INSTANTONS AND SATURATION IN THE COLOUR DIPOLE PICTURE*

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We pursue the intriguing possibility that larger-size instantons build up diffractive scattering, with the marked instanton-size scale $\langle\rho\rangle \approx 0.5$ fm being reflected in the conspicuous "geometrization" of soft QCD. As an explicit illustration, the known instanton contribution to DIS is transformed into the intuitive colour dipole picture. With the help of lattice results, the $q\bar{q}$-dipole size r is carefully increased towards hadronic dimensions. Unlike pQCD, one now observes a competition between two crucial length scales: the dipole size r and the size ρ of the background instanton that is sharply localized around $\langle\rho\rangle \approx 0.5$ fm. For $r \gtrsim \langle\rho\rangle$, the dipole cross section indeed saturates towards a geometrical limit, proportional to the area $\pi\,\langle\rho\rangle^2$, subtended by the instanton.

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QCD instantons [1] are non-perturbative fluctuations of the gluon fields, with a size distribution *sharply* localized around $\langle\rho\rangle \approx 0.5$ fm according to lattice simulations [2] (Fig. 1(left)). They are well known to induce, chirality-violating processes, absent in conventional perturbation theory [3]. Deep-inelastic scattering (DIS) at HERA has been shown to offer a unique opportunity [4] for discovering such processes induced by *small* instantons (I) through a sizeable rate [5–7] and a characteristic final-state signature [4,8,9]. The intriguing but non-conclusive excess of events, found recently in the first dedicated search for instanton-induced processes in DIS at HERA [10], has also been reported at this meeting.

The validity of I-perturbation theory in DIS is warranted by some (generic) hard momentum scale $\mathcal{Q}$ that ensures a dynamical suppression [5] of contributions from larger size instantons with $\rho \gtrsim \mathcal{O}(1/\mathcal{Q})$. Here, the above mentioned intrinsic instanton-size scale $\langle\rho\rangle \approx 0.5$ fm is correspondingly unimportant.

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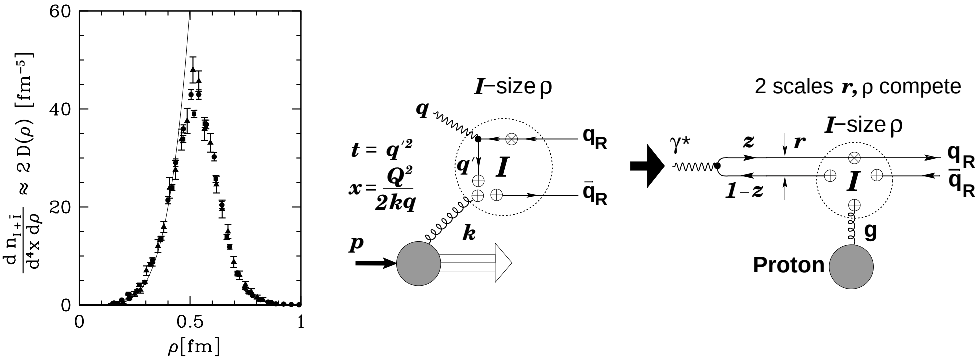

Fig. 1. (Left) $I+\overline{I}$-size distribution from the lattice [2,7]. Both the sharply defined I-size scale $\langle\rho\rangle \approx 0.5$ fm and the parameter-free agreement with I-perturbation theory for $\rho \lesssim 0.35$ fm are apparent. (Right) Transcription of the simplest I-induced process ($n_f = 1,\ n_g = 0$) with variables x and t into the colour dipole picture with the variables z and $\boldsymbol{r}$.

This paper, in contrast, is devoted to the intriguing question about the rôle of *larger-size* instantons and the associated intrinsic scale $\langle\rho\rangle \approx 0.5$ fm, for decreasing (Q^2, $x_{\rm Bj}$) towards the soft regime. We shall briefly report on a detailed study [11] of the interesting possibility that larger-size instantons may well be associated with a dominant part of soft high-energy scattering, or even make up diffractive scattering altogether [12–15]. We shall argue below that the intrinsic instanton scale $\langle\rho\rangle$ is reflected in the conspicuous *geometrization* of soft QCD.

