. J.* **C22**, 655 (2002)). Predictions for various asymmetries are also given.

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Why DVCS? On the theory side, we have an explicitly proven factorisation theorem [3], valid for *asymptotic* Q^2, which expresses the amplitude as a convolution in perturbatively calculable coefficients with generalised parton distributions[1]. A dipole analysis[2] of DVCS is complementary to this formal QCD analysis: going beyond leading twist, we can, in principle, establish quantitatively a regime in Q^2 for which the formal pQCD approach is valid. Furthermore, at the lepton level, the interference of DVCS with the purely real Bethe–Heitler (BH) process offers a unique opportunity to isolate the real and imaginary DVCS amplitude via various azimuthal angle asymmetries [7].

The dipole model is valid at low x, when a factorisation of time-scales allows us to express the forward diffractive amplitude as a convolution in the *photon wave functions* and the *dipole cross-section* as shown in (1)

$$\mathcal{A}(s, t = 0) = s \int dz \; d^2 d_{\mathrm{T}} \; \Psi_{\gamma^*}^*(d_{\mathrm{T}}, z, Q^2) \; \sigma_d \; \Psi_\gamma(d_{\mathrm{T}}, z, 0) \,. \qquad (1)$$

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[1] Leading order and next to leading order calculations have since been done [4,5].

[2] Donnachie and Dosch provided a dipole analysis in [6].

The conserved quantities during the interaction are the dipole size d_{T} and the longitudinal momentum fraction z carried by the quark. Since the final state photon is real, the final photon wave function is evaluated at $Q^2 = 0$. For small dipole size d_{T}, the photon wave function can be calculated perturbatively using the usual $-i\gamma^\mu$ QED vertex (see *e.g* [9]). The challenge is to model the dipole cross-section[3], σ_d, which encodes all the dynamics (perturbative and non-perturbative) of the dipole–proton interaction. Colour transparency dictates σ_d to vanish as r^2 as $r^2 \to 0$. On purely geometrical grounds, we expect a monotonic increase of σ_d with d_{T} and σ_d to become *hadron-like* at large d_{T}. We assume no flavour and z dependence. As for the energy dependence, dipole models fall into two main classes: either direct dependence on W or via x. It is an ongoing issue whether saturation effects should be incorporated in dipole models. *One of the models presented here does include saturation effects while the other does not.*

The FKS (Forshaw, Kerley, Shaw) dipole cross-section [10] is a sum of a soft and a hard term, each with Regge-like energy dependence.

$$\hat{\sigma}\left(W^2, d_{\mathrm{T}}\right) \;=\; a_0^{\mathrm{S}} P^{\mathrm{s}}\left(a_n^{\mathrm{s}}, d_{\mathrm{T}}\right) \left(d_{\mathrm{T}}^2 W^2\right)^{\lambda_{\mathrm{S}}} + P^{\mathrm{h}}\left(a_n^{\mathrm{h}}, d_{\mathrm{T}}\right)$$

$$\times \exp\left(-\nu_{\mathrm{H}} d_{\mathrm{T}}\right) \left(d_{\mathrm{T}}^2 W^2\right)^{\lambda_{\mathrm{H}}} , \tag{2}$$

where $P^{\mathrm{s}}(a_n^{\mathrm{s}}, d_{\mathrm{T}})$ and $P^{\mathrm{h}}(a_n^{\mathrm{h}}, d_{\mathrm{T}})$ are polynomials in d_{T}. A distinctive feature of the FKS model is that the authors modify the photon wave function at large d_{T} using a shifted Gaussian [10]

$$f(d_{\mathrm{T}}): \; \left|\psi\left(z, d_{\mathrm{T}}, Q^2\right)\right|^2 \to \left|\psi\left(z, d_{\mathrm{T}}, Q^2\right)\right|^2 f(d_{\mathrm{T}}).$$

All the free parameters in the FKS have been successfully fitted to F_2 and real photoabsorption data [10] and the model has been used to predict F_2^{D} [11]. In its present form, the FKS model does not include saturation effects. We use this fitted dipole cross-section to make a no-free parameter predictions for DVCS.

The MFGS (McDermott, Frankfurt, Guzey and Strikman) model [12] makes direct contact with pQCD, as for small d_{T} $(d_{\mathrm{T}} < d_{\mathrm{T,C}})$, it is directly related to the gluon distribution in the proton, through the well-known equation [13]

$$\hat{\sigma}_{\mathrm{pQCD}}(x, d_{\mathrm{T}}) = \frac{\pi^2 d_{\mathrm{T}}^2}{3} \; \alpha_s\left(\bar{Q}^2\right) \; xg\left(x_g, \bar{Q}^2\right) , \qquad \bar{Q}^2 = \frac{\lambda}{d_{\mathrm{T}}^2}. \tag{3}$$

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[3] A number of authors have proposed different models, see *e.g* [8] for an overview.

