

GVMD MODEL PREDICTIONS
FOR THE LOW Q^2 BEHAVIOUR
OF THE SPIN STRUCTURE FUNCTION $g_1(x, Q^2)$
AND OF THE DHGHY INTEGRAL $I(Q^2)^*$

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Predictions for $g_1(x, Q^2)$ at low Q^2 are obtained in the framework of the GVMD model. Contributions from both light and heavy vector mesons are evaluated. The DHGHY sum rule is employed to fix the magnitude of the light vector meson contribution to g_1 , using the recent measurements in the region of baryonic resonances. The DHGHY moment function is calculated. Predictions are compared to the data.

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1. Introduction

Data on polarised nucleon structure function $g_1(x, Q^2)$ are now available at low values of (negative) four-momentum transfer Q^2 [1, 2]. This is of particular interest since non-perturbative mechanisms dominate the particle dynamics there and a transition from soft- to hard physics may be studied.

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In the previous attempt [3] g_1 at low x and low Q^2 was described within a formalism based on the unintegrated spin dependent parton distributions, incorporating the leading order Altarelli–Parisi evolution and the double $\ln^2(1/x)$ resummation at low x . A VMD-type non-perturbative part of g_1 was also included, its unknown normalisation was extracted from the data and turned out to be nonzero and negative.

In this paper we apply the Generalised Vector Meson Dominance (GVMD) model. To evaluate the non-perturbative contributions to the polarised structure function $g_1(x, Q^2)$ at low values of Q^2 . The heavy meson ($M_V > Q_0$) contribution is directly related to the structure function in the scaling region, g_1^{AS} , described by the QCD improved parton model, suitably extrapolated to the low Q^2 region. The contribution of light ($M_V < Q_0$) vector mesons describes non-perturbative effects and vanishes as $1/Q^4$ for large Q^2 . At low Q^2 these effects are large and predominant. Here M_V denotes the mass of a vector meson. Then the Drell–Hearn–Gerasimov–Hosoda–Yamamoto (DHGHY) sum rule [4] together with measurements in the resonance region are employed to fix the magnitude of the light vector meson contribution to g_1 .

2. The GVMD representation of the structure function $g_1(x, Q^2)$ and the DHGHY sum rule

In the GVMD model, g_1 has the following representation, valid for fixed $W^2 \gg Q^2$, *i.e.* small values of x ,

$$\begin{aligned}
 x &= \frac{Q^2}{(Q^2 + W^2 - M^2)}, \\
 g_1(x, Q^2) &= g_1^{\text{L}}(x, Q^2) + g_1^{\text{H}}(x, Q^2) \\
 &= \frac{M\nu}{4\pi} \sum_V \frac{M_V^4 \Delta\sigma_V(W^2)}{\gamma_V^2 (Q^2 + M_V^2)^2} + g_1^{\text{AS}}(\bar{x}, Q^2 + Q_0^2). \quad (1)
 \end{aligned}$$

The first term sums up contributions from light vector mesons, $M_V < Q_0$ where $Q_0^2 \sim 1 \text{ GeV}^2$ [5]. Here W is the invariant mass of the electro-produced hadronic system, $\nu = Q^2/2Mx$, and M is the nucleon mass. The constants γ_V^2 are determined from the leptonic widths of the vector mesons and the cross sections $\Delta\sigma_V(W^2)$ are combinations of the total cross sections for the scattering of polarised mesons and nucleons. They are not known and have to be parametrised. Following Ref. [3], we assume that they can be expressed through the combinations of non-perturbative parton distributions, $\Delta p_j^{(0)}(x)$, evaluated at fixed Q_0^2 .

The second term in (1), $g_1^H(x, Q^2)$, which represents the contribution of heavy ($M_V > Q_0$) vector mesons to $g_1(x, Q^2)$ can also be treated as an extrapolation of the QCD improved parton model structure function, $g_1^{\text{AS}}(x, Q^2)$, to arbitrary values of Q^2 . Here the scaling variable x is replaced by $\bar{x} = (Q^2 + Q_0^2)/(Q^2 + Q_0^2 + W^2 - M^2)$, [5]. It follows that $g_1^H(x, Q^2) \rightarrow g_1^{\text{AS}}(x, Q^2)$ as Q^2 is large. We thus get

$$g_1(x, Q^2) = C \left[\frac{4}{9} \left(\Delta u_{\text{val}}^{(0)}(x) + \Delta \bar{u}^{(0)}(x) \right) + \frac{1}{9} \left(\Delta d_{\text{val}}^{(0)}(x) + \Delta \bar{d}^{(0)}(x) \right) \right] \\ \times \frac{M_\rho^4}{(Q^2 + M_\rho^2)^2} + C \left[\frac{1}{9} \left(2\Delta \bar{s}^{(0)}(x) \right) \right] \frac{M_\phi^4}{(Q^2 + M_\phi^2)^2} \\ + g_1^{\text{AS}}(\bar{x}, Q^2 + Q_0^2) . \quad (2)$$

