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SPECTRUM OF THE MULTI-REGGEON COMPOUND STATES IN MULTI-COLOUR QCD*

Jan Kotański

M. Smoluchowski Institute of Physics, Jagellonian University
Reymonta 4, 30-059 Kraków, Poland

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We study the properties of the colour-singlet compound states of reggeized gluons in multi-colour QCD. Applying the methods of integrable models, we calculate their spectrum and discuss the application of the obtained results to high-energy asymptotics of the scattering amplitudes in perturbative QCD.

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The high-energy asymptotics of the scattering amplitude is governed by the t-channel exchange of arbitrary number $N = 2, 3, ...$ of reggeized gluons described by the so-called "fish-net" or "checker-board" Feynman diagrams. The problem of resuming such diagrams formulated in sixties [1]. The lowest non-trivial contribution for $N = 2$ was calculated by Balitski, Fadin, Kuraev and Lipatov in 1978 [2], who derived and solved an equation for intercept of the compound state of $N = 2$ reggeized gluons — the BFKL Pomeron. The equation for three and more reggeized gluons was formulated in the 1980's by Bartels, Kwieciński, Praszałowicz [3] and Jaroszewicz [4]. However, it took almost twenty years to obtain the solution for $N = 3$, which corresponds to the QCD odderon [5, 6]. Finally, the solutions for higher $N = 4, ..., 8$ and their generalizations to arbitrary N were obtained recently in the multi-colour limit in series of papers [7–9] written in collaboration with S.É. Derkachov[1], G.P. Korchemsky[2], A.N. Manashov[3].

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[1] Department of Mathematics, St. Petersburg Technology Institute, St. Petersburg, Russia.

[2] Laboratoire de Physique Théorique, Université de Paris XI, 91405 Orsay Cédex, France.

[3] Department d'ECM, Universitat de Barcelona, 08028 Barcelona, Spain.

The leading contribution to the total elastic scattering amplitude of two hadrons (A,B) can be written in QCD in the Regge limit: $s\to\infty$, $t=$const., as a power series in a strong coupling constant $\overline{\alpha}_{\rm s}=\alpha_{\rm s}N_c/\pi$

$$\mathcal{A}(s,t)\sim -i\sum_{N=2}^{\infty}(i\overline{\alpha}_{\rm s})^N\frac{s^{1+\overline{\alpha}_{\rm s}\varepsilon_N}}{\sqrt{\overline{\alpha}_{\rm s}\ln s}}\,\beta_A^{(N)}(t)\beta_B^{(N)}(t)\,. \tag{1}$$

Above, N denotes the number of reggeized gluons, called Reggeons, propagating in the t-channel and the residue factors, $\beta^{(N)}_{A(B)}$, measure the overlap of the wave function of the compound state of N reggeized gluons with the wave functions of two scattered particles. The parameter ε_N is defined as the maximal energy for the Schrödinger (BKP) equation

$$\mathcal{H}_N\varPsi\left(\{\vec{z}_k\}\right)=\varepsilon_N\varPsi\left(\{\vec{z}_k\}\right)\,, \tag{2}$$

where $\varPsi\left(\{\vec{z}_k\}\right)$ is the Reggeon wave function and $\vec{z}_k$ denotes two-dimensional transverse coordinates of $k^{\rm th}$ reggeized gluon. $\mathcal{H}_N$ is the effective QCD Hamiltonian describing pair-wise interaction between N Reggeons.

In the multi-colour limit, this Hamiltonian simplifies significantly [10,11] leading to

$$\mathcal{H}_N=\sum_{k=0}^{N-1}H(\vec{z}_k,\vec{z}_{k+1})\,,\qquad \text{where}\qquad \vec{z}_0\equiv\vec{z}_N\,. \tag{3}$$

It describes [8] the nearest neighbor interaction of the Reggeons and has a hidden cyclic and mirror permutational symmetry. Moreover, it possesses the set of the $(N-1)$ integrals of motion, which are the eigenvalues of conformal charges, $\hat{q}_k$ and $\hat{\overline{q}}_k$ [4] [7],

$$[\mathcal{H}_N,\hat{q}_n]=[\hat{q}_n,\hat{q}_m]=\left[\mathcal{H}_N,\hat{\overline{q}}_n\right]=\left[\hat{\overline{q}}_n,\hat{\overline{q}}_m\right]=0,\quad n,m=2,\dots,N. \tag{4}$$

Thus, this system is completely integrable. The lowest integral may be expressed in terms of the conformal SL(2) weight of the state h as $q_2=-h(h\text{–}1)$. The Hamiltonian (3) is equivalent to the XXX Heisenberg spin magnet [7].