There are two immediate qualitative reasons for this idea.

First of all, instantons represent truely non-perturbative gluons that naturally bring in an intrinsic size scale $\langle\rho\rangle \approx 0.5$ fm of hadronic dimension (Fig. 1(left)). The instanton-size scale happens to be surprisingly close to a corresponding "diffractive"-size scale, $R_{\mathbb{P}} = R\sqrt{\alpha^\prime_{\mathbb{P}}/\alpha^\prime} \approx 0.5$ fm, resulting from simple dimensional rescaling along with a generic hadronic size $R \approx 1$ fm and the abnormally small $\mathbb{P}$omeron slope $\alpha^\prime_{\mathbb{P}} \approx (1/4)\,\alpha^\prime$ in terms of the normal, universal Regge slope $\alpha^\prime$.

Secondly, we know already from I-perturbation theory that the instanton contribution tends to strongly increase towards the infrared regime [4,6,8,16]. The mechanism for the decreasing instanton suppression with increasing energy is known since a long time [15, 17]: Feeding increasing energy into the scattering process makes the picture shift from one of tunneling between vacua ($E \approx 0$) to that of the actual creation of the sphaleron configuration [18] on top of the potential barrier of height [4] $E = M_{\rm sphaleron} \propto \frac{1}{\alpha_{\rm s}\rho_{\rm eff.}}$. In a second step, the action is real and the sphaleron then decays into a multiparton final state.

The familiar colour dipole picture [19] represents a convenient and intuitive framework for investigating the transition from hard to soft physics (diffraction) in DIS at small $x_{\rm Bj}$. At the same time, this picture is very well suited for studying the crucial interplay between the $q\bar{q}$-dipole size r and the instanton size ρ in an explicit and well-defined manner, as we shall summarize next. The details may be found in Ref. [11].

The large difference of the $\gamma^* \to q\overline{q}$–dipole formation and $(q\overline{q})$–P interaction times in the proton's rest frame at small $x_{\rm Bj}$ is at the root of the familiar factorized expression of the inclusive photon–proton cross sections,

$$\sigma_{\rm L,T}(x_{\rm Bj},Q^2) = \int\limits_0^1 dz \int d^2\boldsymbol{r}\, \left|\Psi_{\rm L,T}(z,r)\right|^2 \sigma_{\rm dipole}(r,\ldots)\,, \tag{1}$$

in terms of the modulus squared of the (light-cone) wave function of the virtual photon, calculable in pQCD ($\hat{Q} = \sqrt{z(1-z)}Q;\ r = \mid \boldsymbol{r} \mid$),

$$\left|\Psi_{\rm L,T}(z,r)\right|^2 = e_q^2 \frac{6\alpha}{4\pi^2} N_{\rm L,T}(z)\, \hat{Q}^2\, {\rm K}_{0,1}\left(\hat{Q}r\right)^2; \qquad \begin{aligned} N_{\rm L} &= 4z(1-z)\,, \\ N_{\rm T} &= z^2 + (1-z)^2\,, \end{aligned} \tag{2}$$

and the dipole-P cross section $\sigma_{\rm dipole}(r,\ldots)$. The variables in Eq. (1) denote the transverse $(q\overline{q})$-size $\boldsymbol{r}$ and the photon's longitudinal momentum fraction z carried by the quark. $\Psi_{\rm L,T}(z,r)$ contains the dependence on the γ^*-helicity. Moreover, one derives [19,20] and expects, respectively,

$$\sigma_{\rm dipole} \begin{cases} \sim \quad \pi r^2, & r^2 \lesssim \mathcal{O}\left(\frac{1}{Q^2}\right), \quad \text{“colour transparency” [19,20]}, \\ \approx \quad \text{const.}, & r \gtrsim 0.5\ \text{fm}, \quad \text{“hadron-like, saturation”}. \end{cases}$$

The strategy is now to transform the known results on I-induced processes in DIS into this intuitive colour dipole picture. Here, for reasons of space, we restrict the discussion to the most transparent case of the simplest I-induced process [5], $\gamma^*\, g \Rightarrow q_{\rm R}\, \overline{q}_{\rm R}$, for one flavor and no final-state gluons (Fig. 1(right)). The more realistic case with gluons and three light flavors, using the $I\overline{I}$-valley approximation, may be found in Ref. [11].