The only free parameter in (2) is the constant C . Its value may be fixed in the photoproduction limit where the first moment of $g_1(x, Q^2)$ is related to the anomalous magnetic moment of the nucleon via the DHGHY sum rule, *cf.* [6, 7]

$$I(0) = I_{\text{res}}(0) + M \int_{\nu_t(0)}^{\infty} \frac{d\nu}{\nu^2} g_1(x(\nu), 0) = \frac{-\kappa_{p(n)}^2}{4}, \quad (3)$$

where the DHGHY moment before taking the $Q^2=0$ limit has been split into two parts, corresponding to $W < W_t \sim 2$ GeV (baryonic resonances) and $W > W_t$

$$I(Q^2) = I_{\text{res}}(Q^2) + M \int_{\nu_t(Q^2)}^{\infty} \frac{d\nu}{\nu^2} g_1(x(\nu), Q^2) . \quad (4)$$

Here $\nu_t(Q^2) = (W_t^2 + Q^2 - M^2)/2M$. Substituting $g_1(x(\nu), 0)$ in Eq. (3) by Eq. (2) at $Q^2 = 0$ we may obtain the value of C from (3) if $I_{\text{res}}(0)$, the contribution from resonances, is known *e.g.* from measurements.

3. Numerical calculations for the proton

To obtain the value of C from Eq. (3), $I_{\text{res}}(0)$ was evaluated using the preliminary data taken at ELSA/MAMI by the GDH Collaboration [8] at the photoproduction, for $W_t = 1.8$ GeV. The g_1^{AS} was parametrised using GRSV2000 fit [9] for the “standard scenario” at the NLO accuracy. The $\Delta p_j^{(0)}(x)$ in Eq. (2) were evaluated at fixed $Q^2 = Q_0^2$, using, either

(i) the GRSV2000 fit, or (ii) a simple, “flat” input, $\Delta p_i^{(0)}(x) = N_i(1-x)^{\eta_i}$ with $\eta_{u_V} = \eta_{d_V} = 3$, $\eta_{\bar{u}} = \eta_{\bar{s}} = 7$ and $\eta_g = 5$, [10]. We have assumed $Q_0^2 = 1.2 \text{ GeV}^2$ as in the analysis of F_2 , [5]. As a result the constant C was found to be -0.30 in case (i) and -0.24 in case (ii). These values change at most by 13% when Q_0^2 changes in the interval $1.0 < Q_0^2 < 1.6 \text{ GeV}^2$.

Negative value of the non-perturbative, vector meson dominance, contribution was also obtained in [3] and from the phenomenological analysis of the sum rules [7, 11].

Our g_1 , Fig. 1(a), reproduces well a general trend in the data; however, experimental errors are too large for a more detailed analysis. To compute the DHGHY moment, Eq. (4), for the proton, we used the preliminary results of the JLAB E91-023 experiment [12] for $0.15 \lesssim Q^2 \lesssim 1.2 \text{ GeV}^2$ and $W < W_t = W_t(Q^2)$ [13]. Results, Fig. 1(b), show that partons contribute significantly even at $Q^2 \rightarrow 0$, where the main part of the $I(Q^2)$ comes from resonances.