Our solution of the Schrödinger equation (2) is based on the method of the Baxter Q-operator [7]. It relies on the existence of the operator $Q(u,\bar{u})$ depending on the pair of complex spectral parameters u and $\bar{u}$ and satisfying the following relations. It commutes with itself for different values of the spectral parameters and with the integrals of motion

$$[\mathbb{Q}(u,\overline{u}),\mathbb{Q}(v,\overline{v})]=\left[\hat{t}_N(u,\{\hat{q}_n\}),\mathbb{Q}(v,\overline{v})\right]=\left[\hat{t}_N(\overline{u},\{\hat{\overline{q}}_n\}),\mathbb{Q}(v,\overline{v})\right]=0\,, \tag{5}$$

[4] *bar* does not denote complex conjugation for which we use an *asterisk*.

where

$$\hat{t}_N(u, \{\hat{q}_n\}) = 2u^N + \hat{q}_2 u^{N-1} + \ldots + \hat{q}_N\,, \tag{6}$$

and u, v are two complex spectral parameters. It also has to satisfy the Baxter equations

$$\begin{aligned}\hat{t}_N(u, \{\hat{q}_n\})\mathbb{Q}(u, \overline{u}) &= u^N \mathbb{Q}(u+i, \overline{u}) + u^N \mathbb{Q}(u-i, \overline{u})\,,\\ \hat{t}_N(\overline{u}, \{\hat{\overline{q}}_n\})\mathbb{Q}(u, \overline{u}) &= (\overline{u}+i)^N \mathbb{Q}(u, \overline{u}+i) + (\overline{u}-i)^N \mathbb{Q}(u, \overline{u}-i)\,.\end{aligned} \tag{7}$$

Furthermore, the Q-operator has prescribed analytical properties, *i.e.* known pole structure, and asymptotic behavior at infinity. The above conditions fix the Q-operator uniquely and allow us to quantize the integrals q_k [7,13–15]. It turns out that it is possible to express the Hamiltonian (3) in terms of the Baxter Q-operator [7]. Combining together the solutions of the Baxter equations and the quantum conditions for q_k with the Schrödinger equation (2) we can calculate the energy spectrum.

For $N = 3$ there exist two integrals of motion, q_2 and q_3. The quantized values of q_3 exhibit some structure that can be seen on Fig. 1 (left panel) [9]. The circled crosses denote the ground states. They have the highest energy for fixed N.

The spectrum of quantized q_3 at $N = 3$ may be approximated by the following formula [12]

$$\left[{q_3}^{\mathrm{approx}}(\ell_1, \ell_2)\right]^{1/3} = \frac{\Gamma^3(2/3)}{2\pi}\left(\frac{1}{2}\ell_1 + i\frac{\sqrt{3}}{2}\ell_2\right)\,, \tag{8}$$

where ℓ_1, $\ell_2 \in \mathbb{Z}$ and $\ell_1 + \ell_2$ is even. Similar structure is observed for $N = 4$, $h = 1/2$ and $q_3 = 0$ (Fig. 1 right panel)

$$\left[q_4^{\mathrm{approx}}(\ell_1, \ell_2)\right]^{1/4} = \frac{\Gamma^2(3/4)}{2\sqrt{\pi}}\left(\frac{1}{\sqrt{2}}\ell_1 + i\frac{1}{\sqrt{2}}\ell_2\right)\,. \tag{9}$$

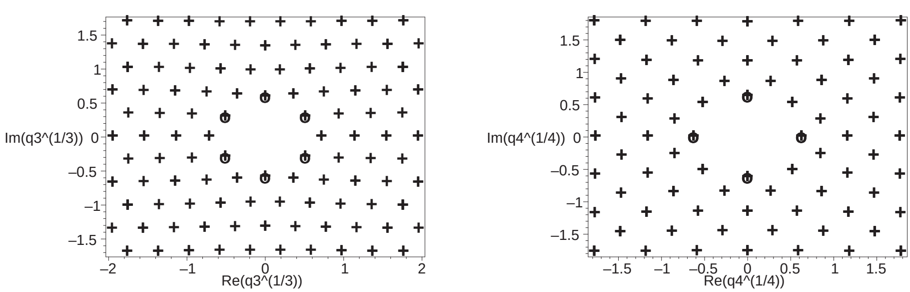

Fig. 1. Quantized values of the integrals of motion at $h = 1/2$ for different number of Reggeons $N = 3$ (left) and $N = 4$ (right).

