UNIVERSAL QED CORRECTIONS TO POLARIZED ELECTRON SCATTERING IN HIGHER ORDERS*

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We derive QED radiators for the universal corrections to polarized electron scattering. To 5^{th} order in the coupling constant the flavor non-singlet and singlet contributions are calculated. We derive the non-singlet and singlet exponentiation of the leading terms $\propto (\alpha \ln^2(x))^k$ to all orders.

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1. Introduction

QED corrections to differential cross sections can be quite large in some kinematic regions both in deep inelastic scattering [1] and e^+e^- scattering. Besides of the process-dependent radiative QED corrections in all the scattering processes classes of universal contributions emerge, which, when having been known once at high precision have not to be calculated again, but can just be used in novel applications. These contributions may be attributed to radiation from outer legs (light fermions) and correspond to those due to *(i)* leading order mass factorization and *(ii)* the resummation of leading order universal contributions $\propto (\alpha \ln^2(x))^k$ to all orders. The latter terms form universal pieces in the higher order anomalous dimension matrices [2–5]. In Ref. [6] we derived the corrections and give a brief summary here.

2. Complete leading order solutions to $O(\alpha^5)$

Unlike the case for parton distributions in QCD the evolution equations cannot be solved simply numerically since the sources are $\propto \delta(1-x)$, the Mellin transforms of which show no damping behaviour as $\mathrm{Re}(N) \to -\infty$.

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Therefore the corresponding Mellin convolutions have to be carried out analytically. All convolution integrals to be used are given in [6]. We define non-singlet and singlet distribution functions for the electron, $D_{\rm NS}(a,x) = D^{e^-}(a,x) - D^{e^+}(a,x)$, $D^e_\Sigma(a,x) = D^{e^-}(a,x) + D^{e^+}(a,x)$, with $a = \alpha/(4\pi)$. The singlet distribution mixes with the photon distribution $D^\gamma(a,x)$ under evolution.

2.1. The non-singlet case

We calculate the Mellin convolution in x space term by term using the Mellin transformation of Nielsen and related functions [6,7]. Using different techniques these contributions were also calculated in [8,9]. A very compact result can be obtained using both soft-photon exponentiation and the asymptotic solution at small x [10] for all orders :

$$D_{\rm NS}(x,\beta) = \left[\frac{\exp[\beta/2(3/4-\gamma_E)]}{\Gamma(1+\beta/2)}\frac{\beta}{2}(1-x)^{\beta/2-1}2\frac{{\rm I}_1\left((-\beta\ln(x))^{1/2}\right)}{[-\beta\ln(x)]^{1/2}} \times\sum_{n=0}^{\infty}\left(\frac{\beta}{2}\right)^n\Psi_n(x)\right]_+ . \tag{1}$$

The functions $\Psi_k(x)$ are given by

$$\Psi_0(x) = 1+x^2\,, \tag{2}$$

$$\Psi_1(x) = -\frac{1}{2}\left[(1-x)^2+x^2\ln(x)\right]\,, \tag{3}$$

$$\Psi_2(x) = \frac{1}{4}(1-x)\left[1-x-x\ln(x)+(1+x){\rm Li}_2(1-x)\right]\,, \tag{4}$$

$$\Psi_3(x) = -\frac{1}{48}\Bigg\{6(1-x^2)[2{\rm Li}_3(1-x)+{\rm Li}_2(1-x)]+5(x-1)^2 +\frac{1}{12}x^2\ln^3(x)+(1+7x^2)[\ln(x){\rm Li}_2(1-x)+2{\rm S}_{1,2}(1-x)] -\left(\frac{1}{2}+6x-\frac{13}{2}x^2\right)\ln(x)\Bigg\}, \tag{5}$$

$$\Psi_4(x) = \frac{1}{96}\Bigg\{(1-x^2)\Bigg[24{\rm Li}_4(1-x)+12{\rm Li}_3(1-x)-\frac{5}{2}{\rm S}_{1,3}(1-x) -12{\rm S}_{2,2}(1-x)-\frac{3}{2}\ln(x){\rm S}_{1,2}(1-x)-\frac{1}{4}\ln^2(x){\rm Li}_2(1-x)\Bigg] +4(1+x^2){\rm Li}_2^2(1-x)+7{\rm Li}_2(1-x)$$

$$+2(1+7x^2)\ln(x)\mathrm{Li}_3(1-x) - \left(\frac{3}{4}+5x-\frac{23}{4}x^2\right)\ln(x)$$
$$-\frac{1}{12}x(1-x)\ln^3(x) - \frac{1}{48}x^2\ln^4(x) + (1-x)^2\left[\frac{7}{2}+\frac{1}{8}\ln^2(x)\right]$$
$$+(1-8x+7x^2)\left[\ln(x)\mathrm{Li}_2(1-x)+2\mathrm{S}_{1,2}(1-x)\right]\Bigg\}. \qquad (6)$$

Here $\beta = (2/\pi)\int_{m_e^2}^{s}(ds'/s')\alpha(s')$. These results agree with Ref. [8].

2.2. The singlet case

Since there is no exponentiation formula in the singlet case, the evolution equations cannot be reduced to a simple form as in the non-singlet case and term by term convolution forms the final result.