In an attempt to go beyond leading log, d_{T}-dependence is included in the scales $\bar{Q}$ and x_g [12]. For large dipole size, $d_{\mathrm{T}} > d_{\mathrm{T},\pi}$, where $d_{\mathrm{T},\pi}$ is the pion size, σ_d is matched onto the pion–proton cross-section. The dipole cross-section is linearly interpolated between $d_{\mathrm{T,C}}$ and $d_{\mathrm{T},\pi}$. What is the appropriate value for $d_{\mathrm{T,C}}$? At moderate x , $d_{\mathrm{T,C}} = 0.246$ fm, corresponding to $Q = Q_0 = 1.6$ GeV. However, for sufficiently low x, the strong rise in the gluon distribution makes the dipole cross-section for small d_{T} exceed that for large d_{T}. To prevent this, $d_{\mathrm{T,C}}$ is made to shift to increasingly small values as x decreases. In this way, saturation effects are included. *For $\lambda = 4$, this correction is not important for the HERA region but does become important above it.* For an exclusive process such as DVCS, it is necessary to use the generalised (or skewed) gluon distribution in (3), which has an additional dependence (as compared to the ordinary gluon distribution) on the skewedness parameter $\delta = x$. For our calculation, we adapted the (leading order) skewed evolution package of Freund and Guzey [14], with CTEQ4L gluon distributions as input.

To see the relative contribution of different dipole sizes to the amplitude, we integrate out the angular and z dependence in Eq. (1). The results are shown in Fig. 1. The FKS model has a larger contribution from large dipoles
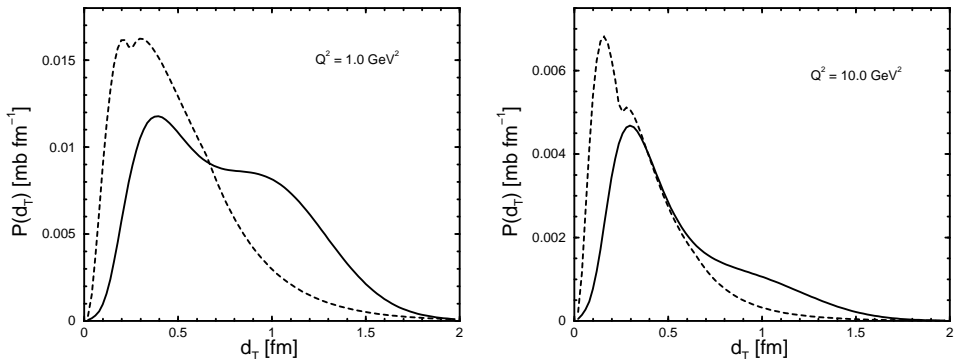

Fig. 1. Profiles in transverse dipole size for different Q^2 values and $W = 75\,\mathrm{GeV}$, FKS (solid line) and MFGS (dashed line).

than the MFGS model although the forward amplitudes obtained by integrating over d_{T} are similar. We assume the usual exponential t dependence of the differential cross-section and use a value[4] of 7 GeV^{-2} for the slope parameter B to calculate the photon-level total cross-section, $\sigma(\gamma^* p \to \gamma p)$. The theoretical predictions, which include the small contribution of the real part of the amplitude, are compared to H1 data points in Fig. 2. We reconstruct the real amplitude using analyticity. The FKS amplitude being

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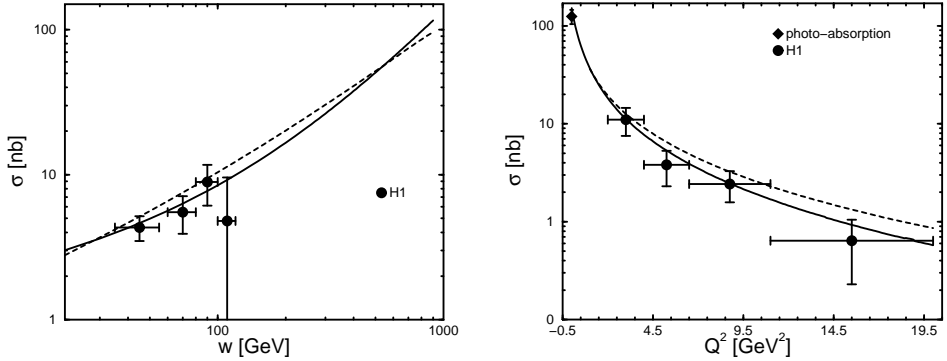[4] This is the value used by H1 in their analysis.