The idea is to consider first large Q^2 and appropriate cuts on the variables z and r, such that I-perturbation theory holds. By exploiting the lattice results on the instanton-size distribution (Fig. 1(left)), we shall then carefully increase the $q\bar{q}$-dipole size r towards hadronic dimensions.

Let us start by recalling the results from Ref. [5],

$$\sigma_{\rm L,T}\left(x_{\rm Bj},Q^2\right) = \int\limits_{x_{\rm Bj}}^{1}\frac{dx}{x}\left(\frac{x_{\rm Bj}}{x}\right)G\left(\frac{x_{\rm Bj}}{x},\mu^2\right)\int dt\frac{d\hat{\sigma}^{\gamma^* g}_{\rm L,T}(x,t,Q^2)}{dt}\,, \quad (3)$$

$$\frac{d\hat{\sigma}^{\gamma^* g}_{\rm L}}{dt} = \frac{\pi^7}{2}\frac{e_q^2}{Q^2}\frac{\alpha}{\alpha_{\rm s}}\left[x(1-x)\sqrt{tu}\,\frac{R(-t)-R(Q^2)}{t+Q^2}-(t\leftrightarrow u)\right]^2 \quad (4)$$

and a similar expression for $d\hat{\sigma}^{\gamma^* g}_{\rm T}/dt$.

Eq. (4) involve the master integral $R(\mathcal{Q})$ with dimensions of a length,

$$R(\mathcal{Q}) = \int\limits_0^\infty d\rho\; D(\rho)\rho^5(\mathcal{Q}\rho){\rm K}_1(\mathcal{Q}\rho)\,. \quad (5)$$

The I-size distribution $D(\rho)$ enters in Eq. (5) as a crucial building block of the I-calculus. For small ρ (probed at large $\mathcal{Q}$) $D(\rho)$ is calculable within I-perturbation theory [3]. For larger I-size ρ (as relevant for smaller $\mathcal{Q}$) $D(\rho)$ is known from lattice simulations (Fig. 1(left)). A striking feature is the strong peaking, whence $R(0) = \int\limits_0^\infty d\rho\; D_{\rm lattice}(\rho)\rho^5 \approx \langle\rho\rangle$.

With an appropriate change of variables (Fig. 1(right)) and a $2d$-Fourier transformation, Eq. (4) may indeed be cast into a colour dipole form,

$$\sigma_{\rm L,T} = \int\limits_{x_{\rm Bj}}^{1}\frac{dx}{x}\int dt\,\frac{d\hat{\sigma}^{\gamma^* g}_{\rm L,T}}{dt}\{\ldots\} \Rightarrow \int dz\int d^2\boldsymbol{r}\;\left(|\Psi_{\rm L,T}|^2\sigma_{\rm dipole}\right)^{(I)}\,. \quad (6)$$

Like in pQCD-calculations [20], we invoke the familiar “leading $\log(1/x_{\rm Bj})$” approximation, $x_{\rm Bj}/xG(x_{\rm Bj}/x,\mu^2) \approx x_{\rm Bj}G(x_{\rm Bj},\mu^2)$. In terms of the familiar pQCD wave function (2) of the photon, we then obtain *e.g.*,

$$\left(|\Psi_{\rm L}|^2\,\sigma_{\rm dipole}\right)^{(I)} \approx \left|\Psi_{\rm L}^{\rm pQCD}(z,r)\right|^2\frac{1}{\alpha_{\rm s}}\,x_{\rm Bj}\,G(x_{\rm Bj},\mu^2)\,\frac{\pi^8}{12}$$

$$\times\left(\int\limits_0^\infty d\rho D(\rho)\,\rho^5\left\{\frac{-\frac{d}{dr^2}\left(2r^2\frac{{\rm K}_1(\hat{Q}\sqrt{r^2+\rho^2/z})}{\hat{Q}\sqrt{r^2+\rho^2/z}}\right)}{{\rm K}_0(\hat{Q}r)}-(z\leftrightarrow 1-z)\right\}\right)^2. \quad (7)$$