The singlet distribution for electrons is given in matrix form as,

$$\boldsymbol{D}_{\Sigma}(a,x) = \boldsymbol{E}_{\mathrm{s}}(a,x)\otimes\begin{pmatrix}\delta(1-x)\\ 0\end{pmatrix}. \qquad (7)$$

Here $\otimes$ indicates both matrix multiplication and Mellin convolution. $\boldsymbol{E}_{\mathrm{s}}(a,x)$ is the evolution operator given by

$$\boldsymbol{E}_{\mathrm{s}}(x,\beta) = \mathbf{1}\,\delta(1-x) + \sum_{k=1}^{\infty}\frac{1}{k!}\boldsymbol{P}_0^{(k)}(x)\left(-\frac{1}{\beta_0}\ln\left(\frac{a}{a_0}\right)\right)^k, \qquad (8)$$

$$\boldsymbol{P}_0^{(k)}(x) = \otimes_{l=1}^{k}\boldsymbol{P}_0\,. \qquad (9)$$

The Mellin convolutions can be separated into the non-singlet part and a pure singlet part for the fermions. The expressions are rather lengthly and are given in Ref. [6] in explicit form.

3. Resummation of small x logarithms

The resummation of the terms $\propto \alpha^n \log^{2n}(z)$ is carried out using infrared evolution equations [11]. Applications to QCD evolution were considered in [2,3] and for unpolarized QED in [5]. The contributions under consideration form the most singular parts of the anomalous dimensions in the respective order as $x \to 0$ and have to be treated as such in the evolution equations.

3.1. Non-singlet case

The leading double-log terms in non-singlet evolution kernel are given in Mellin space by[1]

$$\mathcal{M}[P_{\rm NS}, x \to 0](N,a) = \frac{N}{2}\left\{1 - \sqrt{1 + \frac{8a}{N^2}\left[1 - 2\sqrt{1 - \frac{8a}{N^2}}\right]}\right\}, \qquad (10)$$

which are converted into x space using serial representations. The solution is given by

$$D_{{\rm NS},x\to 0}(z) = \sum_{k=0}^{\infty} c_k \int\limits_{m_e^2}^{s} \frac{dq^2}{q^2} a^{k+1}(q^2) \log^{2k}(z) \ . \qquad (11)$$

The coefficients c_k agree with those of Ref. [5].

3.2. The singlet case

In the singlet case the small x leading double log kernel in given by

$$\boldsymbol{P}(x,a)_{x\to 0} = \sum_{l=0}^{\infty} \boldsymbol{P}^{(l)}_{x\to 0} a^{l+1} \ln^{2l}(x) = \frac{1}{8\pi^2}\mathcal{M}^{-1}\left[\boldsymbol{F}_0(N,a)\right](x) \ , \qquad (12)$$

where the matrix $\boldsymbol{F}_0(N,a)$ is determined by

$$\boldsymbol{F}_0(N,a) \ = \ 16\pi^2 \frac{a}{N}\boldsymbol{M}_0 - \frac{8a}{N^2}\boldsymbol{F}_8(N,a)\boldsymbol{G}_0 + \frac{1}{8\pi^2}\frac{1}{N}\boldsymbol{F}_0^2(N,a) \ , \qquad (13)$$

$$\boldsymbol{F}_8(N,a) \ = \ 4\pi^2\left(1 - \sqrt{1 - \frac{8a}{N^2}}\right)\boldsymbol{M}_8 \ , \qquad (14)$$

with the matrices

$$\boldsymbol{M}_0 = \begin{pmatrix} 1 & -2 \\ 2 & 0 \end{pmatrix}, \qquad \boldsymbol{M}_8 = \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}, \qquad \boldsymbol{G}_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}. \qquad (15)$$

Solving these equations we obtain[2]

$$\boldsymbol{P}^{(0)}_{z\to 0} = \begin{pmatrix} 2 & -4 \\ 4 & 0 \end{pmatrix}, \quad \boldsymbol{P}^{(1)}_{z\to 0} = \begin{pmatrix} 2 & -4 \\ 4 & 0 \end{pmatrix}, \quad \cdots \qquad (16)$$

[1] Note that **none** of these representations exhibits poles in N but only branch cuts.

[2] Similar to a result found for QCD in [3], the off diagonal elements of these matrices are equal up to their sign.

In the singlet case, the resummed evolution equation cannot easily be solved analytically since the kernels in different orders do not commute in general and one has to take into account a larger number of terms. However, unlike the case in the fixed-order iteration, the Mellin transform of $\ln^{2k}(x)$ is suitably damped as $\mathrm{Re}(N) \to -\infty$. The singlet solution is obtained in terms of the $\boldsymbol{U}$–matrix formalism, see *e.g.* [4b]. The $\boldsymbol{U}$-matrix is obtained order by order in Mellin space as,

$$\left[\boldsymbol{U}_k(N), \frac{\boldsymbol{P}_0(N)}{\beta_0}\right] = \frac{\boldsymbol{P}_k}{\beta_0} + \sum_{i=1}^{k-1} \boldsymbol{P}_i(N)\,\boldsymbol{U}_i(N) + k\,\boldsymbol{U}_k(N)\,, \tag{17}$$

where $\boldsymbol{P}_k$ are the most singular anomalous dimension matrices $\propto \ln^{2k}(x)$. After having solved these equations the result is used to perturb around $\boldsymbol{E}_{\rm s}$

$$\boldsymbol{D}_{\rm s}(a,N) = \left(1+\sum_{k=0}^{\infty} a^k\,\boldsymbol{U}_k(N)\right)\,\boldsymbol{E}_{\rm s}(a,a_0,N)\left(1+\sum_{k=0}^{\infty} a_0^k\,\boldsymbol{U}_k(N)\right)^{-1}. \tag{18}$$

4. Summary

We calculated the universal QED corrections to hard scattering processes due to light fermions for the non-singlet and singlet channels. The leading order solution was evaluated up to $O(\alpha^5)$ in analytic form. The universal contributions $\propto (\alpha \ln^2(x))^k$ are found in terms of analytic series expansions to all orders and are combined with the foregoing corrections numerically. More details of the calculation and results are given in Ref. [6].

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