Fig. 2. The energy (at $Q^2 = 4.5\,\mathrm{GeV}^2$) dependence and Q^2 dependence ($W = 75\,\mathrm{GeV}$) of the photon level DVCS cross-section, FKS (solid line) and MFGS (dashed line).

a sum of two Regge terms, the real part is easily computed using the signature factors. As for the MFGS amplitude, we have to do a two power fit to the imaginary amplitude first. The FKS real amplitude shows a steeper energy dependence at very high energies as can be clearly seen in Fig. 3.
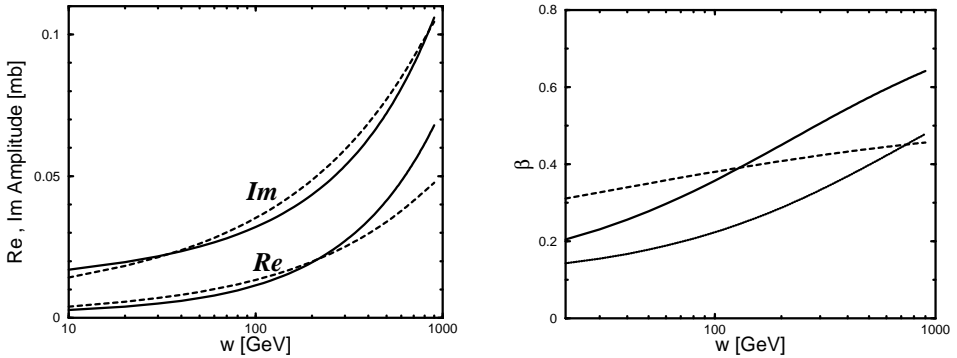

Fig. 3. The energy dependence (at $Q^2 = 4.5\,\mathrm{GeV}^2$) of the real and imaginary parts of the DVCS amplitude, FKS (solid line) and MFGS (dashed line). β is the real to imaginary parts ratio. Dotted line for FKS model, at $Q^2 = 0$.

As mentioned in the introduction, we can isolate the real and imaginary parts of the DVCS amplitude via various azimuthal angle asymmetries, defined and discussed in [1]. I simply highlight here that in the special frame of reference [1, 7] chosen for our calculation, the pure BH cross-section has a residual ϕ dependence. It is necessary to subtract off the BH contribution in defining the Azimuthal Angle Asymmetry (AAA) (see Eq. (37) of [1]) so that AAA becomes directly proportional to the real DVCS amplitude in

the joint limit of low x and high Q^2. In this limit, the charge asymmetry (Eq. (39) in [1]) is also directly proportional to the real DVCS amplitude, while the Single Spin Asymmetry (SSA)(Eq. (38) in [1]) is propotional to the imaginary DVCS amplitude. Our predictions for AAA and CA are shown in Fig. 4 [5].
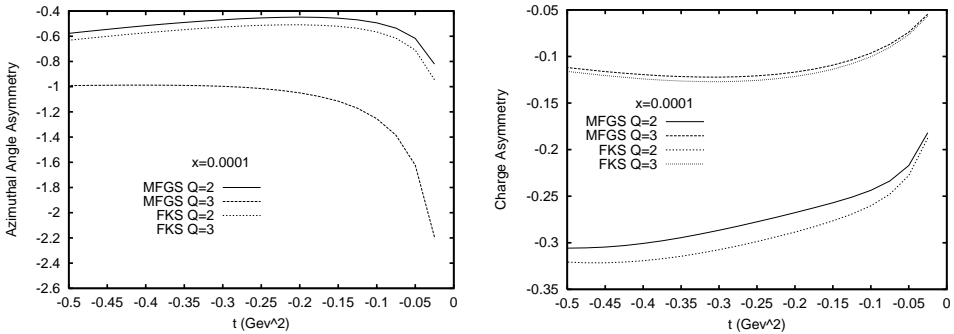

Fig. 4. Asymmetries for fixed $x = 10^{-4}$ at two values of $Q = 2, 3 \, \mathrm{GeV}^2$, accessible in the HERA kinematic range.

We have used two different dipole models to make predictions for DVCS. There is good agreement with H1 data for both models, even beyond the HERA region, despite only one of them including saturation effects. A more pronounced difference is found for the real amplitude, at very high energies. Experimental measurements of these asymmetries will allow us to test our predictions.

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