

QCD, DIFFRACTION AND STRING THEORY*

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(Received July 8, 2002)

Recently, string theory on some specific curved background spacetime geometries has been conjectured to be equivalent to certain gauge theories (AdS/CFT correspondence). This correspondence may be used to investigate the non-perturbative regime of gauge theories. I describe its application to the study of soft scattering amplitudes in a confining gauge theory. I describe two qualitatively different applications: amplitudes with vacuum quantum number exchange (Pomeron-like), amplitudes with Reggeon exchange. The last case requires going beyond eikonal approximation on the gauge theory side.

PACS numbers: 11.25.Sq, 12.38.Lg

1. Introduction

In the phenomenological description of (soft) diffractive processes in the Regge limit a prominent role is played by the various Regge poles and their couplings, like *e.g.* the 3-Pomeron vertex. Experimentally their properties are well established. One has two distinct cases. The dominant trajectory with vacuum quantum number exchange is the Pomeron, leading to amplitudes behaving like $s^{1.08+0.25t}$ in the soft regime. The other family of trajectories correspond to Reggeon exchanges (mesonic trajectories) and involves typically the exchange of flavour. The amplitudes behave here quite differently — with the leading amplitudes like $s^{0.55+1t}$. In particular the slope is almost exactly *four* times larger than for the Pomeron.

* Presented at the X International Workshop on Deep Inelastic Scattering (DIS2002) Cracow, Poland, 30 April–4 May, 2002.

It remains a formidable challenge to understand/derive these properties from more fundamental principles. The major stumbling block is of course the inherently nonperturbative character of these processes. In this talk I will describe an approach [1–3] to calculating the properties of these trajectories¹ within the framework of the AdS/CFT correspondence.

The AdS/CFT correspondence [4] is the conjectured equivalence between certain gauge theories and string theories on appropriate curved backgrounds. The utility of the correspondence comes from the fact that strong coupling problems in gauge theory side are mapped to quasi-classical problems on the string theory side. A precise version of this correspondence does not exist so far for QCD, so we used a generic version for a theory with confinement.

2. Pomeron dominated amplitudes

In the Pomeron channel, since we want to study soft processes and no flavour quantum numbers are exchanged it suffices to use the eikonal approximation [7]. In this approximation the impact parameter $q\bar{q}$ scattering amplitude is given by a correlation function of two Wilson lines which follow classical straight line trajectories:

$$A(s, L) = is \left\langle \exp \left(i \int_{L_1} A \right) \exp \left(i \int_{L_2} A \right) \right\rangle. \quad (1)$$

Technically we performed the calculation of the Wilson line correlator in Euclidean space (using AdS/CFT correspondence) as a function of the impact parameter L and the relative angle θ between the two lines. The result $A(\theta, L)$ was then continued back to *Minkowski* space using the substitutions $\theta \rightarrow -i\chi \sim -i \log s$ and $T \rightarrow iT$. The above procedure was first used within the eikonal approximation in perturbative QED and QCD in [8].

The $q\bar{q}$ amplitude as it stands is IR divergent. We regularized it by introducing a temporal cut-off by taking the Wilson lines to be of finite length, and adding gauge ‘connectors’ at both ends to close the lines into a loop.

Within the AdS/CFT correspondence the expectation value of a Wilson loop at strong coupling is given by [9]

$$\langle W(C) \rangle \sim \text{Fluctuations}(\Sigma_{\text{minimal}}) e^{-\frac{1}{2\pi\alpha'} \text{Area}(\Sigma_{\text{minimal}})}, \quad (2)$$

¹ For other approaches to the soft Pomeron see [5, 6].

where Σ_{minimal} is the surface of minimal area in the bulk of the geometry which is spanned on the contour C , and the prefactor represents the contribution of quadratic fluctuations of the string worldsheet around the minimal surface.

In our case, in the confining regime the relevant surface will be (a sector of) a helicoid. The explicit formula for its area is [1]

$$\text{Area} = \int_{-L/2}^{L/2} d\sigma \left\{ T \sqrt{1 + p^2 T^2} + \frac{1}{p} \log \left(pT + \sqrt{1 + p^2 T^2} \right) \right\}. \quad (3)$$

This result has to be analytically continued to Minkowski space. Naively we would obtain a pure phase. However, due to the presence of logarithmic cuts in the complex plane, we obtain very specific contributions when going through a cut. It is therefore interesting to explore the physical consequences of these contributions. Consequently we have to perform the substitution $\log(\dots) \rightarrow \log(\dots) - 2\pi i n$, with n being some integer number. Under this transformation, the amplitude gets a contribution:

$$\exp \left(\frac{1}{2\pi\alpha'_{\text{eff}}} \frac{L^2}{\theta} 2\pi i n \right) \rightarrow \exp \left(-\frac{1}{\alpha'_{\text{eff}}} \frac{L^2}{\log s} n \right) \quad (4)$$

which is *independent* of the IR cut-off T . In the following we will neglect the T dependent terms assuming that T is small (some justification for this assumption was given in [1]). After Fourier transform we obtain an inelastic amplitude with a linear Regge trajectory:

$$(\text{prefactor}) s^{1 + \frac{\alpha'_{\text{eff}}}{4} t}. \quad (5)$$

The prefactor here includes a $\log s$, further such contributions may come from α' corrections. In the following we concentrate on the dominant terms which give rise to a power-like s^α behaviour.