All these states have continuation in $\nu_h = \Im(h)$, so, the eigenvalues of $\{\hat{q}_k\}$ form linear trajectories in the full q_k space [16,17]. The spectrum of q_N has similar structure for higher N.

For the ground states (Table I), the integrals of motion are either purely real or purely imaginary. Moreover, the odd integrals are equal to zero for even N's. The numbers at $N = 2$ and $N = 3$ agree with the previously published ones [2, 5, 8, 15, 17], while the results for higher N are new.

TABLE I

Quantum numbers q_k and energy, $\varepsilon_N(\{q_k\})$, of the N-Reggeons states in the multi-colour QCD for $h = 1/2$.

N	iq_3	q_4	iq_5	q_6	iq_7	q_8	ε_N
2							2.7726
3	0.20526						−0.2472
4	0	0.15359					0.6742
5	0.26768	0.03945	0.06024				−0.1275
6	0	0.28182	0	0.07049			0.3946
7	0.31307	0.07099	0.12846	0.00849	0.01950		−0.0814
8	0	0.39117	0	0.17908	0	0.03043	0.2810

The energy is positive for even N's and negative for odd N's and it is shown in a Fig. 2. The contribution of these states to the scattering amplitude increases with s for the positive energy ε_N and decreases for the negative energy ε_N. The exact values of the energy are denoted by crosses on the left panel of Fig. 2. The upper and the lower curves stand for the functions $1.8402/(N - 1.3143)$ and $-2.0594/(N - 1.0877)$, respectively.

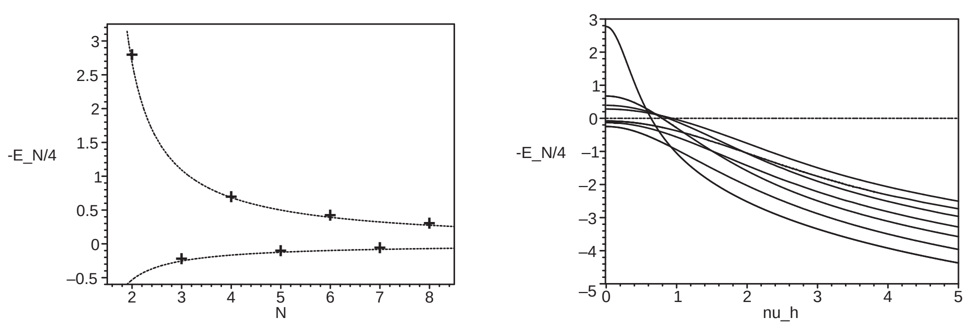

Fig. 2. The dependence of the ground state energy, ε_N, on the number of particles N and on the ν_h.

The right panel in the Fig. 2 shows the dependence of the energy $\varepsilon(\nu_h)$ along the ground state trajectory for different number of particles $2 \leq N \leq 8$. These functions are symmetrical in ν_h. The maximum energy is in $\nu_h = 0$. At large ν_h, $\varepsilon_8 > \ldots > \varepsilon_3 > \varepsilon_2$.

Summarizing we found the spectrum of the multi-Reggeon compound states in QCD. In the Pomeron sector (even N) the intercept of states is bigger than 1 but smaller than the intercept of the BFKL Pomeron: $\alpha_2 > \alpha_4 > \ldots > 1$. In the odderon sector (odd N) the intercept of the states is smaller than 1 but it increases with N: $\alpha_3 < \alpha_5 < \ldots < 1$.

Recently Lipatov and de Vega [18] found another set of the solutions for $N = 3, 4$ which differ from our expressions. This discrepancy requires further studies.

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