As expected, one explicitly observes a *competition* between two crucial length scales in Eq. (7): the size r of the $q\bar{q}$-dipole and the typical size of the background instanton of about $\langle\rho\rangle \approx 0.5$ fm. Like in pQCD, the *asymmetric* configuration, $z \gg 1-z$ or $1-z \gg z$, obviously dominates.

The validity of strict I-perturbation theory, $D(\rho) = D_{I-\mathrm{pert}}(\rho)$ in Eq. (5), requires the presence of a hard scale $\mathcal{Q}$ along with certain cuts. However, after replacing $D(\rho)$ by $D_{\mathrm{lattice}}(\rho)$ (Fig. 1(left)), these restrictions are at least no longer necessary for reasons of convergence of the ρ-integral (5) *etc.*, and one may tentatively increase the dipole size r towards hadronic dimensions.

Next, we note in Eq. (7),

$$-\frac{d}{d\,r^2}\left(2\,r^2\frac{\mathrm{K}_1\left(\hat{Q}\sqrt{r^2+\rho^2/z}\right)}{\hat{Q}\sqrt{r^2+\rho^2/z}}\right) \approx \begin{cases} -\frac{\mathrm{K}_1\left(Q\,\rho\sqrt{1-z}\right)}{Q\,\rho\sqrt{1-z}} & \frac{r^2\,z}{\rho^2} \Rightarrow 0, \\ \mathrm{K}_0\left(\hat{Q}\,r\right) & \frac{r^2\,z}{\rho^2}\ \text{large}. \end{cases} \tag{8}$$

Due to the strong peaking of $D_{\mathrm{lattice}}(\rho)$ around $\rho \approx \langle\rho\rangle$, one finds from Eqs. (7), (8) ($z \gg 1-z$ without restriction) for the limiting cases of interest

r	$\left(\mid \Psi_{\mathrm{L,T}} \mid^2 \sigma_{\mathrm{dipole}}\right)^{(I)}$
$r^2 \Rightarrow 0$	$\mathcal{O}(1)$, but exponentially small for large $\hat{Q}$,
	$\left\lvert\Psi^{\mathrm{pQCD}}_{\mathrm{L,T}}\right\rvert^2 \sigma^{(I)}_{\mathrm{dipole}}$ with
$r^2 \gtrsim \langle\rho\rangle^2$:	$\sigma^{(I)}_{\mathrm{dipole}} = \frac{1}{\alpha_{\mathrm{s}}}\,x_{\mathrm{Bj}}\,G(x_{\mathrm{Bj}},\mu^2)\,\frac{\pi^8}{12}\left(\int\limits_0^\infty d\rho\,D_{\mathrm{lattice}}(\rho)\,\rho^5\right)^2.$

(9)

In conclusion: As apparent in Eq. (9), the dipole cross section indeed *saturates* for large $r^2/\rho^2 \approx r^2/\langle\rho\rangle^2$ towards a *geometrical limit*, proportional to the area $\pi\,R(0)^2 = \pi\left(\int_0^\infty d\rho\,D_{\mathrm{lattice}}(\rho)\,\rho^5\right)^2$, subtended by the instanton. Clearly, without the crucial information about $D(\rho)$ from the lattice (Fig. 1(left)), the result would be infinite. Note the inverse power of α_{s} in front of $\sigma^{(I)}_{\mathrm{dipole}}$ in Eq. (9), signaling its non-perturbative nature.

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