An interesting feature of this result is that the linear slope $\alpha'_{\text{eff}}/4$ characteristic of soft Pomeron exchange arose through the analytic structure of the helicoid area.

The contribution of quadratic fluctuations was evaluated in [2] using the fact that the dual string theory in the AdS/CFT picture is *critical*. The piece that dominates after continuation to Minkowski space for high energies is

$$\text{Fluctuations} = \exp \left(n_\perp \frac{\pi}{24} \frac{\theta}{2 \log \left(pT + \sqrt{1 + p^2 T^2} \right)} \right). \quad (6)$$

Let us now perform the same analytical continuation to Minkowski space, keeping in mind the substitution $\log \rightarrow \log - 2\pi i n$. Furthermore we neglect the logarithmic T dependent terms. The outcome is

$$\text{Fluctuations} = \exp\left(\frac{n_{\perp}}{96} \log s\right) = s^{n_{\perp}/96}. \quad (7)$$

Putting together the above results we obtain finally for the trajectory

$$(\text{prefactors}) s^{1+\frac{n_{\perp}}{96}+\frac{\alpha'_{\text{eff}}}{4}t}. \quad (8)$$

The values for $n_{\perp} = 7$ suggested by the AdS/CFT correspondence [10] would give an intercept of 1.073 (or 1.083 for $n_{\perp} = 8$), very close to the observed soft Pomeron intercept of 1.08. The phenomenological value of α'_{eff} extracted from the static quark–antiquark potential is $\alpha'_{\text{eff}} \sim 0.9 \text{ GeV}^{-2}$. This gives the slope 0.225 in comparison with the observed one of 0.25.

3. Reggeon dominated amplitudes

In order to be able to isolate an amplitude where the dominant contribution will be given by a Reggeon exchange, we have to consider a mesonic scattering amplitude with an *exchange* of two quarks (see [3] for a detailed discussion). The (position-space) amplitude typically involves four fermionic propagators, two of which can be calculated in the eikonal approximation (the spectator quarks), while the exchanged ones are described within the worldline formalism:

$$S(x, y|\mathcal{A}) = \int \mathcal{D}x^{\mu}(\tau) e^{-m \text{Length}} \{\text{Spin Factor}\} \exp\left(i \int_{\text{trajectory}} A\right), \quad (9)$$

where the {Spin Factor} keeps track of the spin 1/2 nature of the quarks. In the above expression the colour and spin parts do factorize, which is very convenient for calculations using various models of the nonperturbative gluonic vacuum.

The amplitude then becomes a path integral over the exchanged quark trajectories, the integrand being the spin factors and a Wilson loop formed by the spectators and the exchanged quarks [3]. In order to perform the Wilson loop average we use the AdS/CFT correspondence. In the approximation when the exchanged quarks are light, one can assume that the dominant contribution will come from the helicoid minimal surface spanned by the spectator quarks whose upper and lower boundaries are formed by the

exchanged quark trajectories. At this stage the expression for the amplitude has the following structure (in the $m \rightarrow 0$ limit):

$$is \int \mathcal{D}x(\tau) \{\text{Spin Factor}\} e^{-S_{\text{area}}[x(\tau)]}. \quad (10)$$

The $\{\text{Spin Factor}\}$ essentially gives a $1/s$ suppression characteristic of an exchange of two spin $1/2$ particles. The form of the effective action $S_{\text{area}}[x(\tau)]$ for the quark trajectories follows from the helicoid geometry. We evaluated the path integral using the saddle point configuration $\delta S_{\text{area}}[x(\tau)]/\delta x(\tau') = 0$. The saddle point turned out to be imaginary, consistent with an inelastic amplitude, and gave a linear trajectory with a slope of α'_{eff} , exactly four times larger as in the Pomeron case. The final result including the fluctuations of the string worldsheet around the helicoid gives the following result:

$$s^{0 + \frac{n+1}{24} + \alpha'_{\text{eff}} t}. \quad (11)$$

4. Discussion

The AdS/CFT correspondence lays down a framework for addressing nonperturbative properties of gauge theories including those intrinsically linked with confinement. Linear trajectories with the experimentally observed slopes arise naturally, although they are encoded in analytical properties of the helicoid minimal surface. String fluctuations lead to a rise of the intercepts. We observed a surprising quantitative agreement with the experimentally observed soft Pomeron intercept, while the Reggeon intercept, while qualitatively reasonable is lower than the observed value. It would be interesting to understand this better.

The vast majority of the results reported here were obtained in collaboration with Robi Peschanski. This work was supported in part by the Polish State Committee for Scientific Research (KBN) grant 2P03B09622 (2002–2004).

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