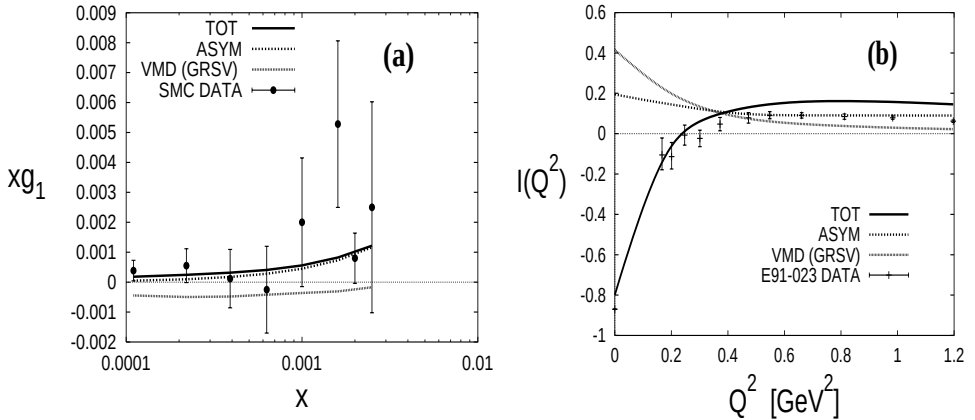


Fig. 1. (a) Values of xg_1 for the proton as a function of x at the measured values of Q^2 in the non-resonant region, $x < x_t = Q^2/2M\nu_t(Q^2)$. Both the VMD input and g_1^{AS} have been evaluated using the GRSV fit for standard scenario at the NLO accuracy [9]. Contributions of the VMD and of the xg_1^{AS} are shown separately. Points are the SMC measurements at $Q^2 < 1 \text{ GeV}^2$, [2]; errors are total. The curves have been calculated at the measured x and Q^2 values. (b) The DHGHY moment $I(Q^2)$ for the proton. Details as in Fig. 1(a). Points mark the contribution of resonances as measured by the JLAB E91-023, [12] at $W < W_t(Q^2)$.

In Fig. 2 we show our DHGHY moment together with the results of calculations of Refs. [11, 14] as well as with the E91-023 measurements in the resonance region used as an input to our $I(Q^2)$ calculations. We also show the E91-023 data corrected by their authors for the deep inelastic

contribution. Our calculations are slightly larger than the DIS-corrected data and then the results of [11] but clearly lower than the results of [14] which overshoot the data.

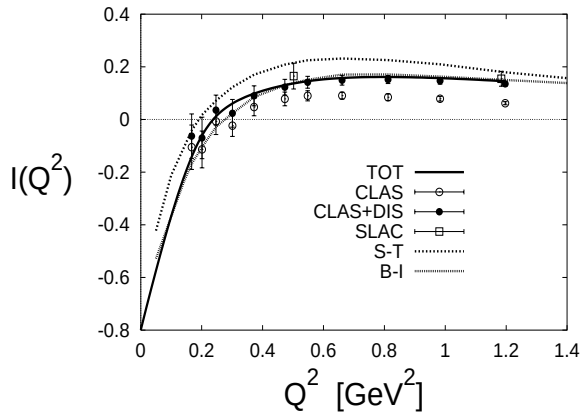


Fig. 2. The DHGHY moment $I(Q^2)$ for the proton with the VMD part parametrized using the GRSV fit [9]. Shown are also calculations of [11] (“B–I”) and [14] (“S–T”). Points marked “CLAS” are from the JLAB E91-023 experiment [12]: the open circles refer to the resonance region, $W < W_t(Q^2)$ and the full circles contain a correction for the DIS contribution. Errors are total.

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REFERENCES

- [1] SLAC, E143, K. Abe *et al.*, *Phys. Rev.* **D58**, 112003 (1998); SMC, B. Adeva *et al.*, *Phys. Rev.* **D58**, 112001 (1998); HERMES, U. Stössl, *Acta Phys. Pol.* **B33**, 2813 (2002), this proceedings part I.
- [2] SMC, B. Adeva *et al.*, *Phys. Rev.* **D60**, 072004 (1999).
- [3] B. Badelek, J. Kiryluk, J. Kwieciński, *Phys. Rev.* **D61**, 014009 (2000).
- [4] S.D. Drell, A.C. Hearn, *Phys. Rev. Lett.* **16**, 908 (1966); S.B. Gerasimov, *Sov. J. Nucl. Phys.* **2**, 430 (1966); M. Hosoda, K. Yamamoto, *Prog. Theor. Phys. Lett.* **36**, 425 (1966).

- [5] B. Badelek, J. Kwieciński, *Phys. Lett.* **B295**, 263 (1992).
- [6] B.L. Ioffe, *Surv. High Energ. Phys.* **8**, 107 (1995).
- [7] B.L. Ioffe, *Phys. At. Nucl.* **60** (1997) 1707.
- [8] K. Helbing, GDH Collaboration, *Nucl. Phys. Proc. Suppl.* **105**, 113 (2002).
- [9] M. Glück, E. Reya, M. Stratmann, W. Vogelsang, *Phys. Rev.* **D63**, 094005 (2001).
- [10] M. Stratmann, [hep-ph/9710379](http://arxiv.org/abs/hep-ph/9710379).
- [11] V. Burkert, B.L. Ioffe, *Phys. Lett.* **B296**, 223 (1992); V. Burkert, B.L. Ioffe, *Sov. Phys. JETP* **78**, 619 (1994).
- [12] JLAB E91-023, R. De Vita, talk at BARYONS 2002, Jefferson Lab., March 3–8, 2002, <http://www.jlab.org/baryons2002/program.html>.
- [13] JLAB E91-023, R. Fatemi, private communication (June 2002).
- [14] J. Soffer, O. Teryaev, *Phys. Rev. Lett.* **70**, 3373 (1993).